合肥学院
HEFEI UNIVERSITY

Optimization Algorithms

2. Structure

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Outline

Introduction

The Structure of Optimization

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- We will first learn about the basic ingredients that make up an optimization task.
- Then we will step-by-step work our way from stupid to good metaheuristics for solving it.

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- Instead, we will directly take the abstract concepts and look how they are implemented on one concrete problem.
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- The example we will use is just an example – the concepts can be implemented differently for almost all optimization problems.

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  4. a search space $\mathbb{X}$, i.e., a simpler data structure for internal use, which can more efficiently be processed by an optimization algorithm than $\mathbb{Y}$

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  7. a termination criterion.
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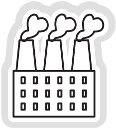
- We want to get an understanding of the structure of optimization problems from the metaheuristic perspective by looking at one concrete problem from production planning.

Example Problem: Job Shop Scheduling

Job Shop Problem



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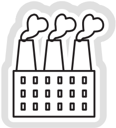

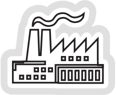

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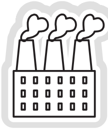

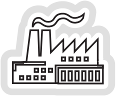

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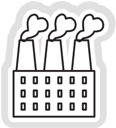

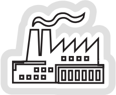


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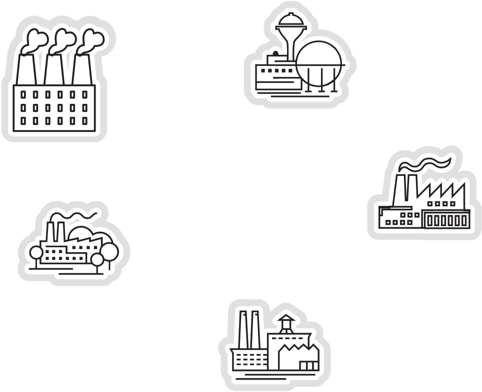

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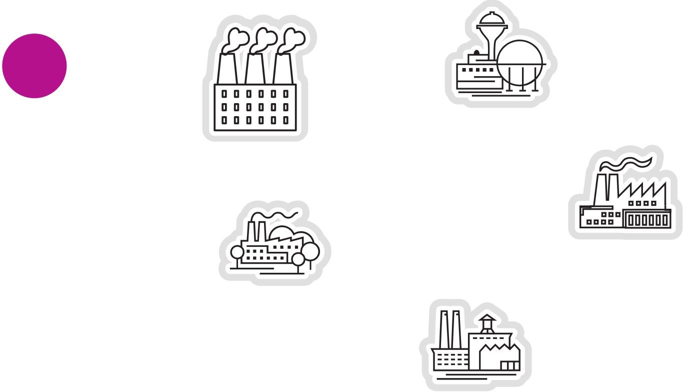

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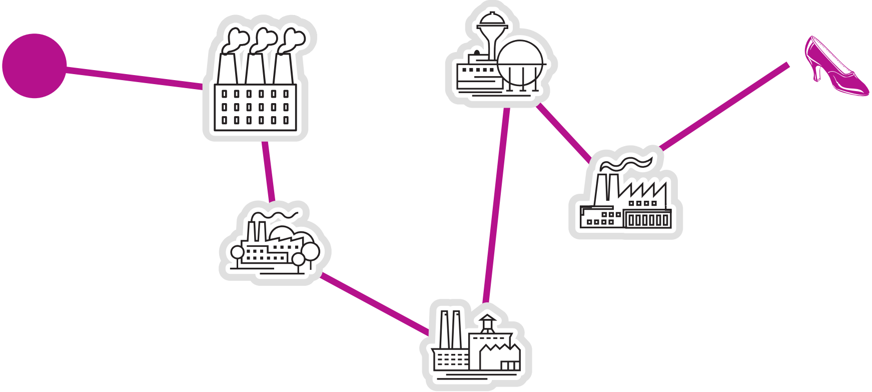

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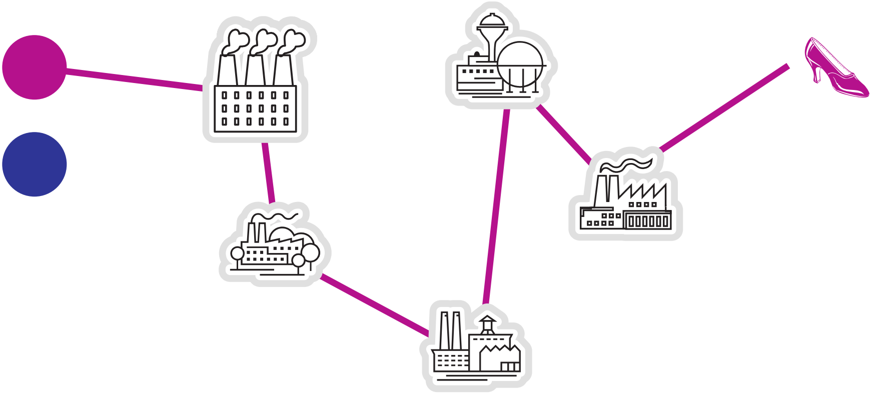

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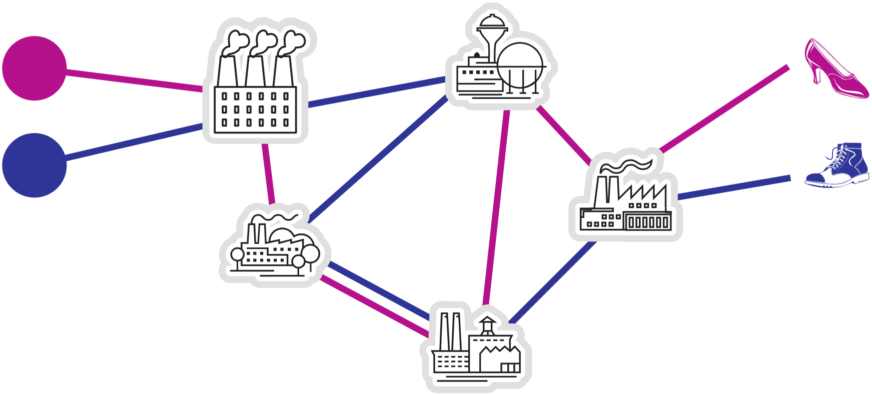

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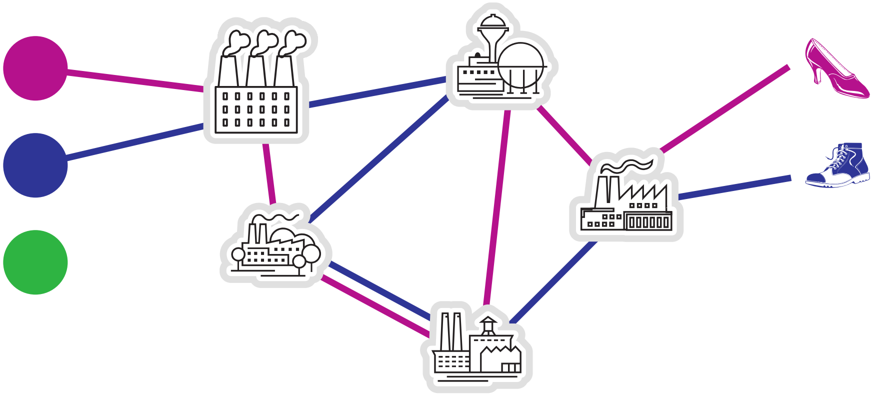

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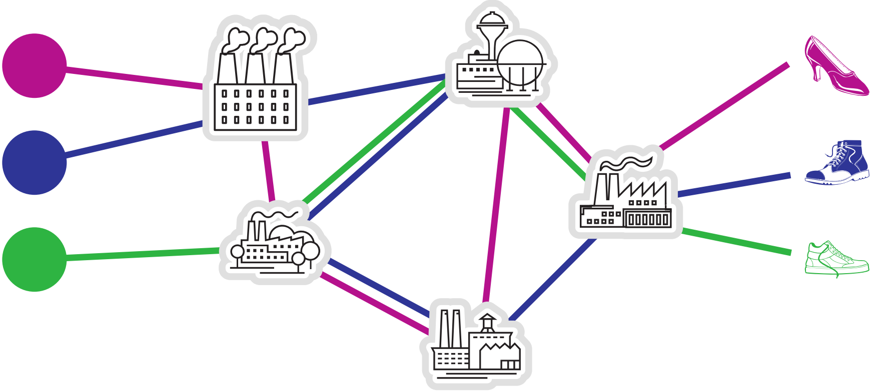

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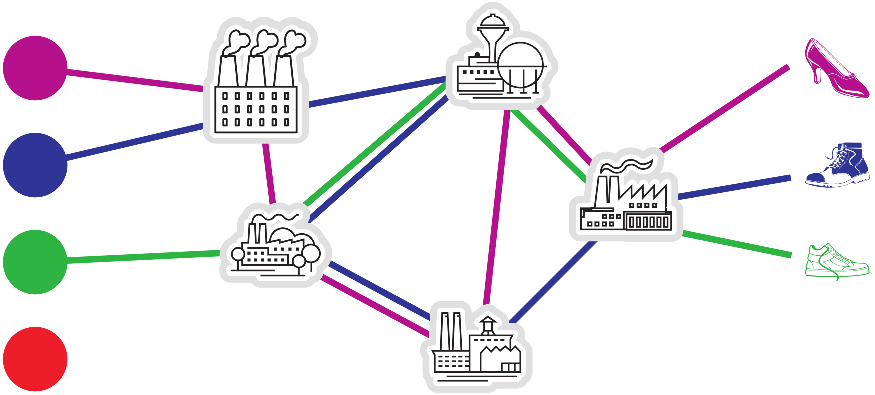

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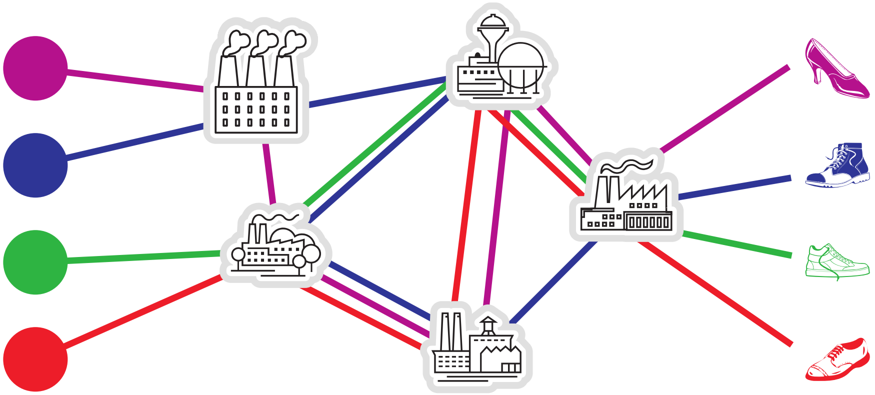

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- This problem is $\mathcal{NP}$-hard.[10 11]

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Problem Instance

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- What do such JSSP instances look like?

Demo Instance

```
++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
++++++++++++++++++++++++
```

Demo Instance

number n of jobs

```
++++++++++++++++++++++++
A simple demo
4  5
0  10   1  20   2  20   3  40   4  10
1  20   0  10   3  30   2  50   4  30
2  30   1  20   4  12   3  40   0  10
4  50   3  30   2  15   0  20   1  15
++++++++++++++++++++++++
```

Demo Instance

number m of machines

number n of jobs

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2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++
```

Demo Instance

number m of machines

number n of jobs

job 0

```
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A simple demo
4  5
0  10   1  20   2  20   3  40   4  10
1  20   0  10   3  30   2  50   4  30
2  30   1  20   4  12   3  40   0  10
4  50   3  30   2  15   0  20   1  15
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number m of machines

number n of jobs

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4  50   3  30   2  15   0  20   1  15
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```

job 1

Demo Instance

number m of machines

number n of jobs

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4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++++
```

job 2

Demo Instance

number m of machines

number n of jobs

```
+++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++++
```

job 3

Demo Instance

number m of machines

number n of jobs

job 0

```
++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
++++++++++++++++++++++++
```

Demo Instance

number m of machines

number n of jobs

```
++++++++++++++++++++++++
A simple demo
4  5
0  10   1  20   2  20   3  40   4  10
1  20   0  10   3  30   2  50   4  30
2  30   1  20   4  12   3  40   0  10
4  50   3  30   2  15   0  20   1  15
++++++++++++++++++++++++
```

job 0

Job 0 first needs to be processed by machine 0 for 10 time units

Demo Instance

number m of machines

number n of jobs

job 0

```
+++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++++
```

Job 0 first needs to be processed by machine 0 for 10 time units, it then goes to machine 1 for 20 time units

Demo Instance

number m of machines

number n of jobs

job 0

```
+++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++
```

Job 0 first needs to be processed by machine 0 for 10 time units, it then goes to machine 1 for 20 time units, it then goes to machine 2 for 20 time units

Demo Instance

number m of machines

number n of jobs

job 0

```
++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
++++++++++++++++++++++++
```

Job 0 first needs to be processed by machine 0 for 10 time units, it then goes to machine 1 for 20 time units, it then goes to machine 2 for 20 time units, it then goes to machine 3 for 40 time units

Demo Instance

number m of machines

number n of jobs

job 0

```
+++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++++
```

Job 0 first needs to be processed by machine 0 for 10 time units, it then goes to machine 1 for 20 time units, it then goes to machine 2 for 20 time units, it then goes to machine 3 for 40 time units, and finally it goes to machine 4 for 10 time units.

Demo Instance

number m of machines

number n of jobs

```
+++++++++++++++++++++++++
A simple demo
4  5
0  10   1  20   2  20   3  40   4  10
1  20   0  10   3  30   2  50   4  30
2  30   1  20   4  12   3  40   0  10
4  50   3  30   2  15   0  20   1  15
+++++++++++++++++++++++++
```

job 1

Similarly, Job 1 first needs to be processed by machine 1 for 20 time units

Demo Instance

number m of machines

number n of jobs

```
+++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++++
```

job 1

Similarly, Job 1 first needs to be processed by machine 1 for 20 time units, it then goes to machine 0 for 10 time units

Demo Instance

number m of machines

number n of jobs

```
++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
++++++++++++++++++++++++
```

job 1

Similarly, Job 1 first needs to be processed by machine 1 for 20 time units, it then goes to machine 0 for 10 time units, it then goes to machine 3 for 30 time units

Demo Instance

number m of machines

number n of jobs

job 1

```
++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
++++++++++++++++++++++++
```

Similarly, Job 1 first needs to be processed by machine 1 for 20 time units, it then goes to machine 0 for 10 time units, it then goes to machine 3 for 30 time units, it then goes to machine 2 for 50 time units

Demo Instance

number m of machines

number n of jobs

job 1

```
+++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++++
```

Similarly, Job 1 first needs to be processed by machine 1 for 20 time units, it then goes to machine 0 for 10 time units, it then goes to machine 3 for 30 time units, it then goes to machine 2 for 50 time units, and finally it goes to machine 4 for 30 time units.

Demo Instance

number m of machines

number n of jobs

job 2

```
+++++++++++++++++++++++++
A simple demo
4  5
0  10   1  20   2  20   3  40   4  10
1  20   0  10   3  30   2  50   4  30
2  30   1  20   4  12   3  40   0  10
4  50   3  30   2  15   0  20   1  15
+++++++++++++++++++++++++
```

Job 2 first needs to be processed by machine 2 for 30 time units, it then goes to machine 1 for 20 time units, it then goes to machine 4 for 12 time units, it then goes to machine 3 for 40 time units, and finally it goes to machine 0 for 10 time units.

Demo Instance

number m of machines

number n of jobs

```
+++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++++
```

job 3

And Job 3 first needs to be processed by machine 4 for 50 time units, it then goes to machine 3 for 30 time units, it then goes to machine 2 for 15 time units, it then goes to machine 0 for 20 time units, and finally it goes to machine 1 for 15 time units.

Demo Instance

number m of machines

number n of jobs

```
+++++++++++++++++++++++++
A simple demo
4 5
0 10 1 20 2 20 3 40 4 10
1 20 0 10 3 30 2 50 4 30
2 30 1 20 4 12 3 40 0 10
4 50 3 30 2 15 0 20 1 15
+++++++++++++++++++++++++
```

job 0
job 1
job 2
job 3

Each of the n jobs has m operations, each consisting of a machine index and a time requirement.

Instance `abz7`

Instance `abz7` by Adams et al.[15]

20 jobs

15 machines

```
+++++++++++++++++++++++++++++
Adams, Balas, and Zawack 15 x 20 instance (Table 1, instance 7)
20 15
  2 24  3 12  9 17  4 27  0 21  6 25  8 27  7 26  1 30  5 31 11 18 14 16 13 39 10 19 12 26
  6 30  3 15 12 20 11 19  1 24 13 15 10 28  2 36  5 26  7 15  0 11  8 23 14 20  9 26  4 28
  6 35  0 22 13 23  7 32  2 20  3 12 12 19 10 23  9 17  1 14  5 16 11 29  8 16  4 22 14 22
  9 20  6 29  1 19  7 14 12 33  4 30  0 32  5 21 11 29 10 24 14 25  2 29  3 13  8 20 13 18
 11 23 13 20  1 28  6 32  7 16  5 18  8 24  9 23  3 24 10 34  2 24  0 24 14 28 12 15  4 18
  8 24 11 19 14 21  1 33  7 34  6 35  5 40 10 36  3 23  2 26  4 15  9 28 13 38 12 13  0 25
 13 27  3 30  6 21  8 19 12 12  4 27  2 39  9 13 14 12  5 36 10 21 11 17  1 29  0 17  7 33
  5 27  4 19  6 29  9 20  3 21 10 40  8 14 14 39 13 39  2 27  1 36 12 12 11 37  7 22  0 13
 13 32 11 29  8 24  3 27  5 40  4 21  9 26  0 27 14 27  6 16  2 21 10 13  7 28 12 28  1 32
 12 35  1 11  5 39 14 18  7 23  0 34  3 24 13 11  8 30 11 31  4 15 10 15  2 28  9 26  6 33
 10 28  5 37 12 29  1 31  7 25  8 13 14 14  4 20  3 27  9 25 13 31 11 14  6 25  2 39  0 36
  0 22 11 25  5 28 13 35  4 31  8 21  9 20 14 19  2 29  7 32 10 18  1 18  3 11 12 17  6 15
 12 39  5 32  2 36  8 14  3 28 13 37  0 38  6 20  7 19 11 12 14 22  1 36  4 15  9 32 10 16
  8 28  1 29 14 40 12 23  4 34  5 33  6 27 10 17  0 20  7 28 11 21  2 21 13 20  9 33  3 27
  9 21 14 34  3 30 12 38  0 11 11 16  2 14  5 14  1 34  8 33  4 23 13 40 10 12  6 23  7 27
  9 13 14 40  7 36  4 17  0 13  5 33  8 25 13 24 10 23  3 36  2 29  1 18 11 13  6 33 12 13
  3 25  5 15  2 28 12 40  7 39  1 31  8 35  6 31 11 36  4 12 10 33 14 19  9 16 13 27  0 21
 12 22 10 14  0 12  2 20  5 12  1 18 11 17  8 39 14 31  3 31  7 32  9 20 13 29  4 13  6 26
  5 18 10 30  7 38 14 22 13 15 11 20  9 16  3 17  1 12  2 13 12 40  6 17  8 30  4 38  0 13
  9 31  8 39 12 27  1 14  5 33  3 31 11 22 13 36  0 16  7 11 14 14  4 29  6 28  2 22 10 17

+++++++++++++++++++ EOF ++++++++++++++++++++++++++++++++++++
```

Instance `la24`

Instance `la24` by Lawrence[16].

15 jobs

10 machines

```
+++++++++++++++++++++++++++++
Lawrence 15x10 instance (Table 7, instance 4)
15 10
7  8 9 75 0 72 6 74 4 30 8 43 2 38 5 98 1 26 3 19
6 19 8 73 3 43 0 23 1 85 4 39 5 13 9 26 2 67 7  9
1 50 3 93 5 80 4  7 0 55 2 61 6 57 8 72 9 42 7 46
1 68 7 43 4 99 6 60 5 68 0 91 8 11 3 96 9 11 2 72
7 84 2 34 8 40 5  7 1 70 6 74 3 12 0 43 9 69 4 30
8 60 0 49 4 59 5 72 9 63 1 69 7 99 6 45 3 27 2  9
6 71 2 91 8 65 1 90 9 98 4  8 7 50 0 75 5 37 3 17
8 62 7 90 5 98 3 31 2 91 4 38 9 72 1  9 0 72 6 49
4 35 0 39 9 74 5 25 7 47 3 52 2 63 8 21 6 35 1 80
9 58 0  5 3 50 8 52 1 88 6 20 2 68 5 24 4 53 7 57
7 99 3 91 4 33 5 19 2 18 6 38 0 24 9 35 1 49 8  9
0 68 3 60 2 77 7 10 8 60 5 15 9 72 1 18 6 90 4 18
9 79 1 60 3 56 6 91 2 40 8 86 7 72 0 80 5 89 4 51
4 10 2 92 5 23 6 46 8 40 7 72 3  6 1 23 0 95 9 34
2 24 5 29 9 49 8 55 0 47 6 77 3 77 7  8 1 28 4 48
+++++++++++++++++++++++++++++
```

Instance swv15

Instance swv15 by Storer et al.[17]

50 jobs

10 machines

```
+++++++++++++++++++++++++++++
Storer, Wu, and Vaccari hard 50x10 instance (Table 2, instance 15)
50 10
  2  93  4  40  0   1  3  77  1  77  5  16  9  74  8  11  6  51  7  92
  0  92  4  80  1  76  3  59  2  70  5  86  9  17  6  78  7  30  8  93
  1  44  2  92  3  96  4  77  0  53  9  10  7  49  5  84  8  59  6  14
  1  60  2  19  3  76  0  73  4  85  7  13  8  93  5  68  9  50  6  78
  2  20  0  24  3  41  1   2  4   4  9  44  7  79  8  81  5  16  6  39
  3  41  2  35  1  32  4  18  0  15  8  98  6  29  5  19  7  14  9  26
  1  59  0  45  4  53  3  44  2  98  5  84  6  23  7  45  8  39  9  89
  1  30  4  51  3  25  0  51  2  84  6  60  5  45  7  89  8  25  9  97
  0  47  3  18  2  40  4  62  1  58  5  36  7  93  8  77  9  90  6  15
  3  33  1  68  0  41  4  72  2  20  6  69  7  47  5  22  9  47  8  22
  2  28  1 100  4  20  0  35  3  26  5  24  9  41  6  42  7 100  8  32
  0  65  2  12  4  53  3  93  1  40  8  18  7  23  5  60  6  89  9  53
  0  58  1  60  4  97  3  31  2  50  9  85  5  64  7  38  6  85  8  35
  3  64  0  58  1  49  2  45  4   9  8  49  6  22  5  99  9  15  7   7
  0  10  4  85  3  72  2  37  1  77  5  70  7  45  9   8  6  83  8  57
  4  93  0  87  1  87  2  18  3   4  8  78  5  67  9  20  6  17  7  35
  4  72  0  56  3  57  2  15  1  45  6  41  5  40  9  85  8  32  7  81
  0  36  3  63  4  79  2  32  1   5  6  25  7  86  9  91  5  21  8  35
  2  83  4  29  0   9  1  38  3  73  7  50  9  99  5  18  8  29  6  41
  0 100  3  29  2  60  4  63  1  64  8  71  6  35  5  26  9   9  7  22
  1  81  0  60  3  62  4  48  2  68  7  28  5  69  8  92  6  79  9  10
  0  40  4  80  1  41  2  10  3  68  8  28  9  51  7  33  6  82  5  25
  4  30  2  12  0  35  3  17  1  70  9  29  7  18  8  93  6  94  5  37
  1  36  2  41  3  27  4  36  0  78  7  64  6  88  5  25  9  92  8  66
  2  65  3  27  4  74  0  32  1  40  5  88  8  73  6  92  7  83  9  42
  0  48  1  85  2  92  4  95  3  61  8  72  9  76  5  58  7  11  6  89
  3  84  2  50  0  70  4  24  1  42  9  55  5 100  6  70  7   4  8  68
  0  95  4  41  2  11  3  98  1  85  5  64  6   8  7  26  8   6  9   6
  0  84  2  49  1  17  3  69  4  55  8  75  6  45  9  38  7  59  5  28
  2  48  0  29  4   1  1  64  3  41  5  23  7  64  9  31  6  56  8  12
  2  81  4  25  3  33  0  22  1  50  5  74  9  56  8  33  7  85  6  83
  1  62  4  25  0  21  2  20  3   8  6  36  9   9  5  91  8  90  7  49
  1  43  0  16  2  91  3  96  4  24  5  11  9  91  7  41  8  35  6  66
  1  91  2  20  4  44  0  42  3  87  9  57  6  15  5  38  8  42  7  89
  0  33  3  95  4  68  2  22  1  80  7  53  8  13  9  70  5  22  6  69
  0  15  3  47  1  24  2  31  4  41  8  14  9  28  7  59  5  52  6  39
  2  95  0  42  4   5  1  57  3  67  6  30  9  21  8  70  5   9  7  20
  2  54  0  15  1  20  3  64  4  83  9  40  7   6  5  89  6  91  8  48
  0  22  4  27  1  77  3  25  2  16  8  72  9  61  6  75  7  4   5  19
  3  68  1  82  2  16  0  83  4   2  7  10  8  88  5  41  9  21  6  66
  1  64  0  76  2  85  3  71  4  97  5  97  7   8  6  40  8  70  9  35
  0  94  1  45  2  94  4  84  3  44  8  41  5  30  7  47  6  19  9  22
  2  23  1  10  0  82  3  93  4  90  8  67  7   9  9  18  5  22  6  87
  0  75  2  27  4  97  3   9  1  57  9  14  5  50  7  31  8  62  6  23
  1  42  3  41  2  35  0  75  4  18  9  65  7  38  6  38  8  51  5  56
  4  72  1  63  0  33  2  27  3  41  5  52  7  42  9  10  6  14  8  71
  2  91  1  89  0  44  4  91  3  26  6  49  5  22  8  31  9  69  7   5
  3  42  1  34  0   4  4  34  2  16  6  86  7  25  8  99  5  67  9  25
  4  34  1  93  0  26  3  81  2   9  7  96  8  79  9  68  5  76  6  10
  3  19  1  47  4  13  2  98  0  32  7  12  9  45  6  52  8  49  5  34
+++++++++++++++++++++++++++++
```

Instance `yn4`

Instance `yn4` by Yamada and Nakano[18].

```
    +++++++++++++++++++++++++++++
20 jobs Yamada and Nakano 20x20 instance (Table 4, instance 4)
    20 20 20 machines
    16 34 17 38  0 21  6 15 15 42  8 17  7 41 18 10 10 26 11 24  1 31 19 25 14 31 13 33  4 35  9 30  3 16 12 16  5 30  2 13
     5 41 11 33  6 15 16 38  0 40 14 38  3 37  1 20 13 22  4 34  7 16 17 39  9 15  2 19 10 36 12 39 18 26  8 19 15 39 19 34
    17 34  1 12 16 10  7 47 13 28 15 27  0 19  6 34 19 33 12 40  9 37 14 24  8 15 10 34  2 44  3 37 18 22 11 31  4 39  5 26
     5 48  7 46 16 47 10 45 14 15  8 25  0 34  3 24 12 35 18 15  2 48 13 19 11 10  1 48 17 16 15 28  4 18  6 17  9 44 19 41
    12 47  3 23  9 48 16 45 14 39  6 42  8 32 15 11 13 16  5 14 11 19  1 46 19 10 10 17  7 41  2 47 17 32  4 17  0 21 18 17
    18 14 16 20  1 18 12 14 13 10  6 16  5 24  4 18  0 24 11 18 15 42 19 13  3 23 14 40  9 48  8 12  2 24 10 23  7 45 17 30
     0 27 12 15  4 26 13 19 17 14  5 49  7 16 18 28 16 16  8 20  9 36  2 21 14 30  3 36  1 17 15 22  6 43 11 32 10 23 19 17
     0 32 16 15 17 12  7 46  3 37 18 43 11 40 13 43  9 48  4 36 15 24  8 25  1 33 14 32  5 26  6 37 12 24 10 24  2 15 19 22
    10 34  6 33 15 25  8 46  0 20 18 33  4 19 13 45  2 47  1 32  3 12 11 29 16 29  5 46 12 17  7 48 14 39 17 40 19 41  9 37
    13 26  3 47  5 44  6 49  1 22 17 12 10 28 19 36  9 27  4 25 14 48  7 11 16 49 12 24 11 48  2 19  0 47 18 49  8 46 15 36
    13 23 18 48 14 15  0 42  3 36  8 15  6 32 10 18  1 45 15 23 11 45  2 13 17 21 12 32  7 44  5 25 19 34 16 22  9 11  4 43
    17 37  7 49 15 45  2 28  9 15  8 35 12 29 13 44  1 26  4 25  5 30  3 39  0 15 14 28 18 23  6 42 11 33 16 45 10 10 19 20
     0 10  6 37  3 15 13 13 10 11  2 49  1 28 14 28 15 13  8 29 12 21 16 32 11 21  4 48  5 11 17 26  9 33 18 22  7 21 19 49
    18 38  0 41  4 30 13 43  6 11  2 43 14 27  3 26  9 30 15 19 16 36  1 31 17 47  5 41 10 34  8 40 12 32  7 13 11 18 19 27
     6 24  5 30  7 10 10 35  8 28 16 43 19 12  9 44 15 15  3 15  2 35 18 43  0 38  4 16  1 29 17 40 14 49 13 38 12 16 11 30
     3 48  6 35 13 43  2 37 17 18  5 27  9 27  7 41  1 22 15 28 16 18 10 37 18 48  4 10  8 14 11 18 14 43  0 48 12 12 19 49
     0 13 13 38  7 34  6 42  1 36  5 45 18 24  8 35 14 26 19 30 12 47 16 24 11 47  4 40 10 43  3 16 15 10  2 12  9 39 17 22
    16 30 13 47 19 49  8 20  4 40  3 46 17 21 14 33  6 44  7 23  9 24  0 48 10 43 15 41  2 32  5 29 11 36  1 38 12 47 18 12
    13 10  5 36 12 18 16 48  0 27 14 43 10 46  6 27  7 46 19 35 11 31  2 18  8 24  3 23 17 29 18 14  9 19  1 40 15 38  4 13
     9 45 16 44  0 43 17 31 14 35 13 17 12 42  3 14 18 37 10 39  6 48  7 38 15 26  4 49  2 28 11 35  1 42  5 24  8 44 19 38

   +++++++++++++++++++ EOF ++++++++++++++++++++++++++++++++++++
```

Problem Instance Data in Java

- How can we represent such data in Java program code?

Problem Instance Data in Java

- How can we represent such data in Java program code?

```java
package aitoa.examples.jssp;

public class JSSPInstance {

  public final int m; // number of machines

  public final int n; // number of jobs

  public final int[][] jobs; // one row per job

  /** Some stuff that is not relevant here has been omitted.
      You can find it in the full code online. */

}
```

Solution Space

Output: Candidate Solutions and Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like, i.e., the input we get.

Output: Candidate Solutions and Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like, i.e., the input we get.
- But what output should we produce?

Output: Candidate Solutions and Solution Space $\mathbb{Y}$

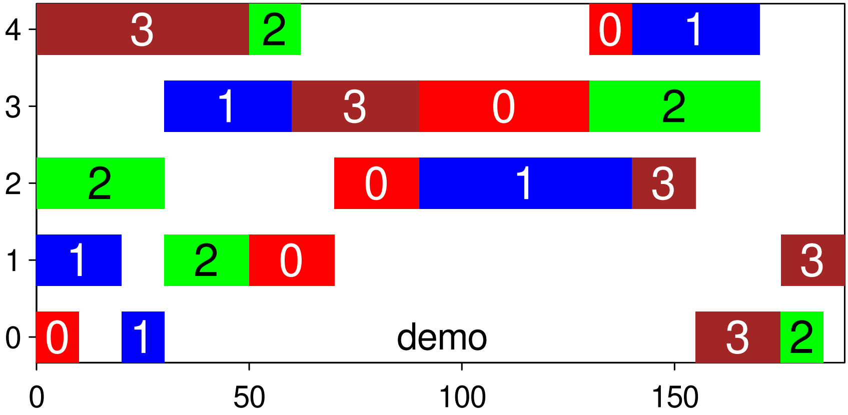
- We now know how a problem instance of the JSSP looks like, i.e., the input we get.
- But what output should we produce?
- In other words, what is a solution for an instance of the JSSP?

Output: Candidate Solutions and Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like, i.e., the input we get.
- But what output should we produce?
- In other words, what is a solution for an instance of the JSSP?
- Basically, a Gantt Chart[19 20].

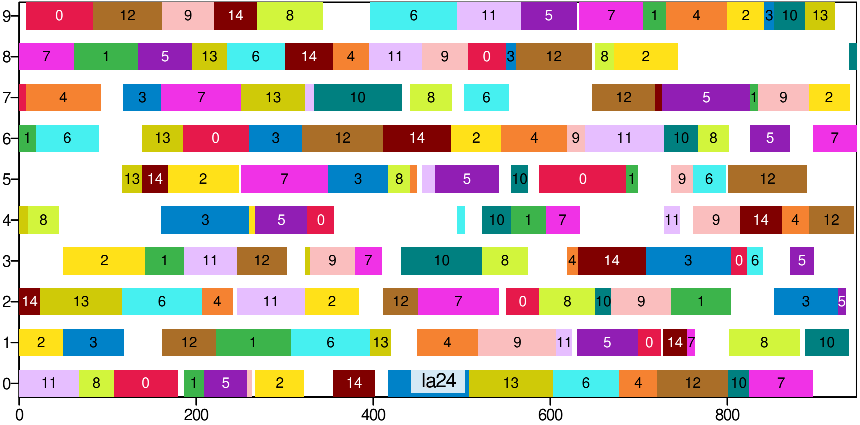
Output: Candidate Solutions and Solution Space $\mathbb{Y}$

one possible solution for the `demo` instance, illustrated as Gantt chart



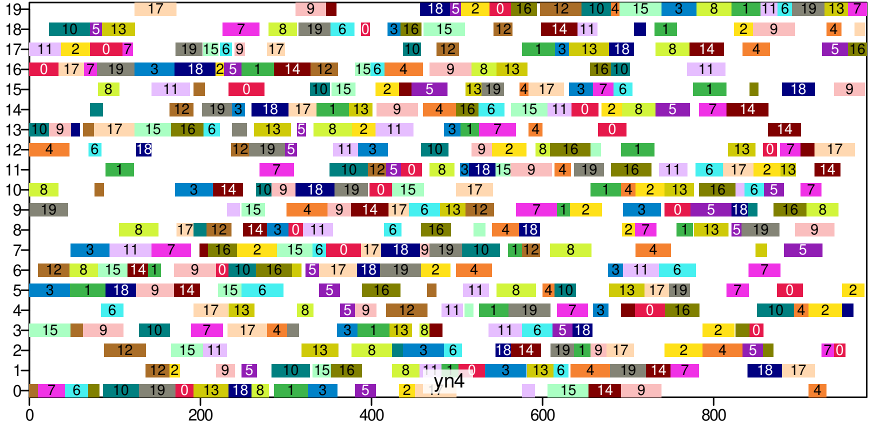
Output: Candidate Solutions and Solution Space $\mathbb{Y}$

one possible solution for the `la24` instance, illustrated as Gantt chart



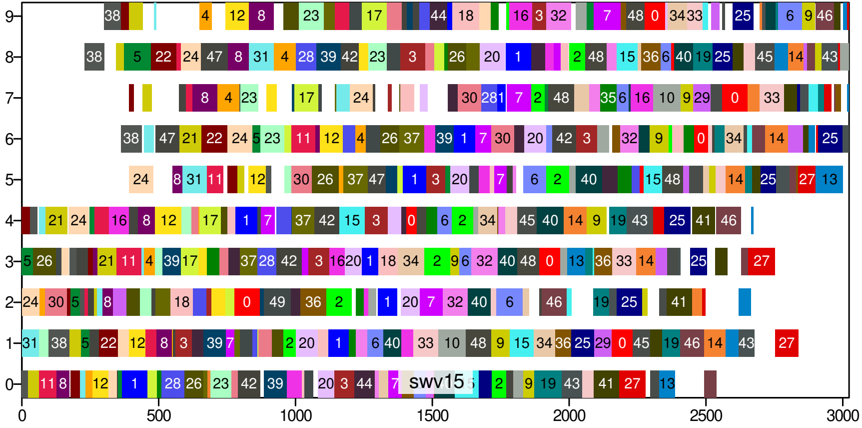
Output: Candidate Solutions and Solution Space $\mathbb{Y}$

one possible solution for the `yn4` instance, illustrated as Gantt chart



Output: Candidate Solutions and Solution Space $\mathbb{Y}$

one possible solution for the `swv15` instance, illustrated as Gantt chart



Output: Candidate Solutions and Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like, i.e., the input we get.
- But what output should we produce?
- In other words, what is a solution for an instance of the JSSP?
- Basically, a Gantt Chart[19] [20].
- A Gantt chart is a diagram which assigns each sub-job on each machine a start and end time.

Output: Candidate Solutions and Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like, i.e., the input we get.
- But what output should we produce?
- In other words, what is a solution for an instance of the JSSP?
- Basically, a Gantt Chart[19] [20].
- A Gantt chart is a diagram which assigns each sub-job on each machine a start and end time.
- The solution space $\mathbb{Y}$ is the set of all possible feasible solutions for one JSSP instance.

Output: Candidate Solutions and Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like, i.e., the input we get.
- But what output should we produce?
- In other words, what is a solution for an instance of the JSSP?
- Basically, a Gantt Chart[19] [20].
- A Gantt chart is a diagram which assigns each sub-job on each machine a start and end time.
- The solution space $\mathbb{Y}$ is the set of all possible feasible solutions for one JSSP instance.
- One possible solution is called candidate solution and it can be illustrated as Gantt chart.

As Java Class

- We now need to represent this information as a Java class.

As Java Class

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```java
package aitoa.examples.jssp;

public class JSSPCandidateSolution {

  public int[][] schedule; // one row per machine

  /** Some stuff that is not relevant here has been omitted.
      You can find it in the full code online. */
}
```

As Java Class

- We now need to represent this information as a Java class.
- Each of the m `int[]` lists in `schedule` holds n operations for each machine as three values jobID, start time, end time, i.e., has length $3n$.

```
package aitoa.examples.jssp;

public class JSSPCandidateSolution {

  public int[][] schedule; // one row per machine

  /** Some stuff that is not relevant here has been omitted.
      You can find it in the full code online. */
}
```

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new int[][] {
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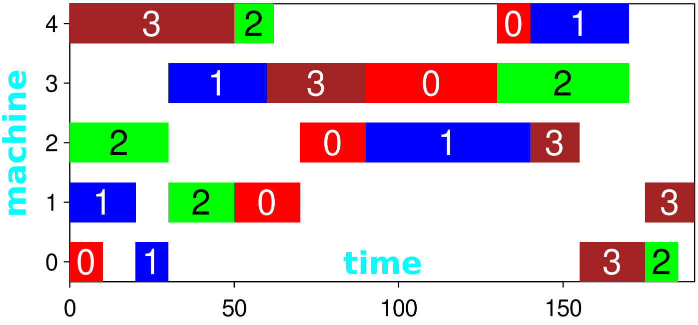
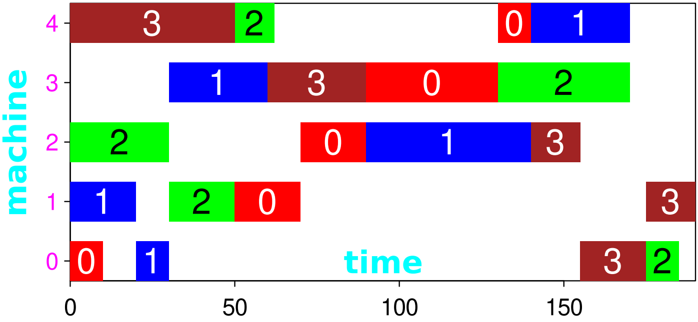
  {2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
  {1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
  {3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
}
```



As Java Class

```
   new int[][] {
M0 {0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 {1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 {2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 {1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 {3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
   }
```



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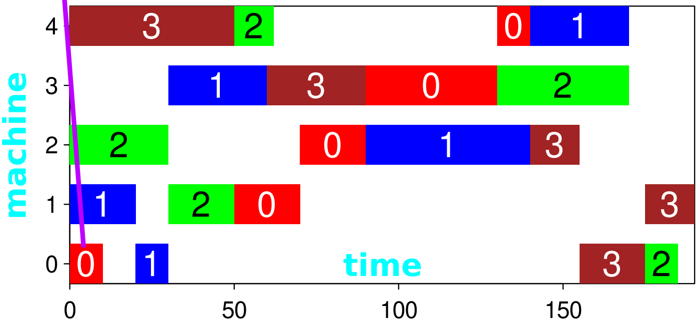

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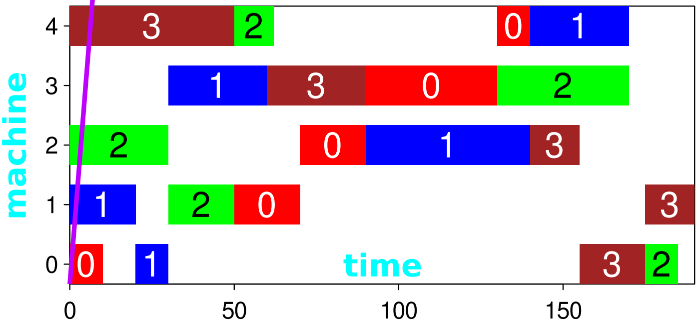

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  {1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
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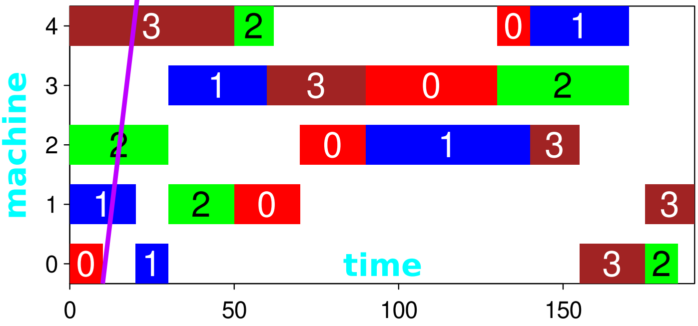

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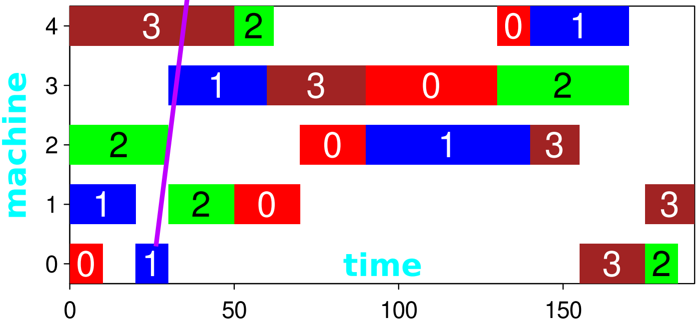

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  {2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
  {1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
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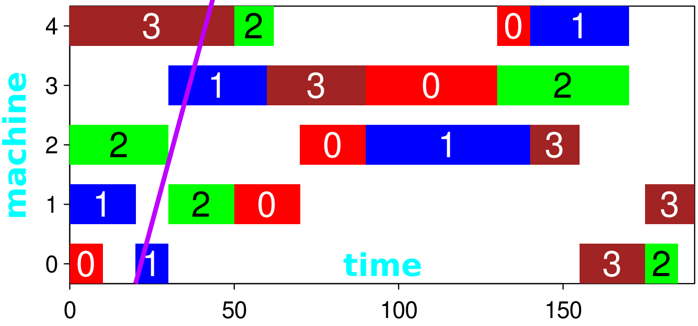

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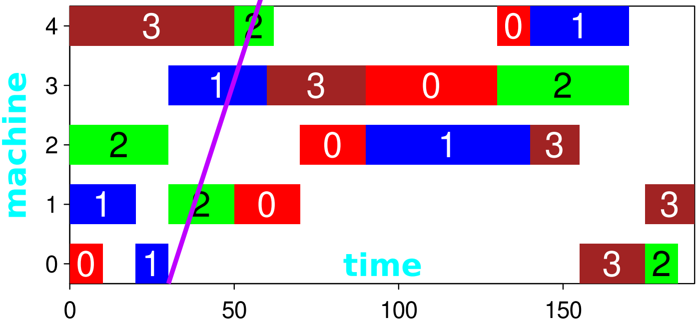

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  {1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
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}
```

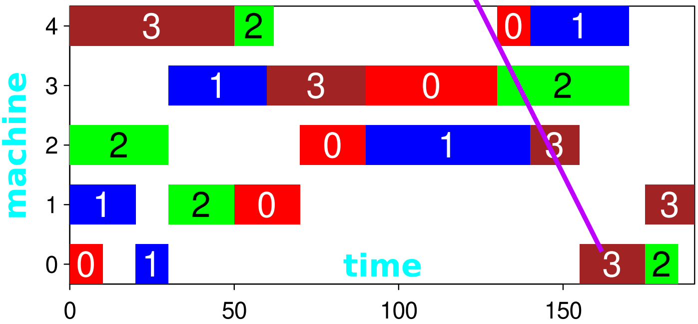

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new int[][] {
  {0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
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  {2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
  {1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
  {3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
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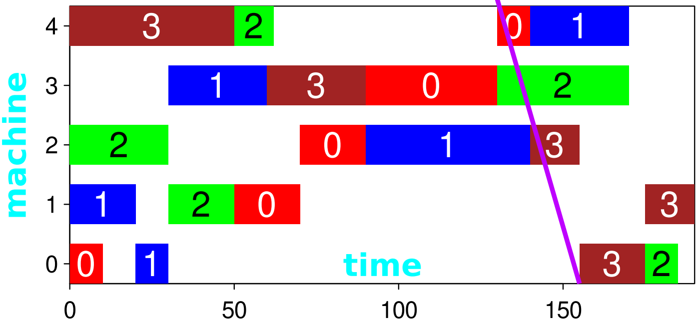

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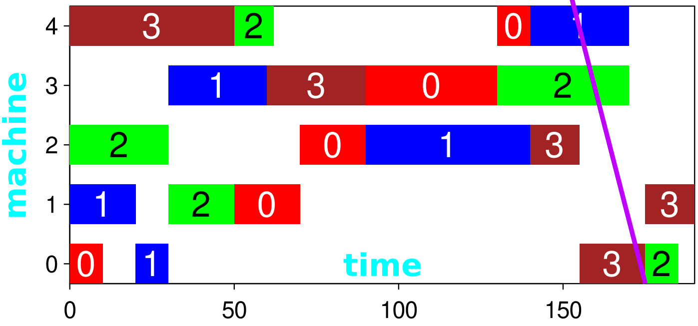

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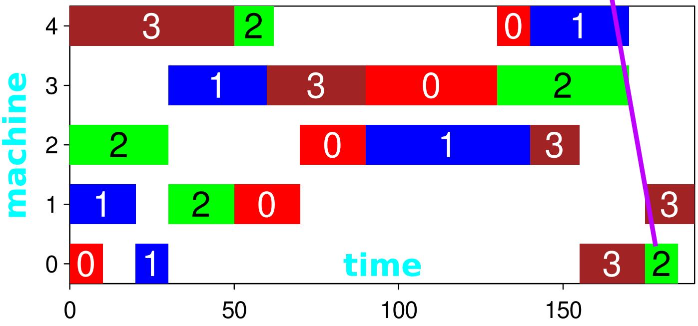

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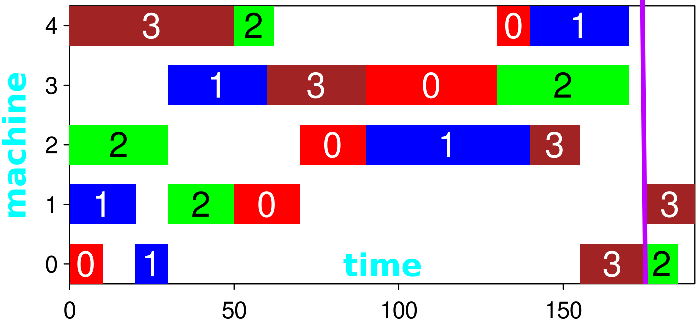

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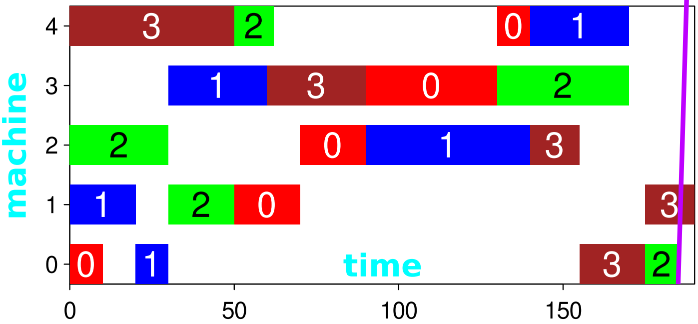

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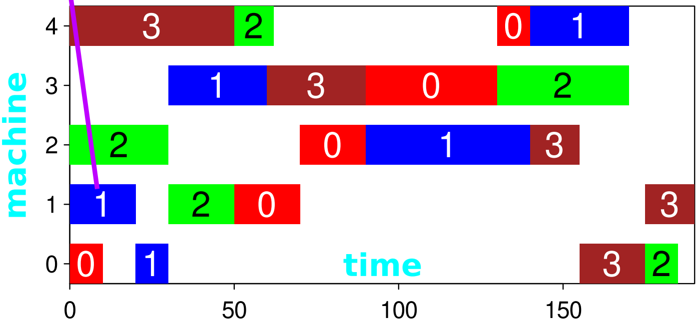

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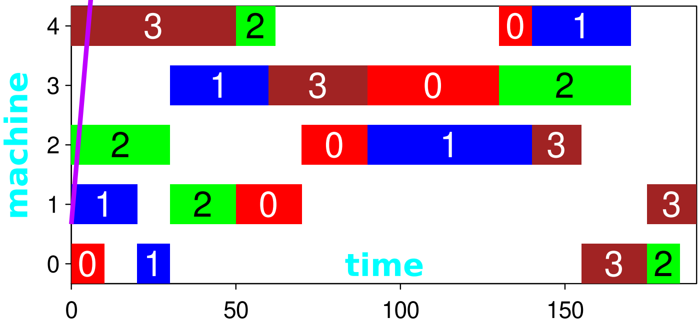

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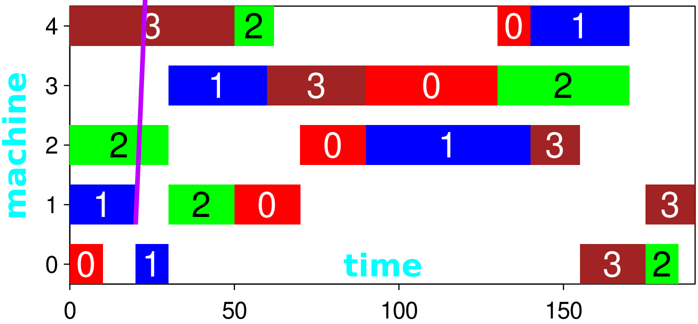

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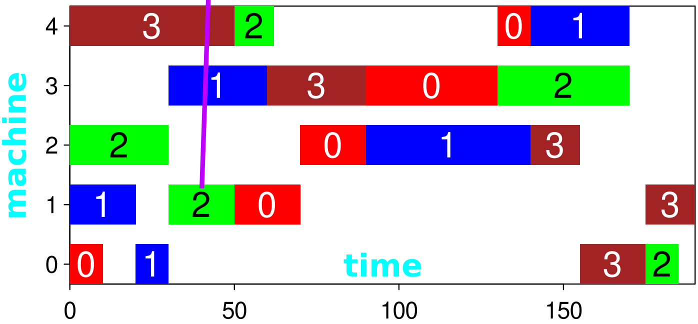

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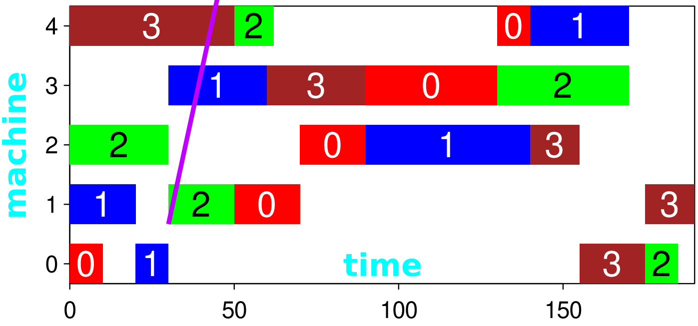

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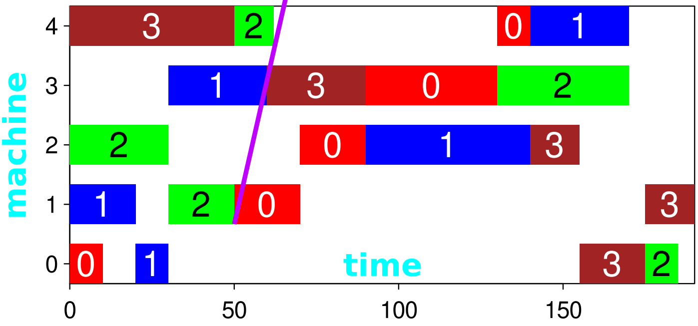

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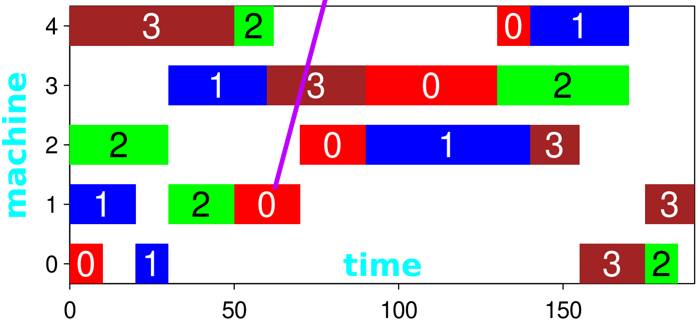

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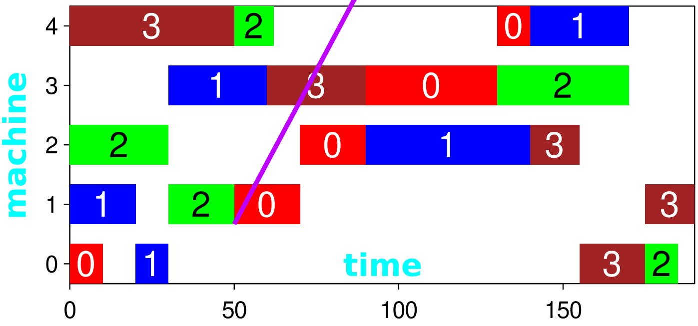

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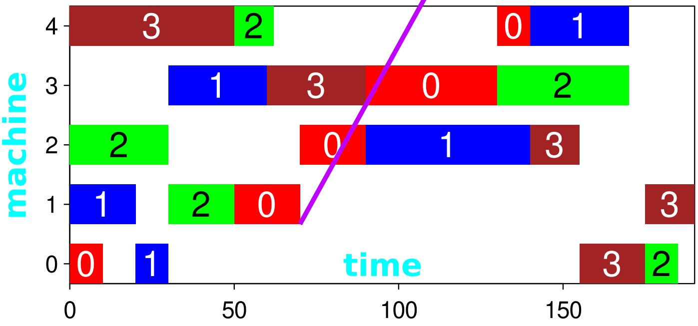

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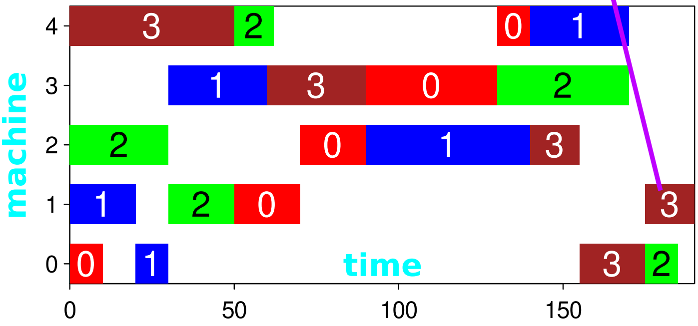

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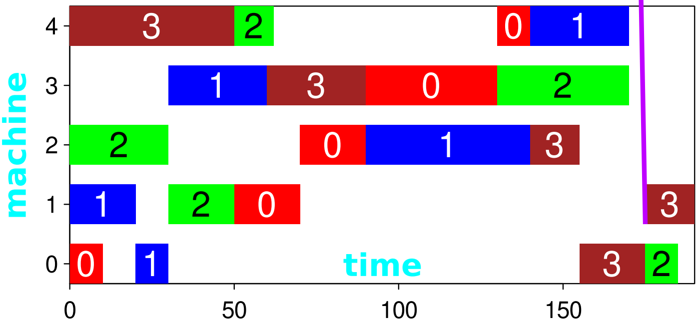

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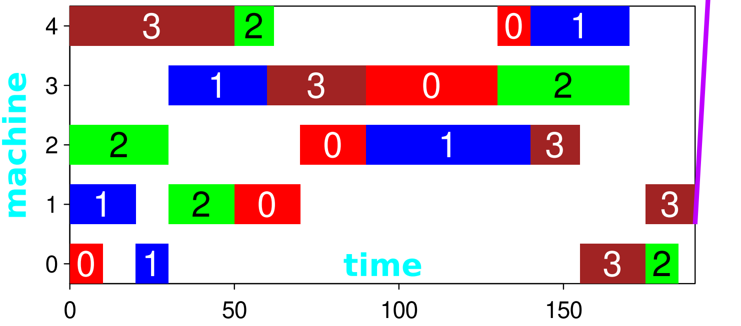

Objective Function

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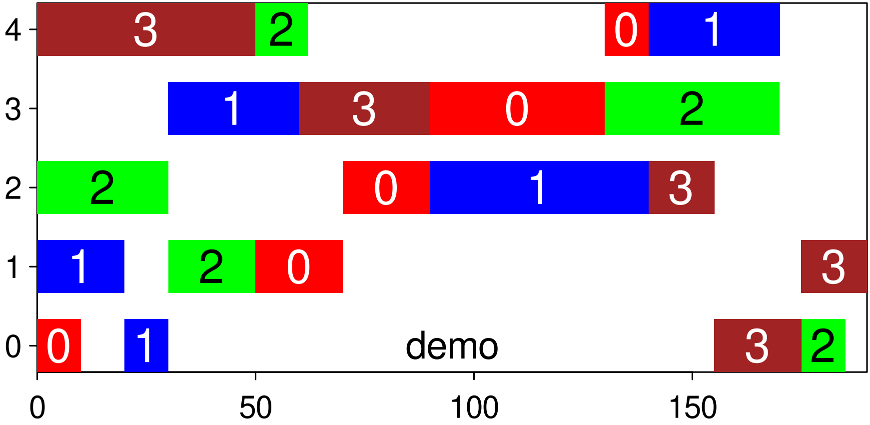

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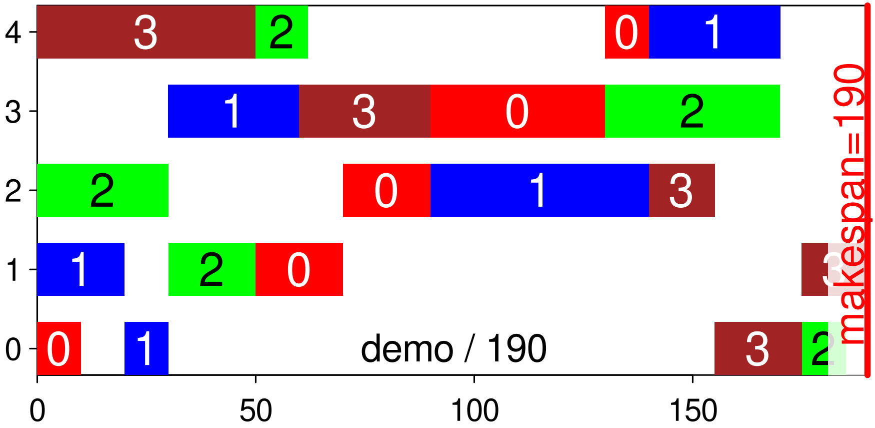

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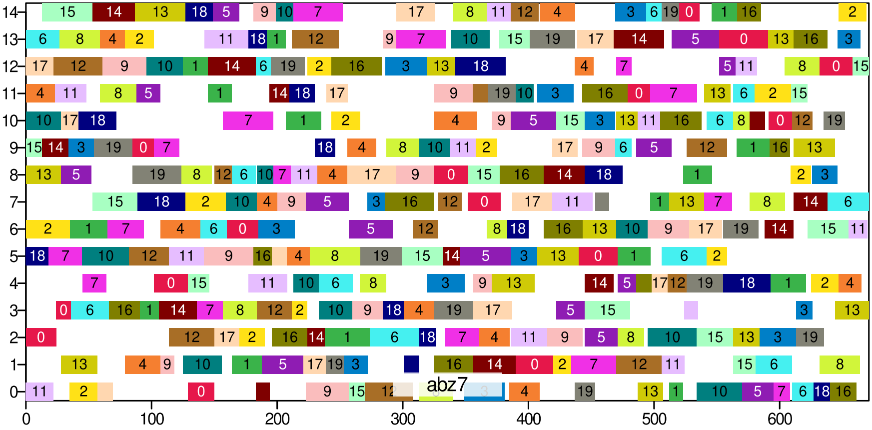

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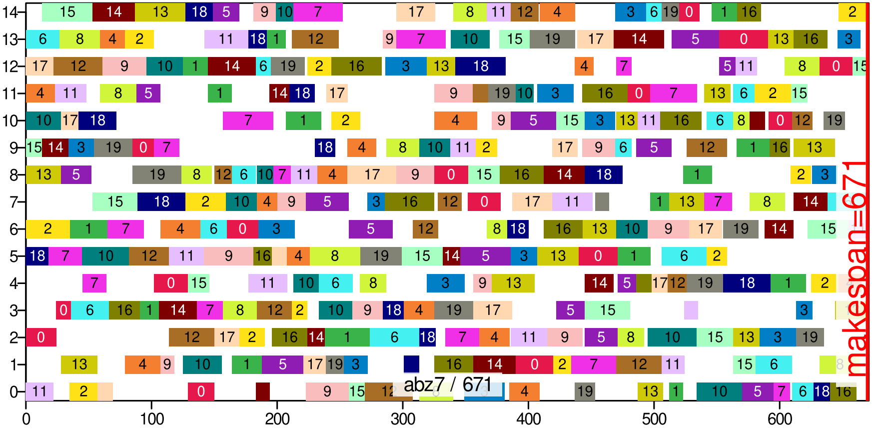

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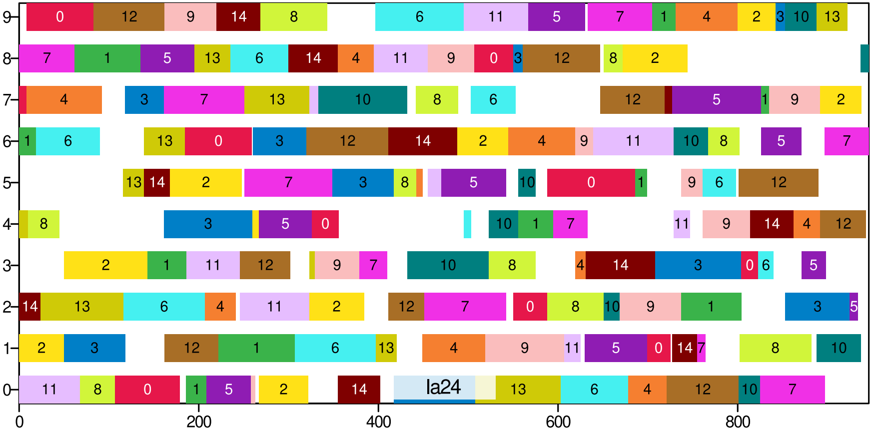

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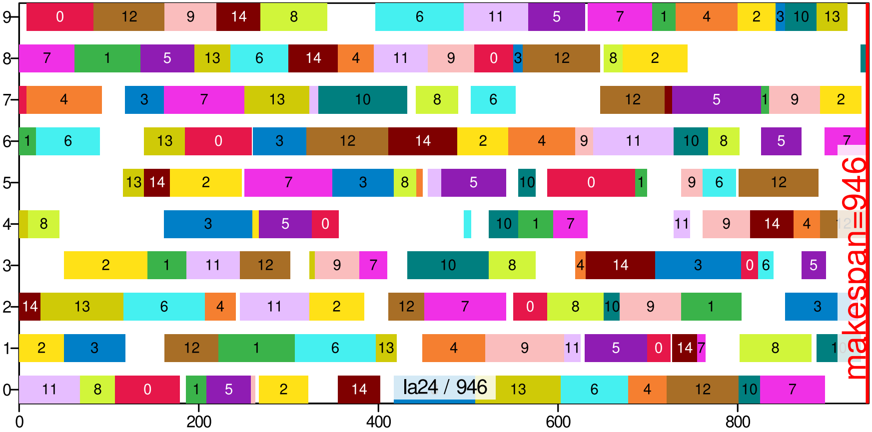

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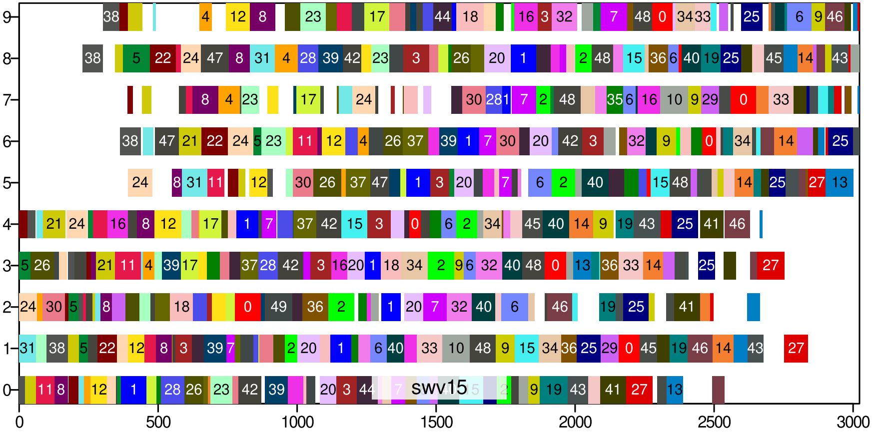

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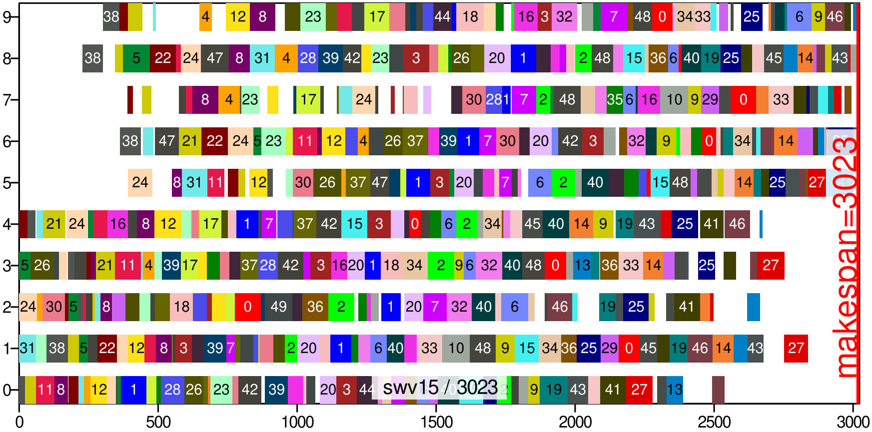

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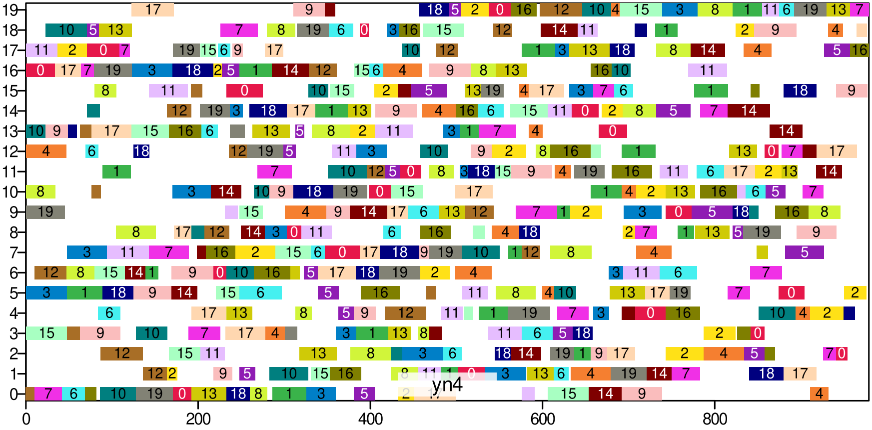

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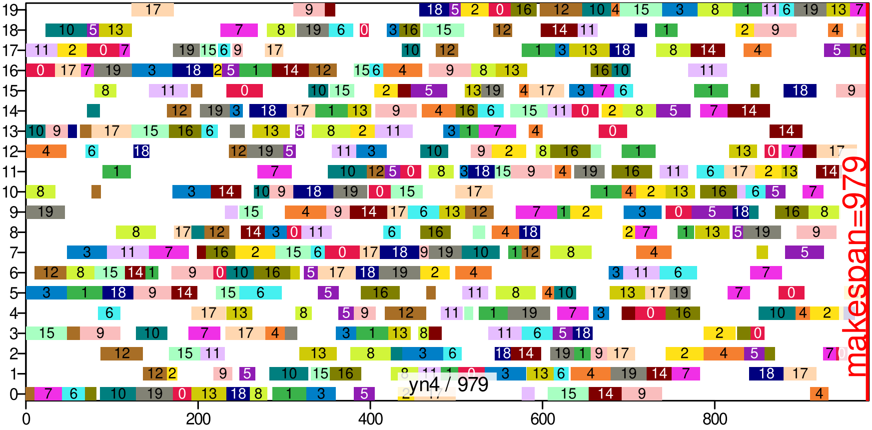

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- How do we rate the quality of a solution?
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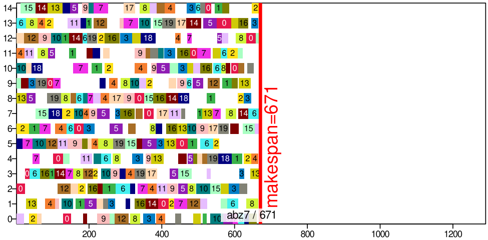

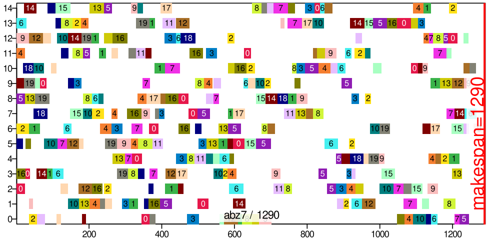

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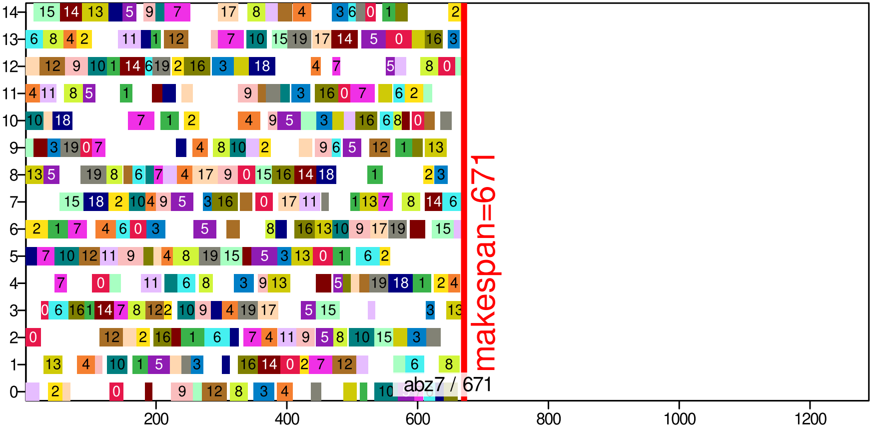

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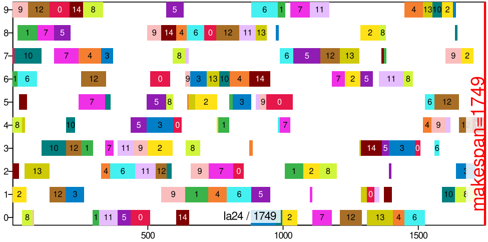

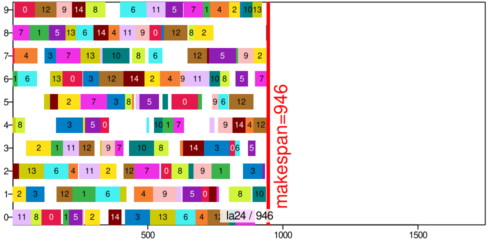

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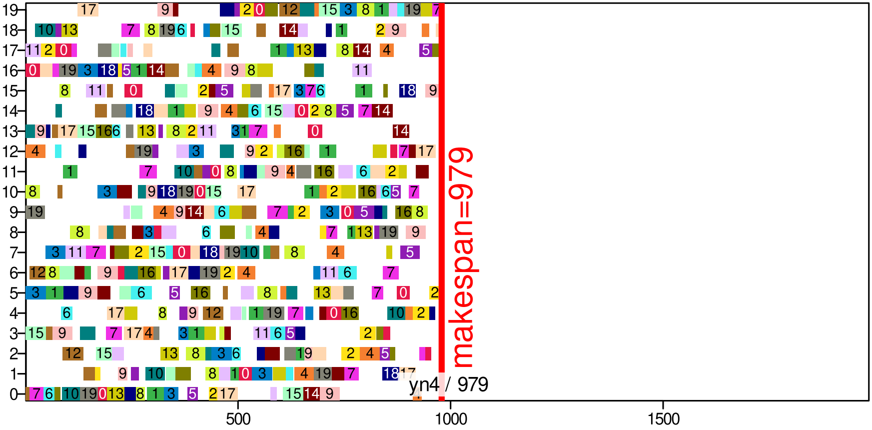

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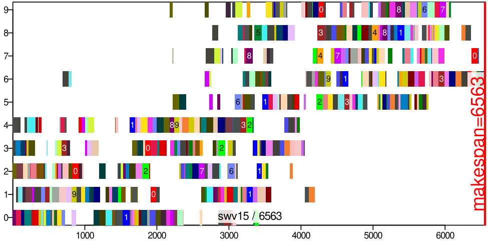

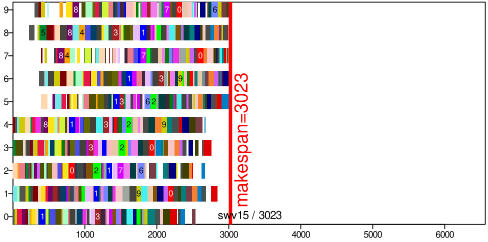

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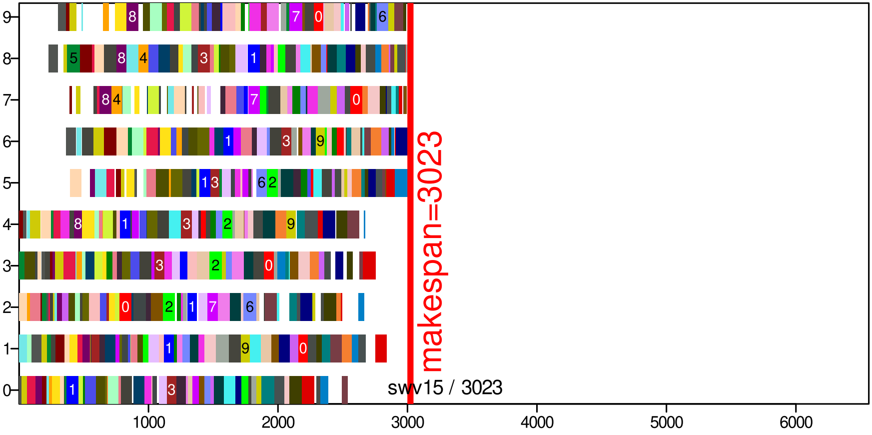

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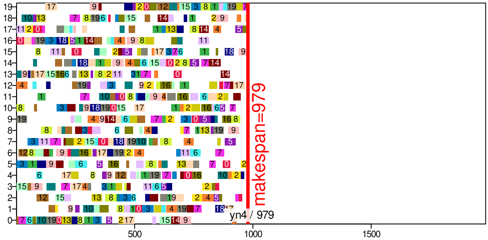

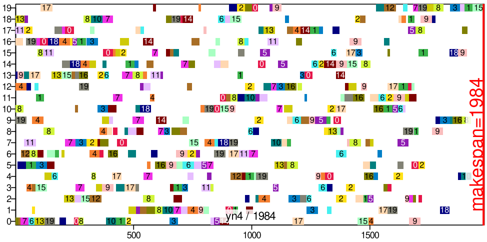

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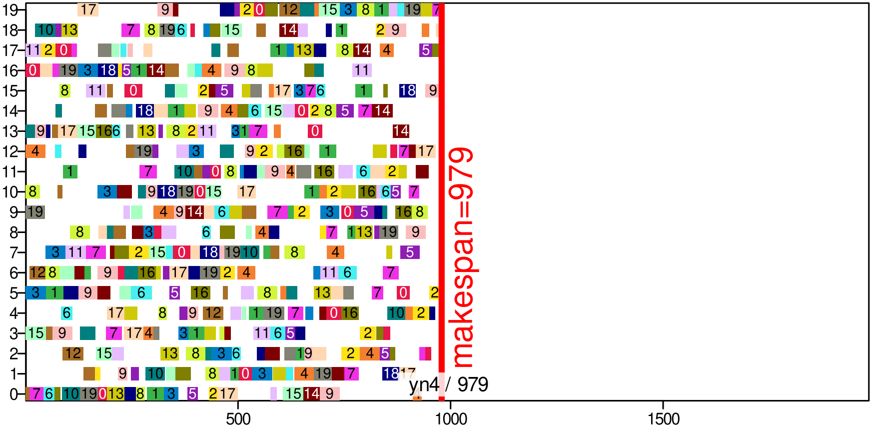

An Interface for Objective Functions in Java

```java
package aitoa.structure;

public interface IObjectiveFunction<Y> {

  double evaluate(Y y);

}
```

The JSSP Objective Function in Java

```java
package aitoa.examples.jssp;

public class JSSPMakespanObjectiveFunction {
//

/** Some stuff that is not relevant here has been omitted.
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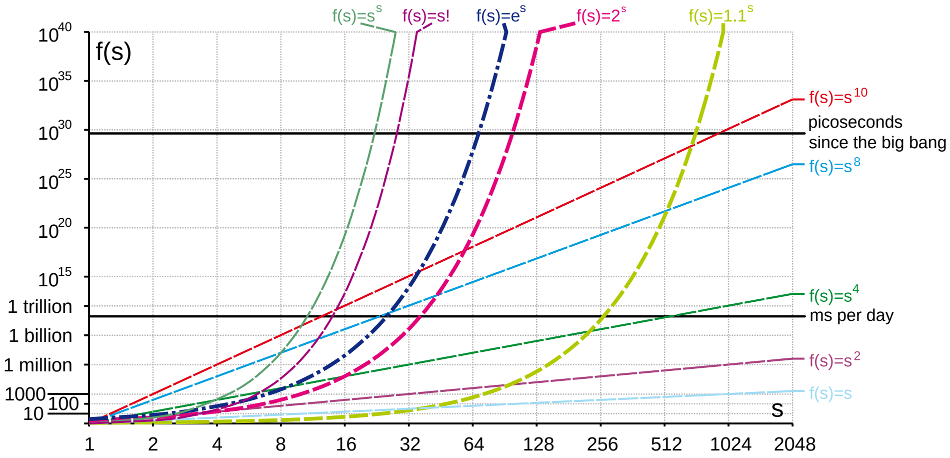

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- If we can find a solution with a slightly larger makespan than the best possible solution, but we can get it within a few minutes, that would also be nice…

From Solution Space to Search Space

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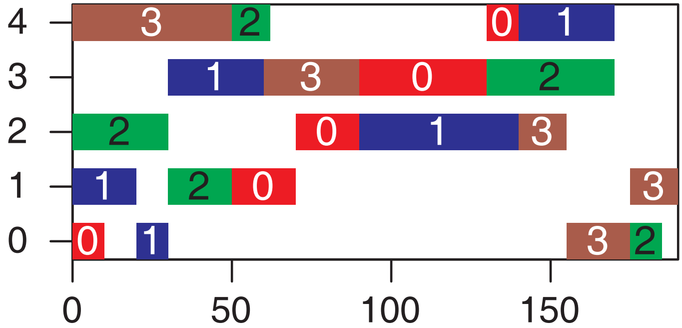

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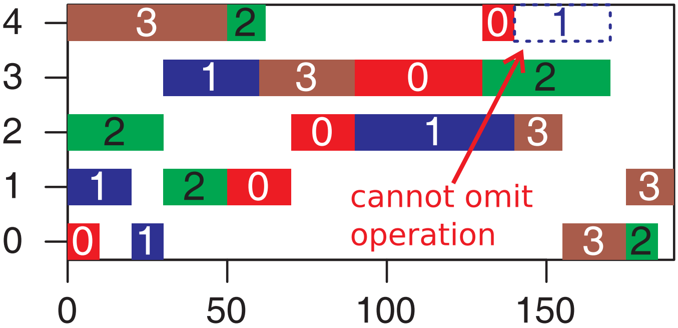

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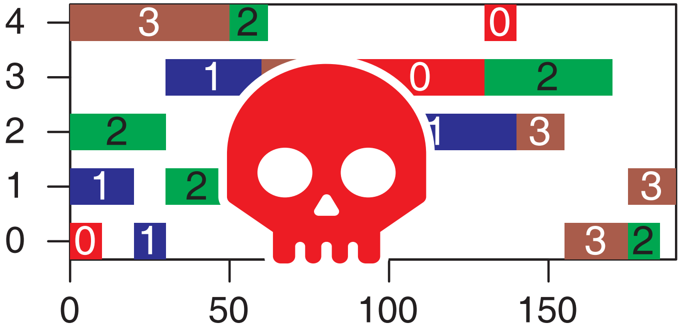

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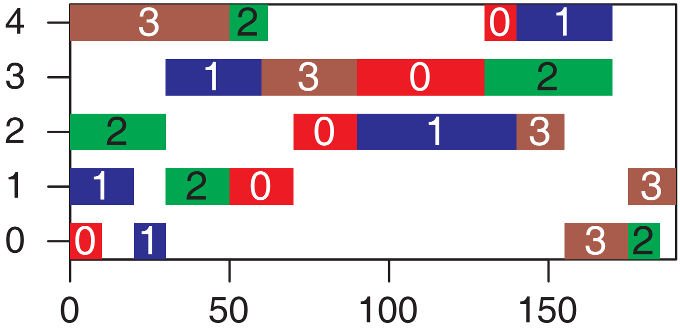

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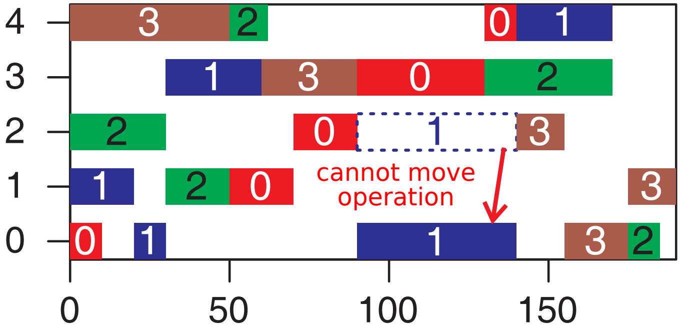

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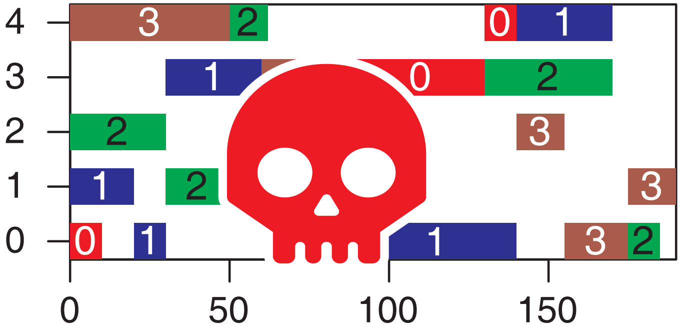

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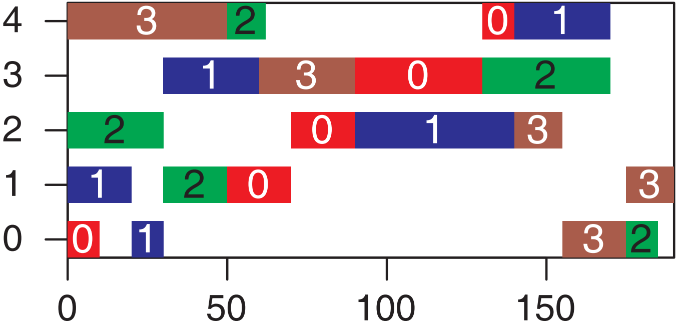

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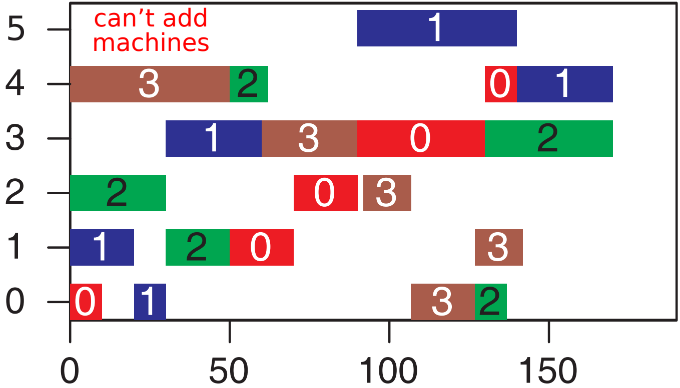

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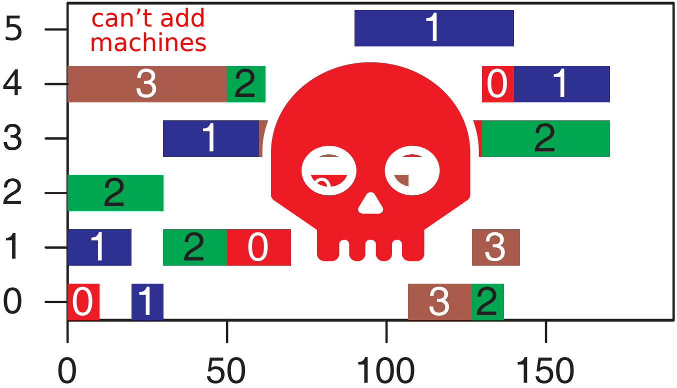

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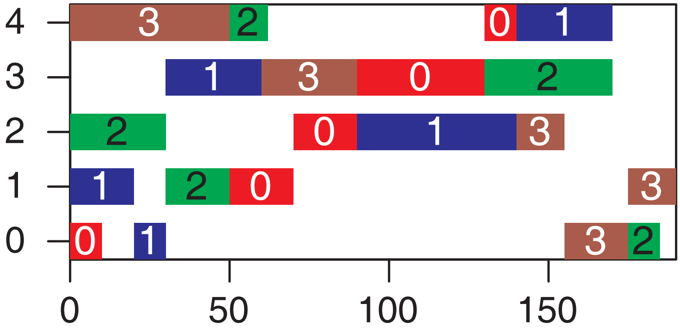

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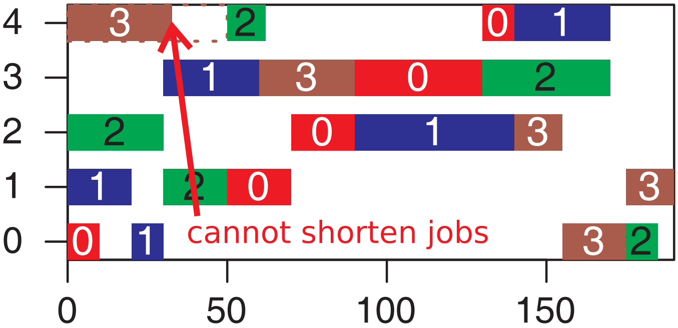

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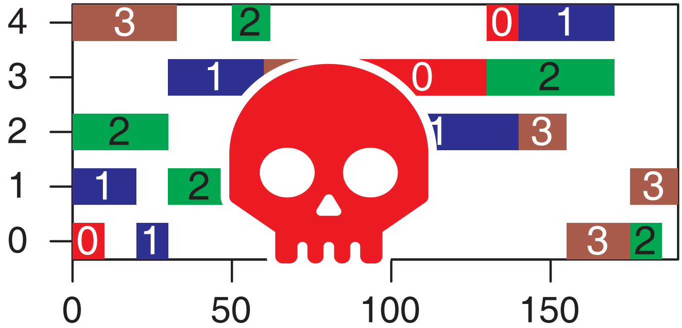

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  4. the operations cannot intersect or overlap, each machine can only carry out one job at a time

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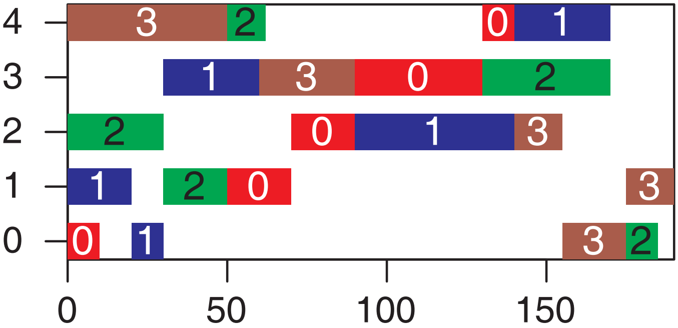

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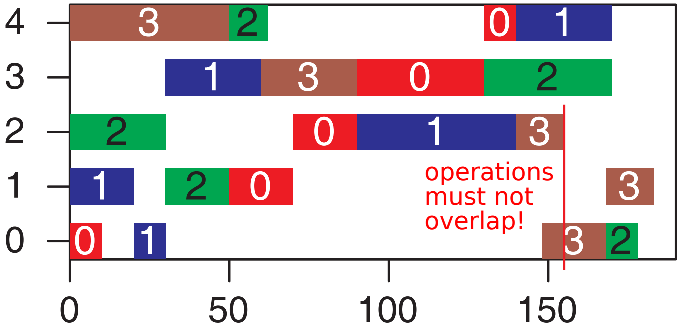

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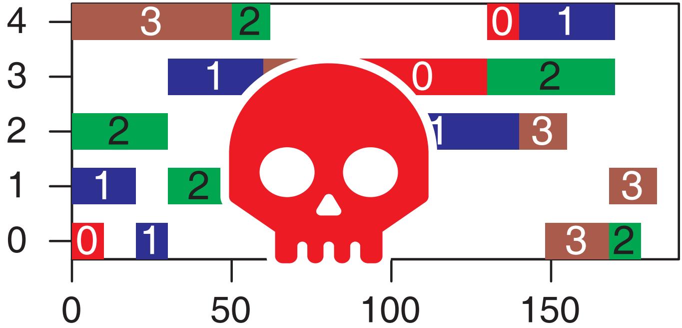
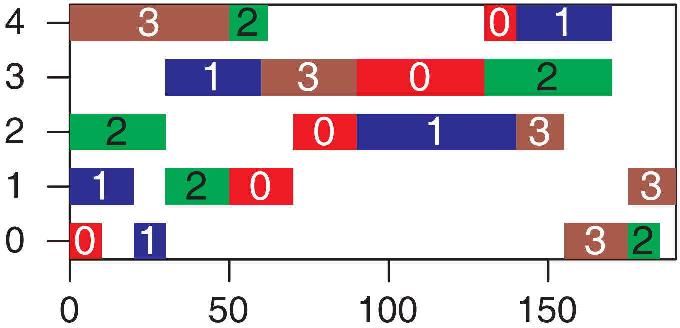


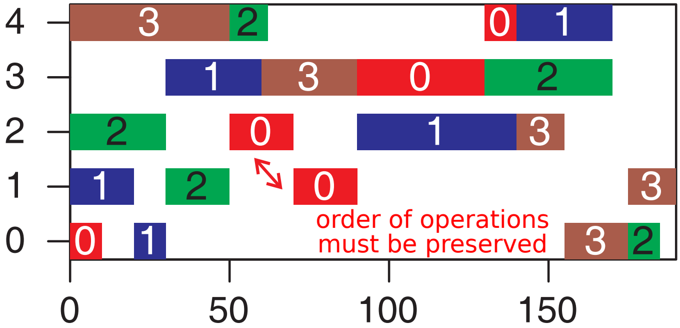
Feasibility of Solutions

- So what do we need to consider when searching in $\mathbb{Y}$?
- A candidate solution $y \in \mathbb{Y}$ is feasible, i.e., can actually be "used," if and only if it fulfills all *constraints*.
- Indeed, there are several constraints we need to impose on our Gantt charts:
  1. all operations of all jobs must be assigned to their respective machines and properly be completed,
  2. only the jobs and machines specified by the problem instance must occur in the chart,
  3. an operations must be assigned a time window on its corresponding machine which is exactly as long as the operation needs on that machine,
  4. the operations cannot intersect or overlap, each machine can only carry out one job at a time, and
  5. the precedence constraints of the operations must be honored.

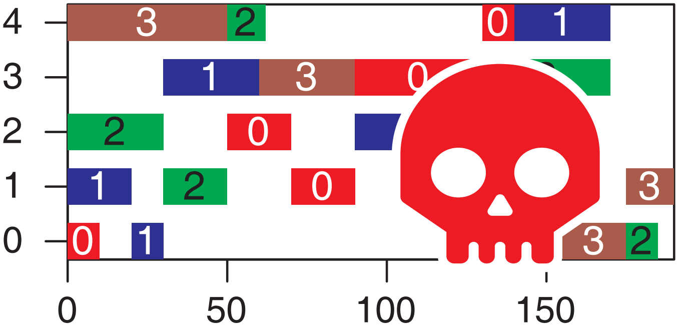
Feasibility of Solutions



Feasibility of Solutions



Feasibility of Solutions



Feasibility of Solutions

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  4. the operations cannot intersect or overlap, each machine can only carry out one job at a time, and
  5. the precedence constraints of the operations must be honored.
- Only a Gantt chart obeying all of these constraints is feasible, i.e., can be implemented in practice.

Hardships when Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?

Hardships when Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?
- We need to create Gantt charts that fulfill all the constraints.

Hardships when Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?
- We need to create Gantt charts that fulfill all the constraints.
- For different instances, different solutions are feasible!

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
++++++++++++++++++++++++
```

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10    1  20
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Hardships when Searching in $\mathbb{Y}$

```
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Hardships when Searching in $\mathbb{Y}$

```
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```

Hardships when Searching in $\mathbb{Y}$

```
job 0
+++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
+++++++++++++++++++++++++
```

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
++++++++++++++++++++++++
```

job 0

job 1

Hardships when Searching in $\mathbb{Y}$

```
job 0
job 1
+++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
+++++++++++++++++++++++
```

M0: Job 0, Job 1

Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
+++++++++++++++++++++++++++
```

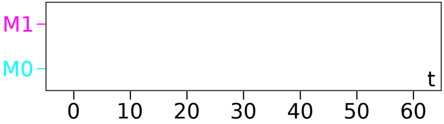
job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1

Hardships when Searching in $\mathbb{Y}$

++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2 2
job 0 | 0 10 1 20
job 1 | 0 10 1 20
++++++++++++++++++++++++

M0: Job 0, Job 1; M1: Job 0, Job 1



Hardships when Searching in $\mathbb{Y}$

job 0
job 1

```
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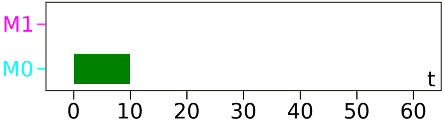

Hardships when Searching in $\mathbb{Y}$

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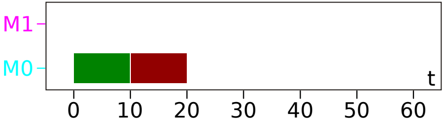

Hardships when Searching in $\mathbb{Y}$

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       +++++++++++++++++++++++
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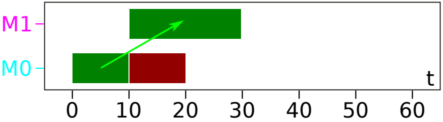
job 1  0  10   1  20
       +++++++++++++++++++++++
```

M0: Job 0, Job 1; M1: Job 0, Job 1



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job 0
job 1

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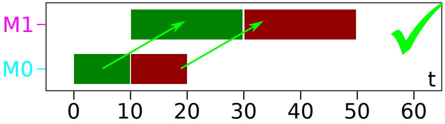

Hardships when Searching in $\mathbb{Y}$

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job 0
job 1

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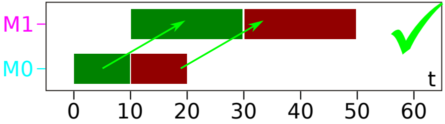

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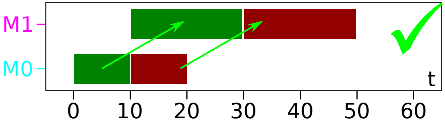

M0: Job 0, Job 1; M1: Job 1, Job 0

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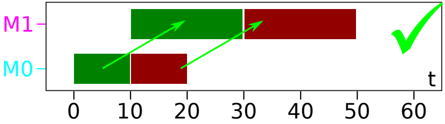

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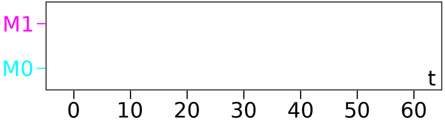

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2  2
0  10    1  20
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```

job 0
job 1

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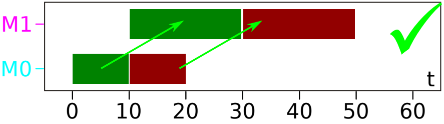
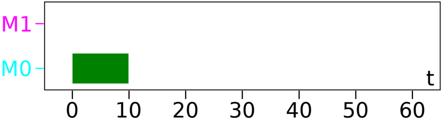


M0: Job 0, Job 1; M1: Job 1, Job 0



Hardships when Searching in $\mathbb{Y}$

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job 1

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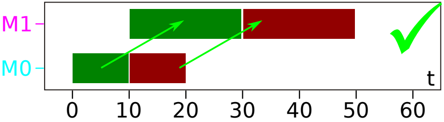
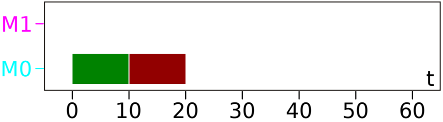


M0: Job 0, Job 1; M1: Job 1, Job 0

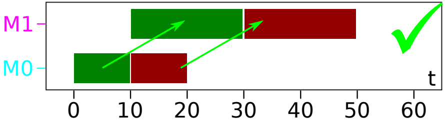


Hardships when Searching in $\mathbb{Y}$

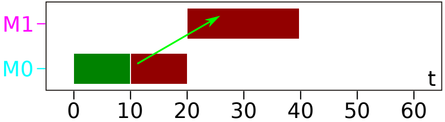
```
+++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10    1  20
0  10    1  20
+++++++++++++++++++++++++
```

job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



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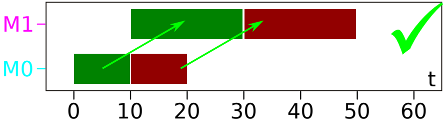

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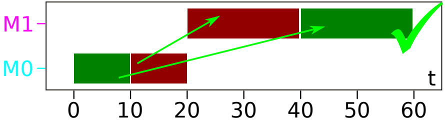

Hardships when Searching in $\mathbb{Y}$

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        ++++++++++++++++++++++++
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        ++++++++++++++++++++++++
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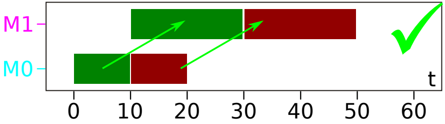

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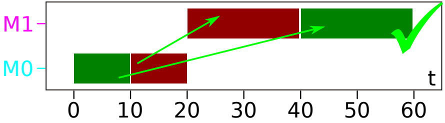

M0: Job 1, Job 0

Hardships when Searching in $\mathbb{Y}$

```
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0  10    1  20
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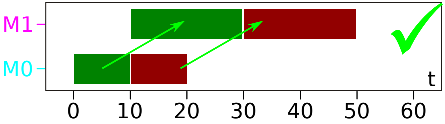
+++++++++++++++++++++++++
```

job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1



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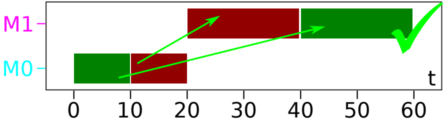

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Hardships when Searching in $\mathbb{Y}$

```
job 0
job 1
```

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2 2
job 0 — 0 10 1 20
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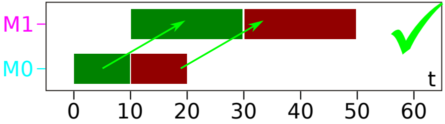

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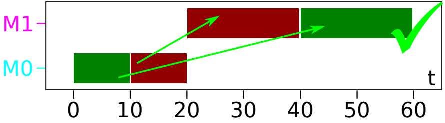

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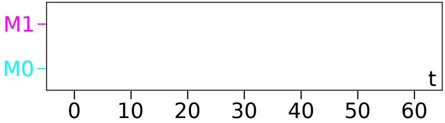

Hardships when Searching in $\mathbb{Y}$

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job 0
job 1

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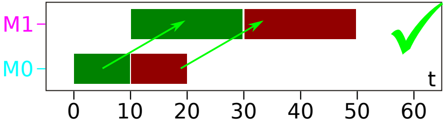

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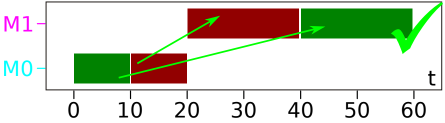

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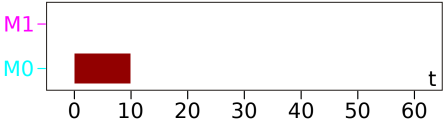

Hardships when Searching in $\mathbb{Y}$

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```

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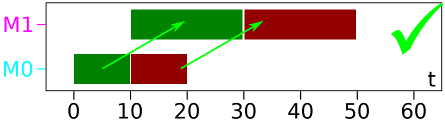

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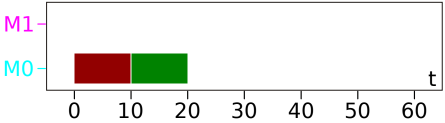

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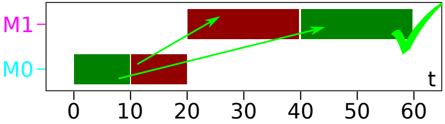

Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
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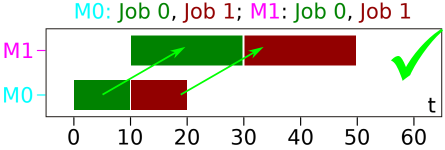
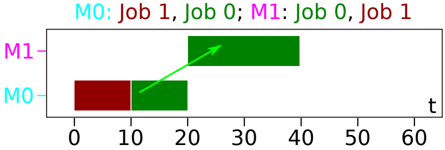
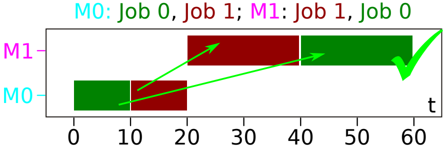
0  10   1  20
+++++++++++++++++++++++
```

job 0
job 1







Hardships when Searching in $\mathbb{Y}$

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```

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job 1 — 0 10 1 20

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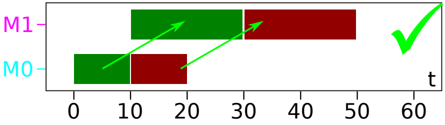

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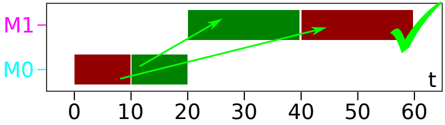

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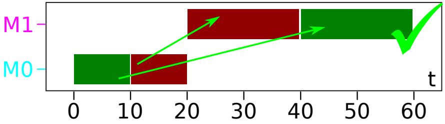

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++++
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0  10   1  20
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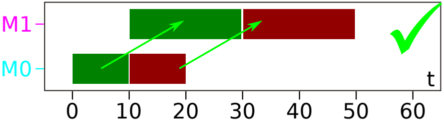
++++++++++++++++++++++++++
```

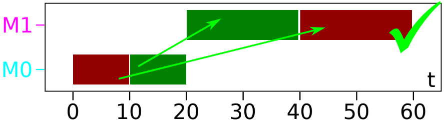
job 0

job 1

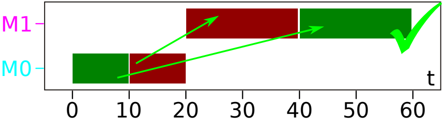
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



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Hardships when Searching in $\mathbb{Y}$

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          ++++++++++++++++++++++++
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          ++++++++++++++++++++++++
```

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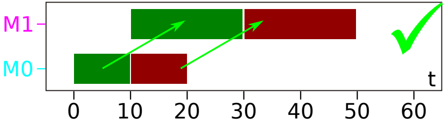
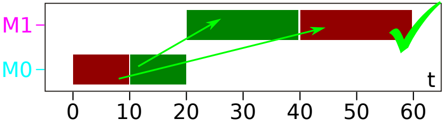
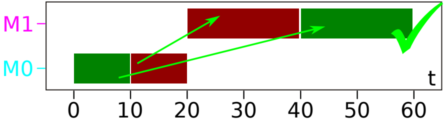


M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0

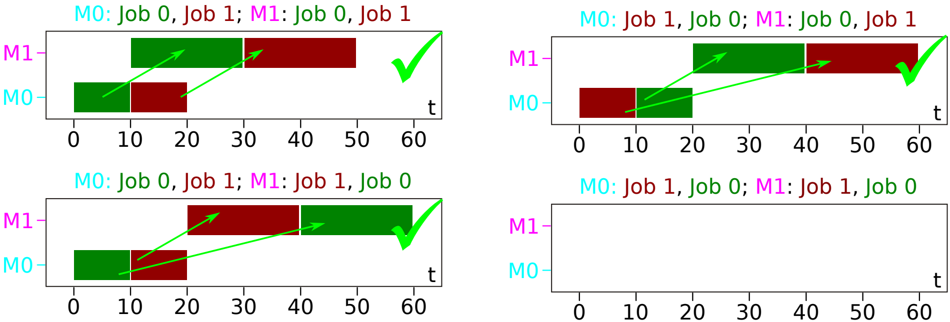


M0: Job 1, Job 0; M1: Job 1, Job 0

Hardships when Searching in $\mathbb{Y}$

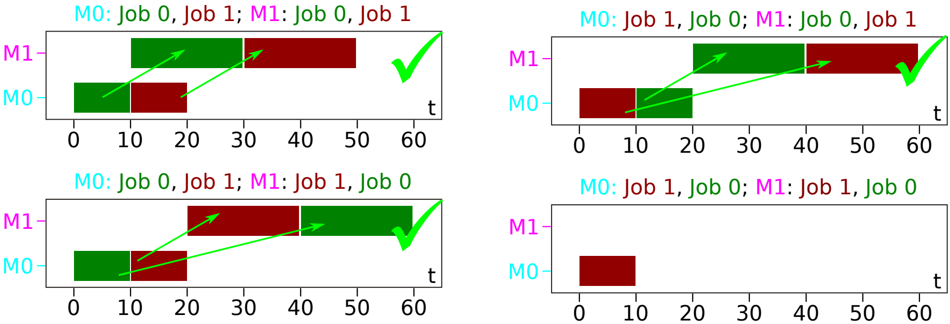
```
++++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
++++++++++++++++++++++++++
```

job 0
job 1



Hardships when Searching in $\mathbb{Y}$

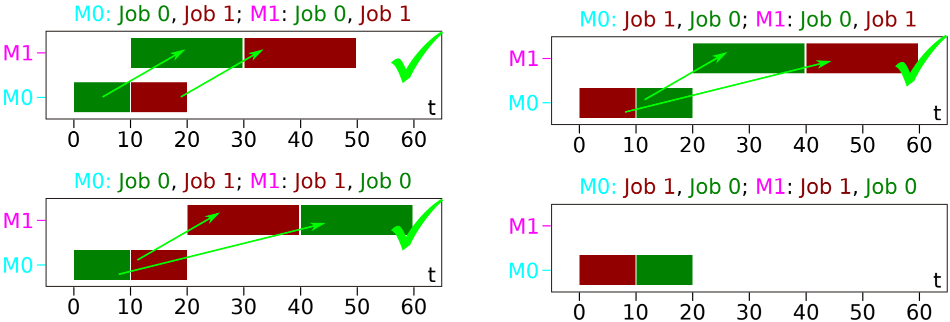
```
        ++++++++++++++++++++++++
        instance A with 2 jobs and 2 machines
        2  2
job 0   0  10    1  20
job 1   0  10    1  20
        ++++++++++++++++++++++++
```



Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
+++++++++++++++++++++++++
```

job 0
job 1

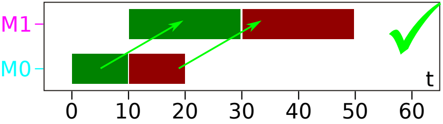


Hardships when Searching in $\mathbb{Y}$

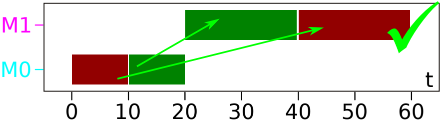
```
+++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
+++++++++++++++++++++++++
```

job 0
job 1

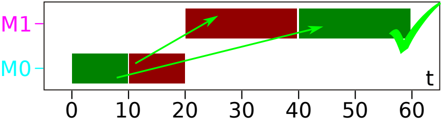
M0: Job 0, Job 1; M1: Job 0, Job 1



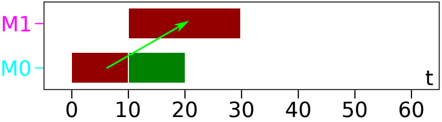
M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



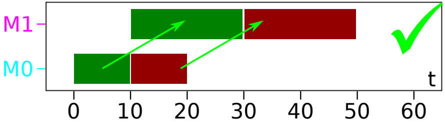
M0: Job 1, Job 0; M1: Job 1, Job 0



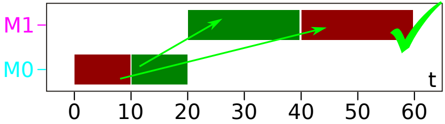
Hardships when Searching in $\mathbb{Y}$

	+++++++++++++++++++++++
	instance A with 2 jobs and 2 machines
	2 2
job 0	0 10 1 20
job 1	0 10 1 20
	+++++++++++++++++++++++

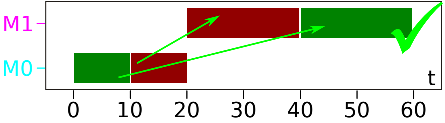
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



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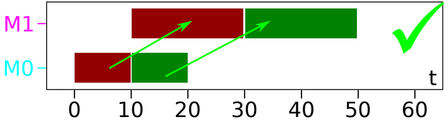

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
++++++++++++++++++++++++
```

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++
```

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
++++++++++++++++++++++++
```

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++
```

Hardships when Searching in $\mathbb{Y}$

job 0

```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

job 0
job 1

Hardships when Searching in $\mathbb{Y}$

```
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instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++
```

job 0
job 1

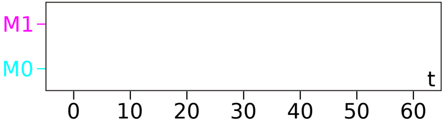
M0: Job 0, Job 1; M1: Job 0, Job 1

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++
```

job 0: 0 10 1 20
job 1: 1 20 0 10

M0: Job 0, Job 1; M1: Job 0, Job 1



Hardships when Searching in $\mathbb{Y}$

```
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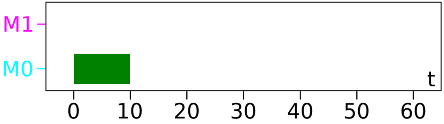
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

job 0
job 1

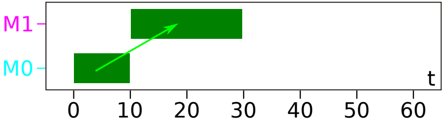
M0: Job 0, Job 1; M1: Job 0, Job 1



Hardships when Searching in $\mathbb{Y}$

```
        ++++++++++++++++++++++++
        instance B with 2 jobs and 2 machines
        2  2
job 0   0  10   1  20
job 1   1  20   0  10
        ++++++++++++++++++++++++
```

M0: Job 0, Job 1; M1: Job 0, Job 1



Hardships when Searching in $\mathbb{Y}$

```
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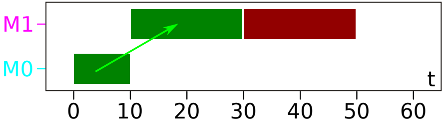
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

job 0

job 1

M0: Job 0, Job 1; M1: Job 0, Job 1

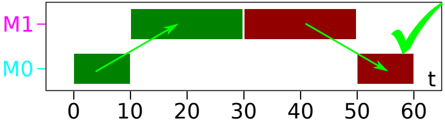


Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1

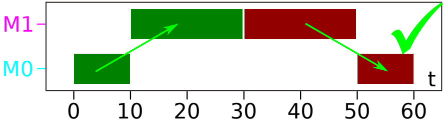


Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++
```

job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1



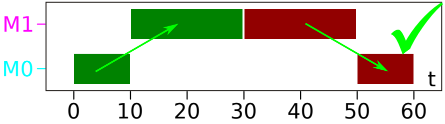
M0: Job 0, Job 1; M1: Job 1, Job 0

Hardships when Searching in $\mathbb{Y}$

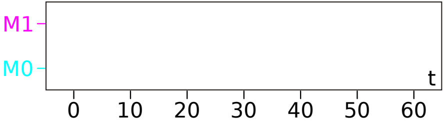
```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++++
```

job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1

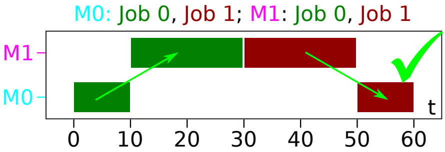


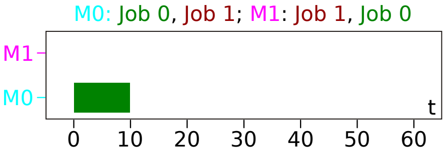
M0: Job 0, Job 1; M1: Job 1, Job 0



Hardships when Searching in $\mathbb{Y}$

```
        +++++++++++++++++++++++++
        instance B with 2 jobs and 2 machines
        2  2
job 0   0  10    1  20
job 1   1  20    0  10
        +++++++++++++++++++++++++
```





Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
job 0   0  10    1  20
job 1   1  20    0  10
+++++++++++++++++++++++
```

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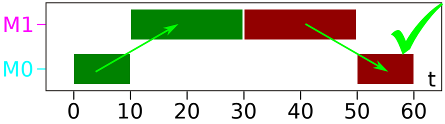
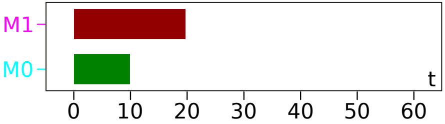


M0: Job 0, Job 1; M1: Job 1, Job 0

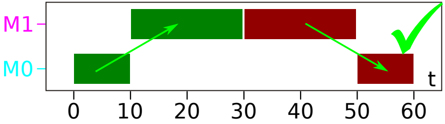


Hardships when Searching in $\mathbb{Y}$

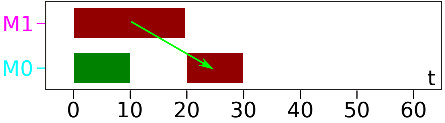
```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

job 0: 0 10 1 20
job 1: 1 20 0 10

M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



Hardships when Searching in $\mathbb{Y}$

```
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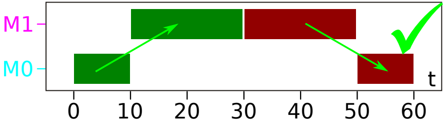
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++++
```

job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1



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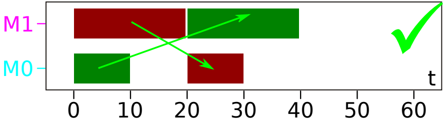
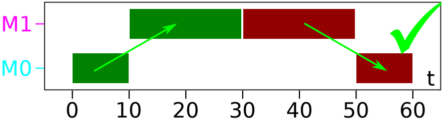


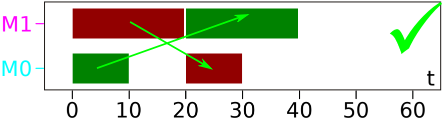
Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20      job 0
1  20   0  10      job 1
+++++++++++++++++++++++
```

M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1

Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++
```

job 0
job 1

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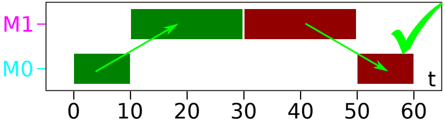
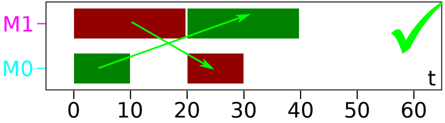
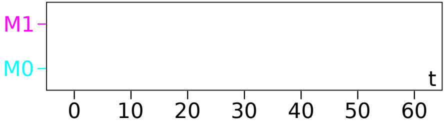


M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1

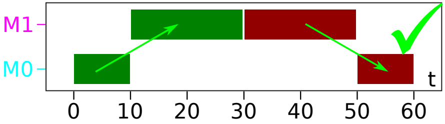


Hardships when Searching in $\mathbb{Y}$

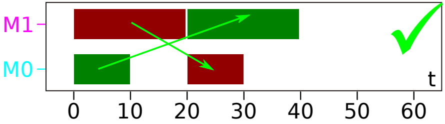
```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++
```

job 0 | job 1

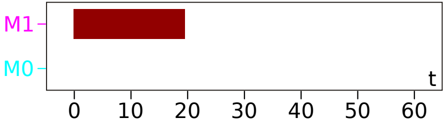
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



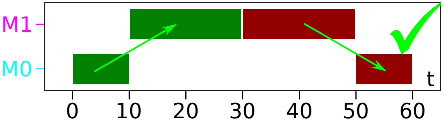
M0: Job 1, Job 0; M1: Job 0, Job 1



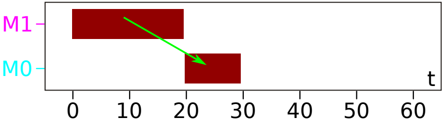
Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
job 0   0  10   1  20
job 1   1  20   0  10
++++++++++++++++++++++++++
```

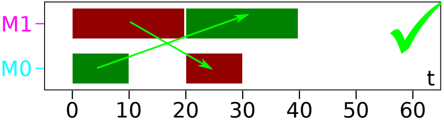
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0

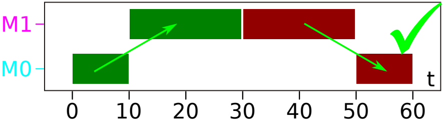


Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++
```

job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1



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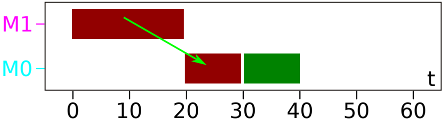
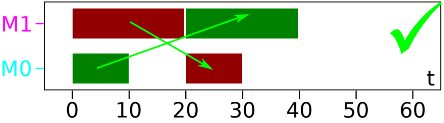


M0: Job 0, Job 1; M1: Job 1, Job 0



Hardships when Searching in $\mathbb{Y}$

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```

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job 1

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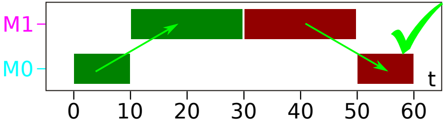

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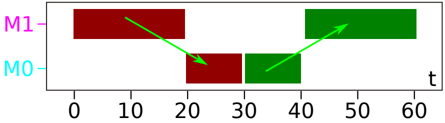
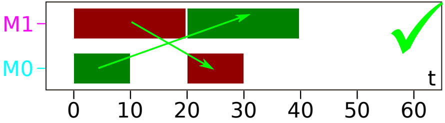


M0: Job 0, Job 1; M1: Job 1, Job 0

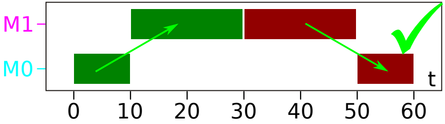


Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

job 0: 0 10 1 20
job 1: 1 20 0 10

M0: Job 0, Job 1; M1: Job 0, Job 1



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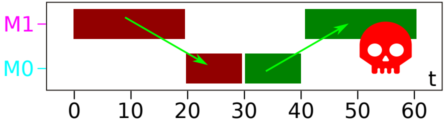

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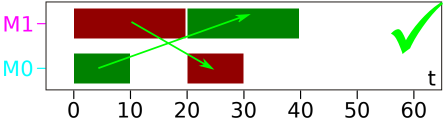
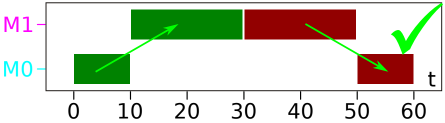


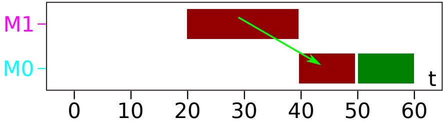
Hardships when Searching in $\mathbb{Y}$

```
      +++++++++++++++++++++++++
      instance B with 2 jobs and 2 machines
      2  2
job 0 0  10   1  20
job 1 1  20   0  10
      +++++++++++++++++++++++++
```

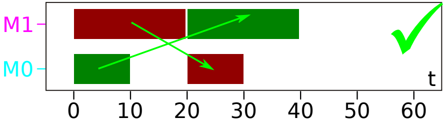
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 1, Job 0; M1: Job 0, Job 1



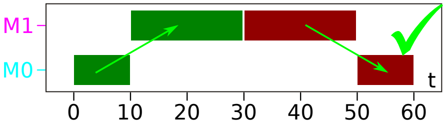
M0: Job 0, Job 1; M1: Job 1, Job 0



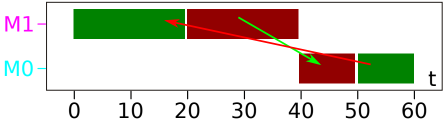
Hardships when Searching in $\mathbb{Y}$

```
        ++++++++++++++++++++++++
        instance B with 2 jobs and 2 machines
        2  2
job 0   0  10   1  20
job 1   1  20   0  10
        ++++++++++++++++++++++++
```

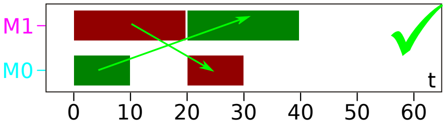
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0

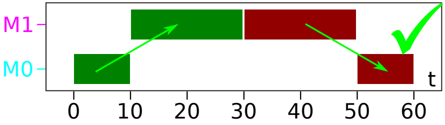


Hardships when Searching in $\mathbb{Y}$

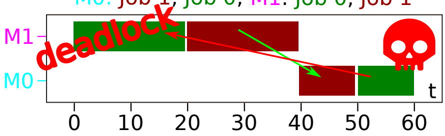
```
+++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++
```

job 0
job 1

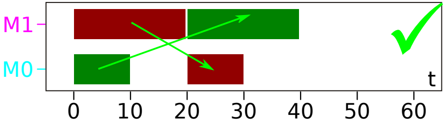
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0

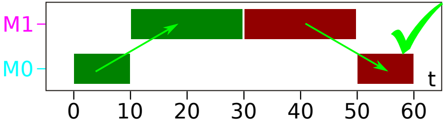


Hardships when Searching in $\mathbb{Y}$

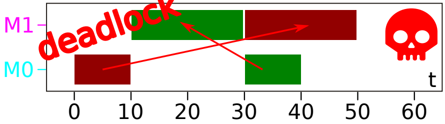
```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

job 0
job 1

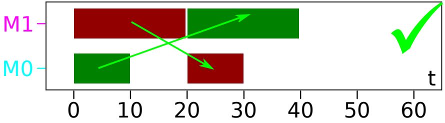
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0

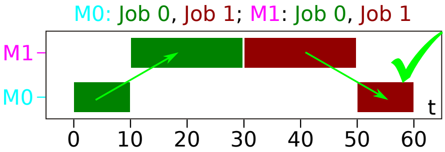


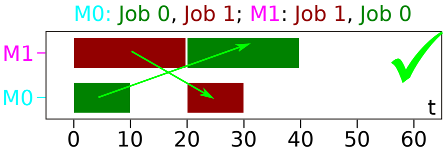
Hardships when Searching in $\mathbb{Y}$

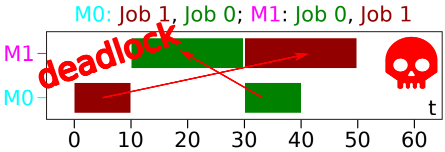
```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2 2
0 10  1 20
1 20  0 10
+++++++++++++++++++++++++
```

job 0
job 1

Machine 0 should begin by doing job 1.





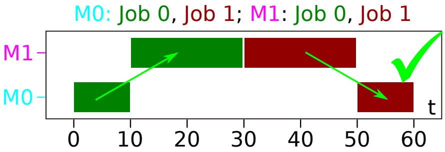


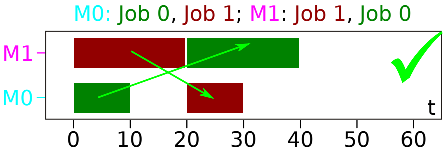
Hardships when Searching in $\mathbb{Y}$

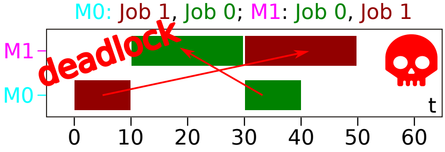
```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

job 0
job 1

Machine 0 should begin by doing job 1.
Job 1 can only start on machine 0 after it has been finished on machine 1.







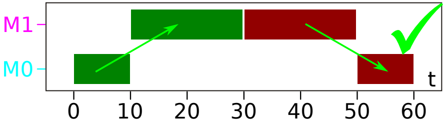
Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

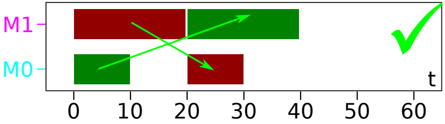
job 0
job 1

Machine 0 should begin by doing job 1. Job 1 can only start on machine 0 after it has been finished on machine 1. At machine 1, we should begin with job 0.

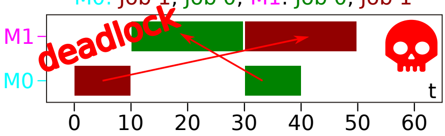
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



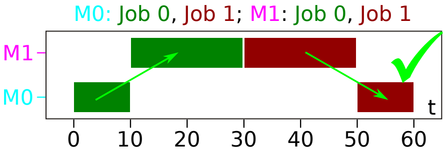
M0: Job 1, Job 0; M1: Job 0, Job 1

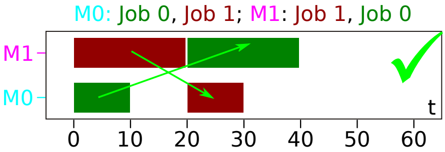


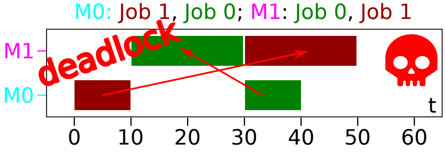
Hardships when Searching in $\mathbb{Y}$

```
job 0
job 1
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++++
```

Job 1 can only start on machine 0 after it has been finished on machine 1. At machine 1, we should begin with job 0. Before job 0 can be put on machine 1, it must go through machine 0.





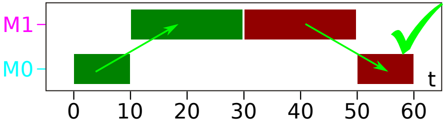


Hardships when Searching in $\mathbb{Y}$

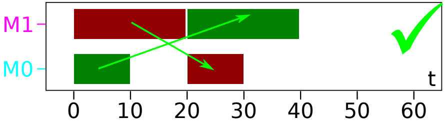
```
          ++++++++++++++++++++++++
          instance B with 2 jobs and 2 machines
          2  2
job 0     0  10   1  20
job 1     1  20   0  10
          ++++++++++++++++++++++++
```

So job 1 cannot go to machine 0 until it has passed through machine 1, but in order to be executed on machine 1, job 0 needs to be finished there first.

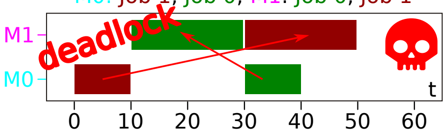
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1

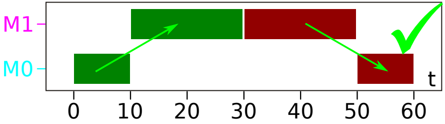


Hardships when Searching in $\mathbb{Y}$

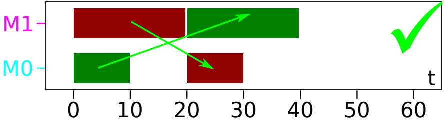
```
++++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2 2
0 10   1 20      (job 0)
1 20   0 10      (job 1)
++++++++++++++++++++++++++
```

Job 0 cannot begin on machine 1 until it has been passed through machine 0, but it cannot be executed there, because job 1 needs to be finished there first.

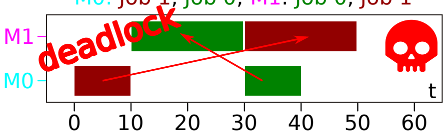
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1

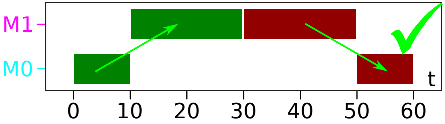


Hardships when Searching in $\mathbb{Y}$

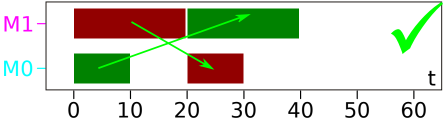
```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2 2
job 0   0 10  1 20
job 1   1 20  0 10
+++++++++++++++++++++++++
```

A cyclic blockage has appeared: no job can be executed on any machine if we follow this schedule.

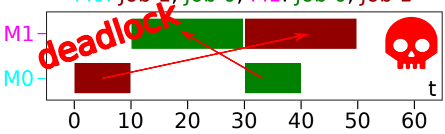
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1

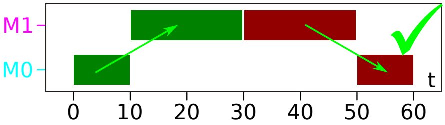


Hardships when Searching in $\mathbb{Y}$

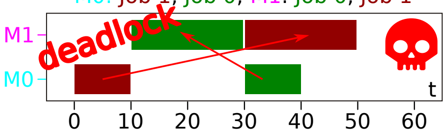
```
      +++++++++++++++++++++++++
      instance B with 2 jobs and 2 machines
      2  2
job 0 0  10   1  20
job 1 1  20   0  10
      +++++++++++++++++++++++++
```

A cyclic blockage has appeared: no job can be executed on any machine if we follow this schedule. This is called a deadlock.

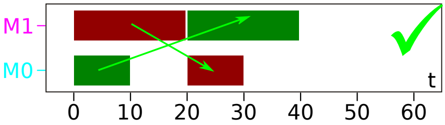
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



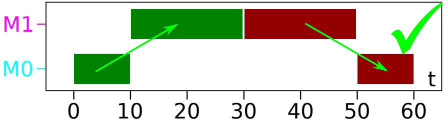
Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++++
```

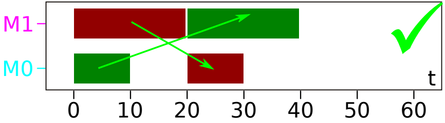
job 0
job 1

This is called a deadlock. The schedule is infeasible, because it cannot be executed or written down without breaking the precedence constraint.

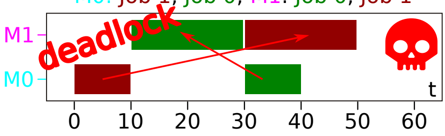
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



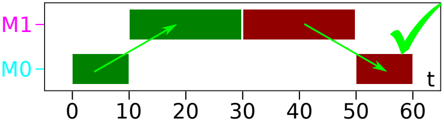
M0: Job 1, Job 0; M1: Job 0, Job 1



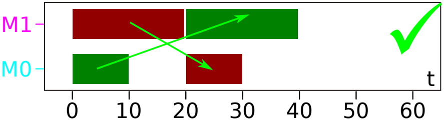
Hardships when Searching in $\mathbb{Y}$

```
    ++++++++++++++++++++++++++
    instance B with 2 jobs and 2 machines
    2  2
job 0  0  10   1  20
job 1  1  20   0  10
    ++++++++++++++++++++++++++
```

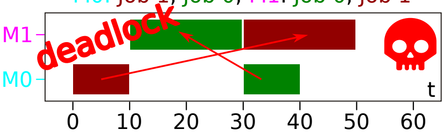
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1

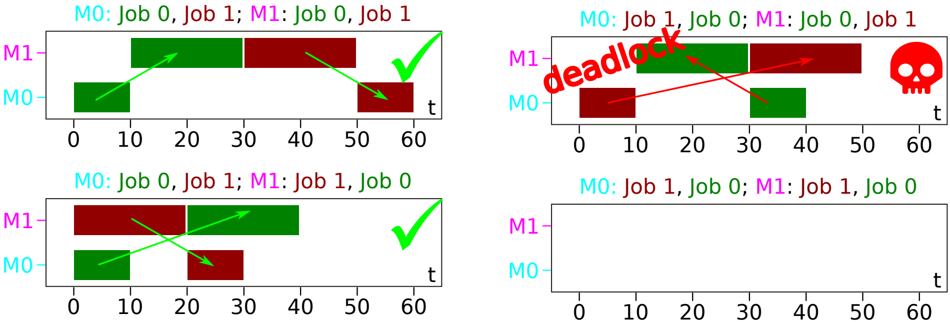


M0: Job 1, Job 0; M1: Job 1, Job 0

Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++
```

job 0
job 1

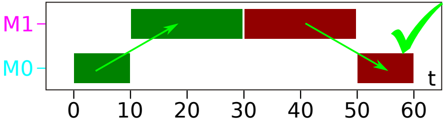


Hardships when Searching in $\mathbb{Y}$

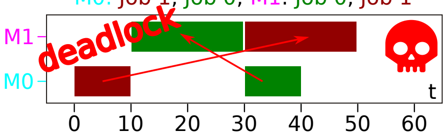
job 0
job 1

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++
```

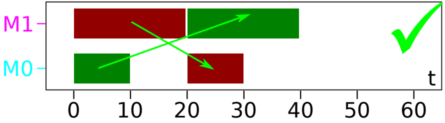
M0: Job 0, Job 1; M1: Job 0, Job 1



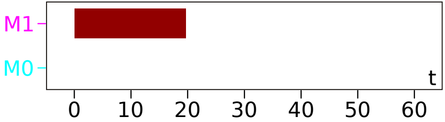
M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 1, Job 0



Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++
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2  2
0  10   1  20
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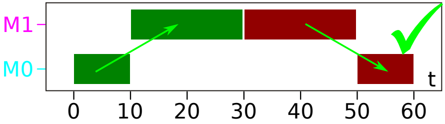
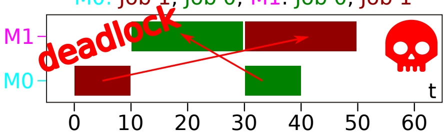
```

job 0
job 1

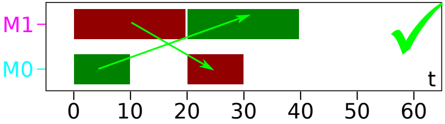
M0: Job 0, Job 1; M1: Job 0, Job 1



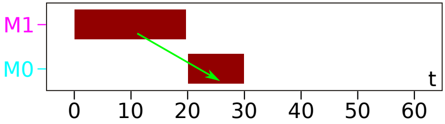
M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



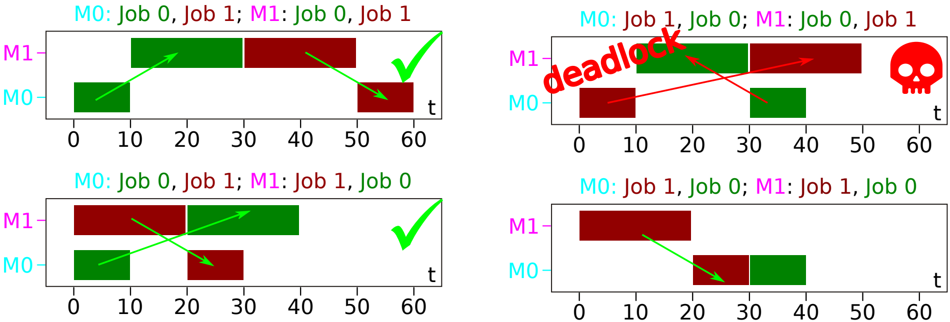
M0: Job 1, Job 0; M1: Job 1, Job 0



Hardships when Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2 2
0 10 1 20
1 20 0 10
++++++++++++++++++++++++
```

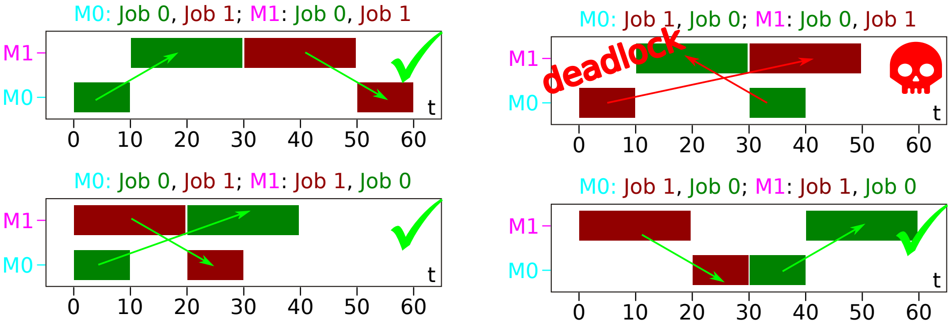
job 0
job 1



Hardships when Searching in $\mathbb{Y}$

```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++++
```

job 0
job 1



Hardships when Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?
- We need to create Gantt charts that fulfill all the constraints.
- For different instances, different solutions are feasible!
- Writing Java code that works directly on the Gantt charts is cumbersome and error-prone.

Hardships when Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?
- We need to create Gantt charts that fulfill all the constraints.
- For different instances, different solutions are feasible!
- Writing Java code that works directly on the Gantt charts is cumbersome and error-prone.
- Actually, the vast majority of possible Gantt charts will often be infeasible and have deadlocks. . .

Hardships when Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?
- We need to create Gantt charts that fulfill all the constraints.
- For different instances, different solutions are feasible!
- Writing Java code that works directly on the Gantt charts is cumbersome and error-prone.
- Actually, the vast majority of possible Gantt charts will often be infeasible and have deadlocks...
- We would like to have a handy representation for Gantt charts.

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- We would like to have a handy representation for Gantt charts.
- The representation should allow us to easy create and modify the candidate solutions.
- Solution: We develop a data structure $\mathbb{X}$ which we can handle easily and which can always be translated to feasible Gantt charts by a mapping $\gamma : \mathbb{X} \mapsto \mathbb{Y}$.

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One Search Space $\mathbb{X}$ for the JSSP

number m of machines

number n of jobs

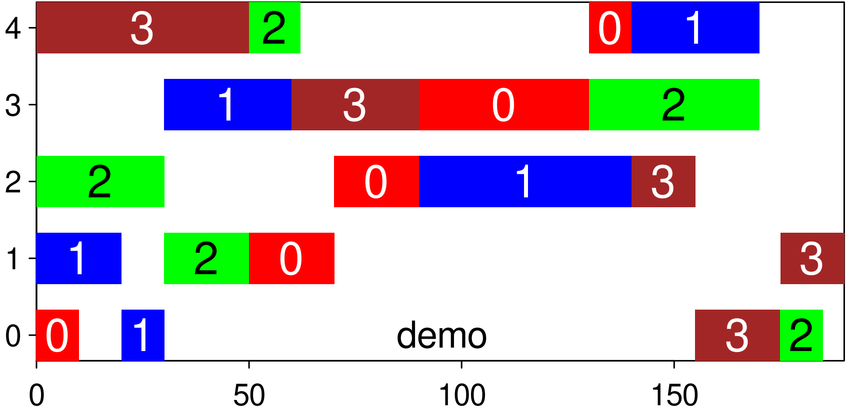
```
+++++++++++++++++++++++++
A simple demo
4 5
0 10  1 20  2 20  3 40  4 10
1 20  0 10  3 30  2 50  4 30
2 30  1 20  4 12  3 40  0 10
4 50  3 30  2 15  0 20  1 15
+++++++++++++++++++++++++
```

job 0
job 1
job 2
job 3

Each of the n jobs has m operations, each consisting of a machine index and a time requirement.

This is information that we have, which does not need to be stored in the elements $x \in \mathbb{X}$.

One Search Space $\mathbb{X}$ for the JSSP



The instance data $\mathcal{I}$ and the data from one point $x \in \mathbb{X}$ should, together, encode such a Gantt chart $y \in \mathbb{Y}$.

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- Ideally, we want to encode this two-dimensional structure in something very simple.

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- Then, a linear string containing a permutation of these IDs could denote the exact processing order of the operations.
- We could easily translate such strings to Gantt charts, but we could end up with infeasible solutions and deadlocks or a string telling us to do the second operation of a job before the first one...

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- We just include each job id m times in the string.[21–24]
- The first occurrence of a job's ID stands for its first operation, the second occurrence for the second operation, and so on.
- This way, we will always have the operations in the right order.

Demo Example for the Search Space

$\boldsymbol{x} \in \mathbb{X}$

`{1, 2, 0, 1, 2, 3, 1, 2, 0, 3, 0, 0, 1, 0, 3, 3, 2, 2, 3, 1}`

Demo Example for the Search Space

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```
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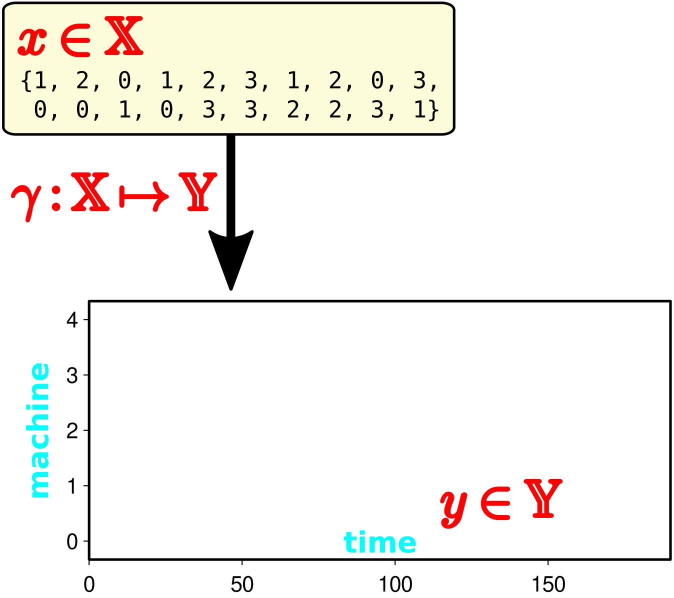

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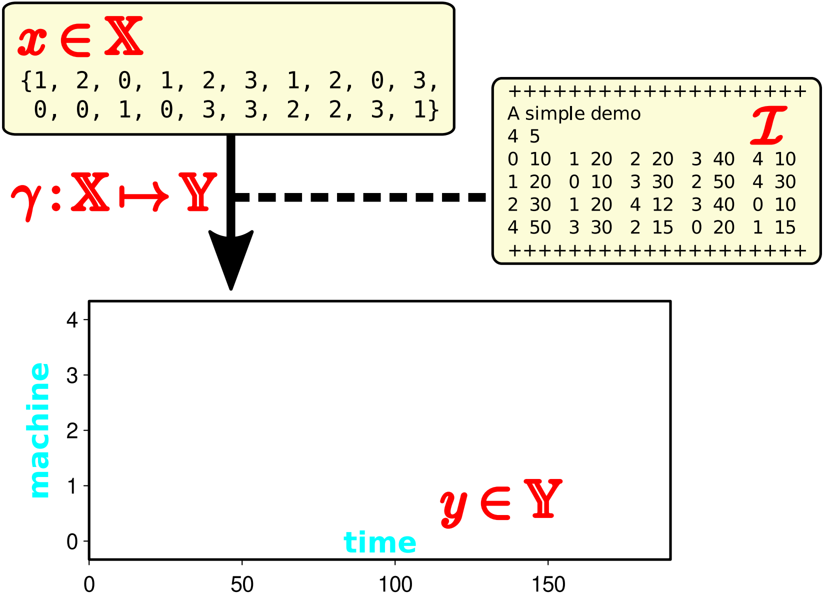

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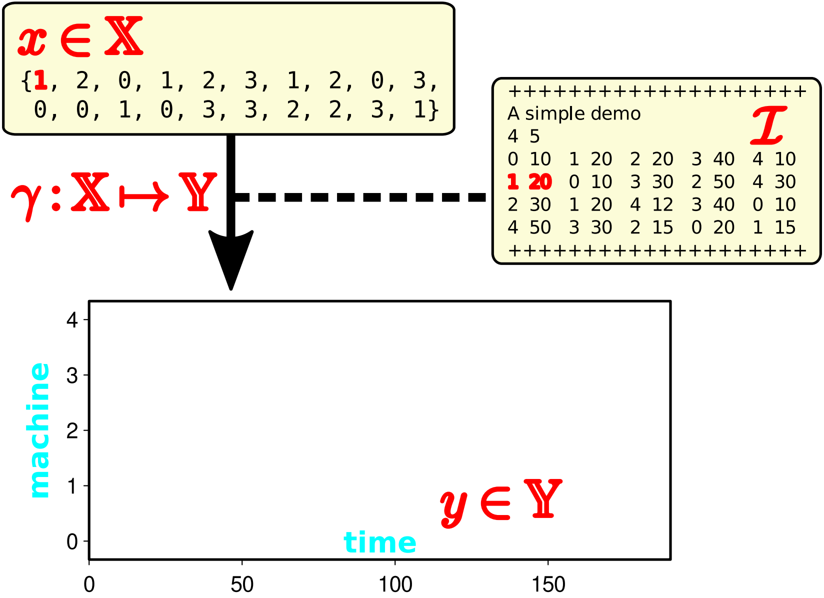

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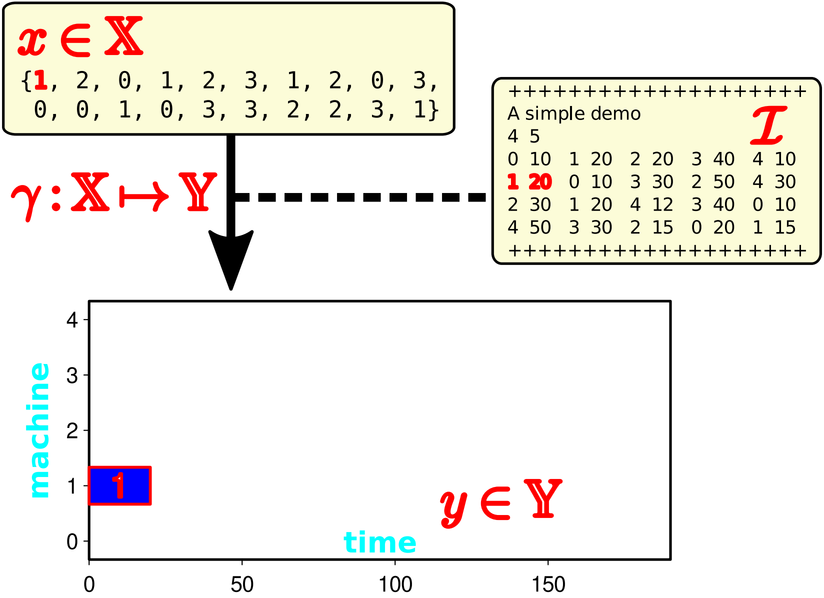

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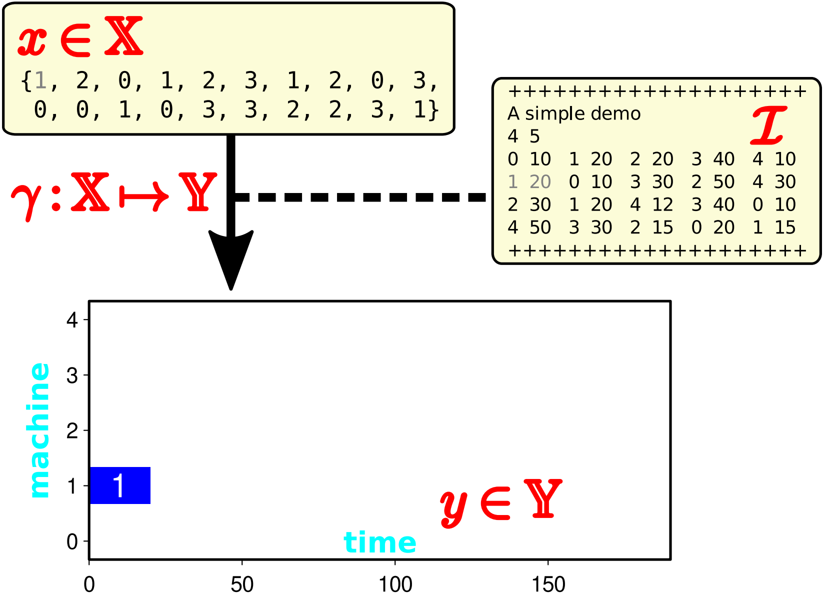

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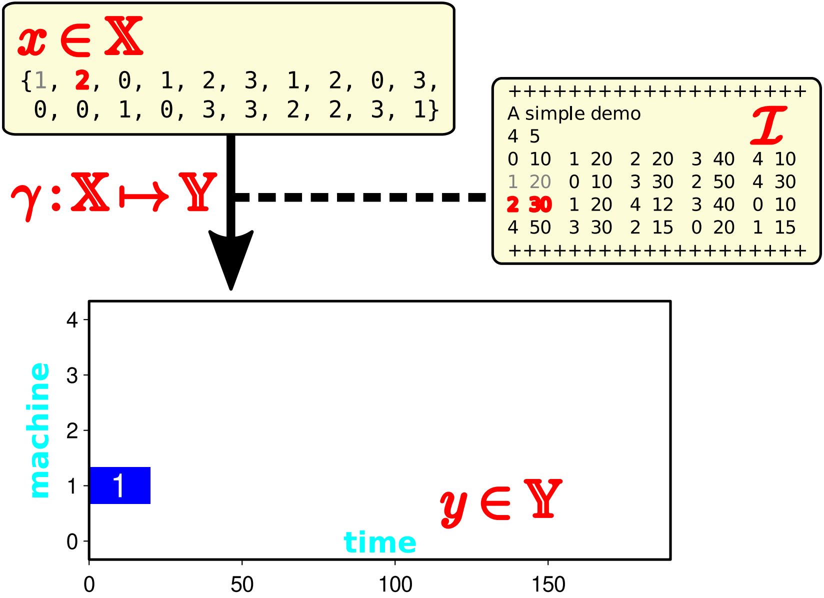

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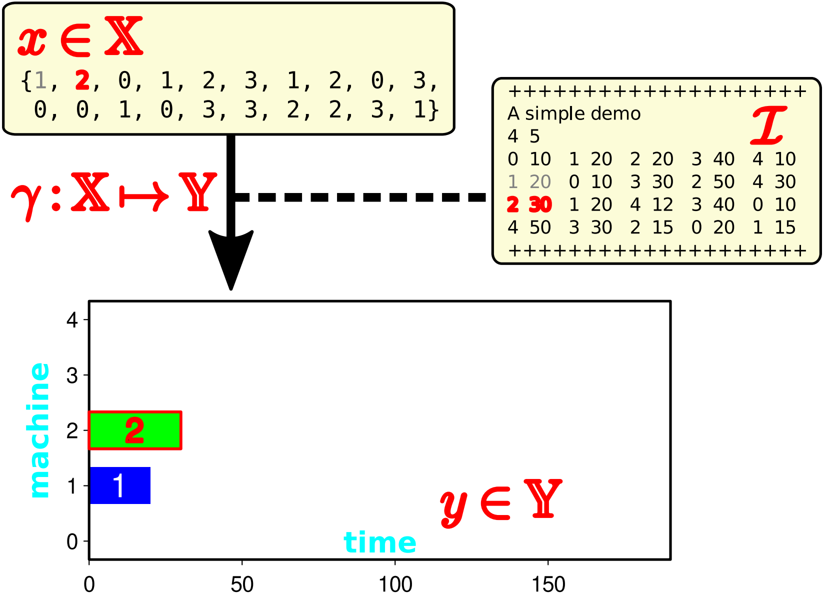

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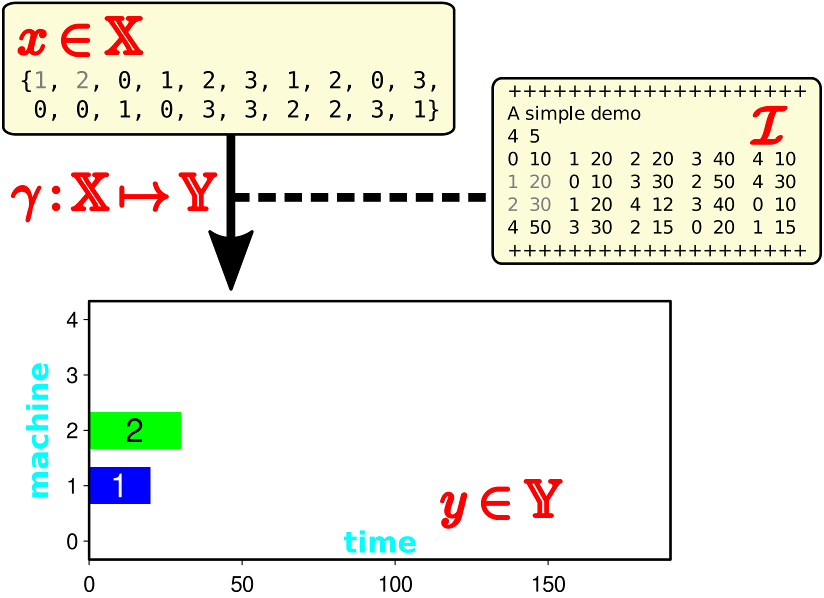

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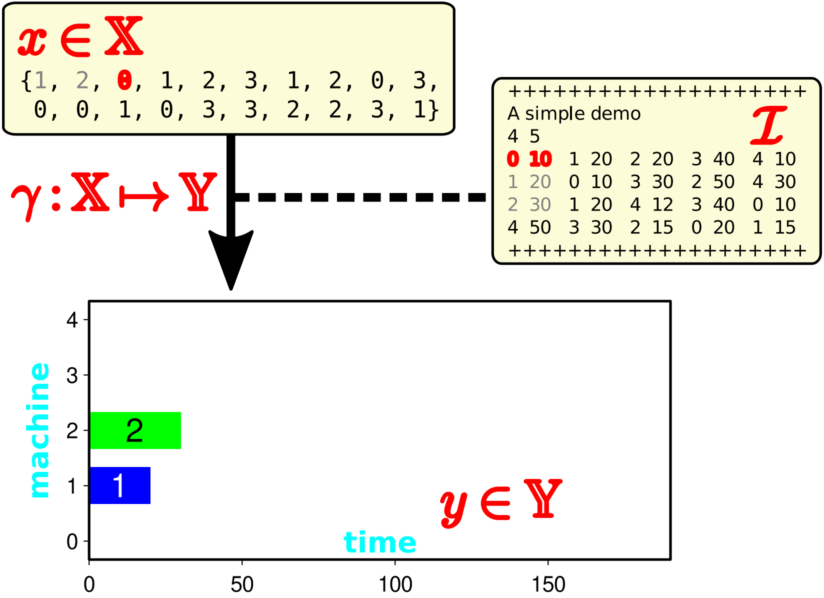

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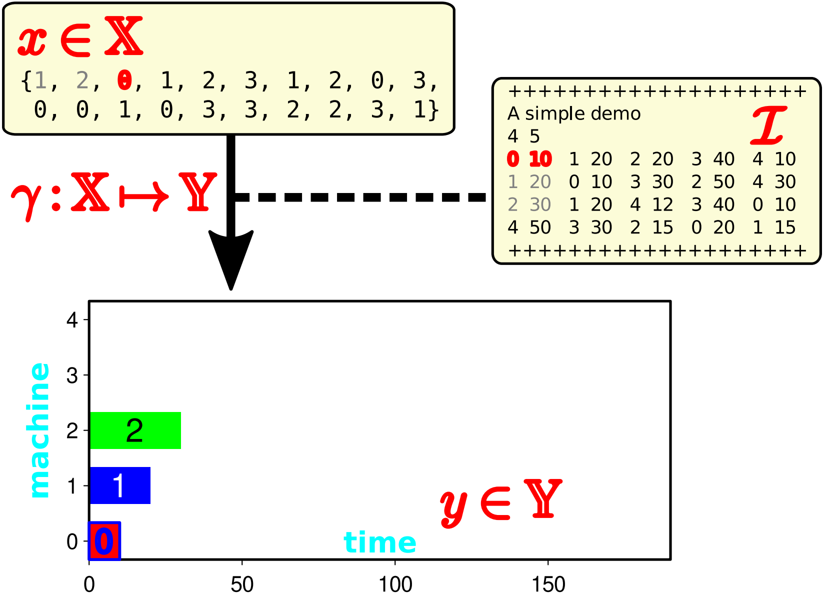

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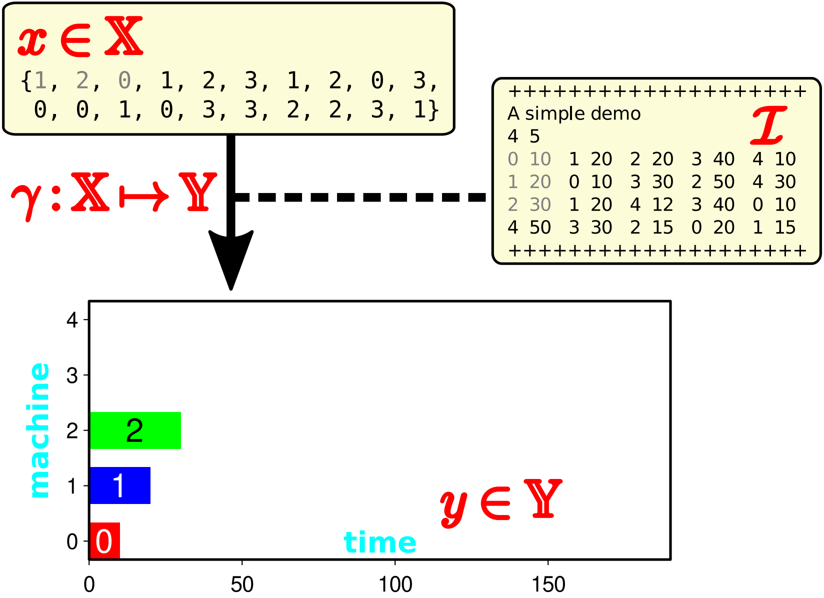

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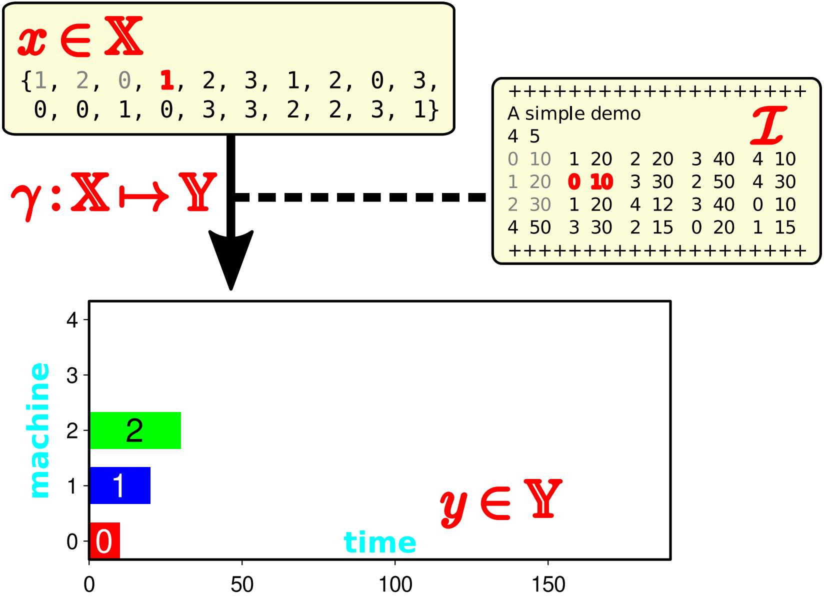

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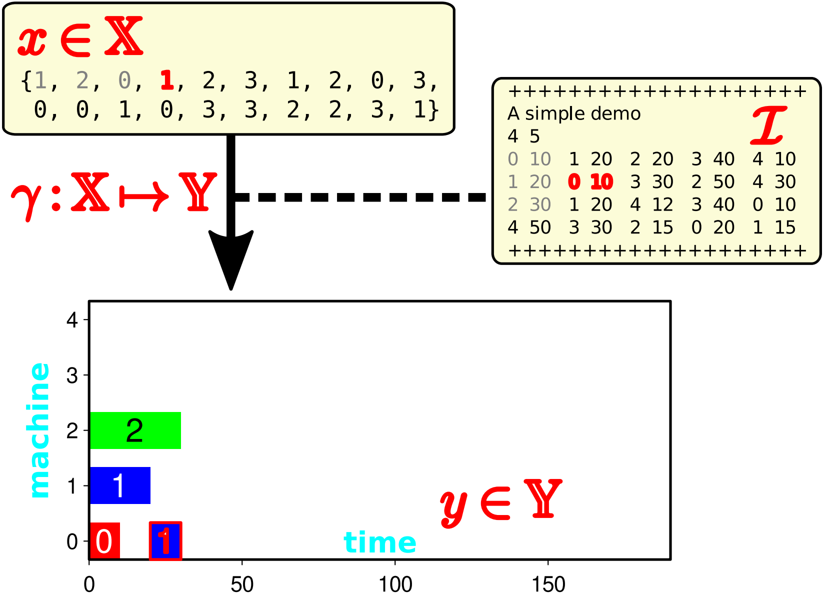

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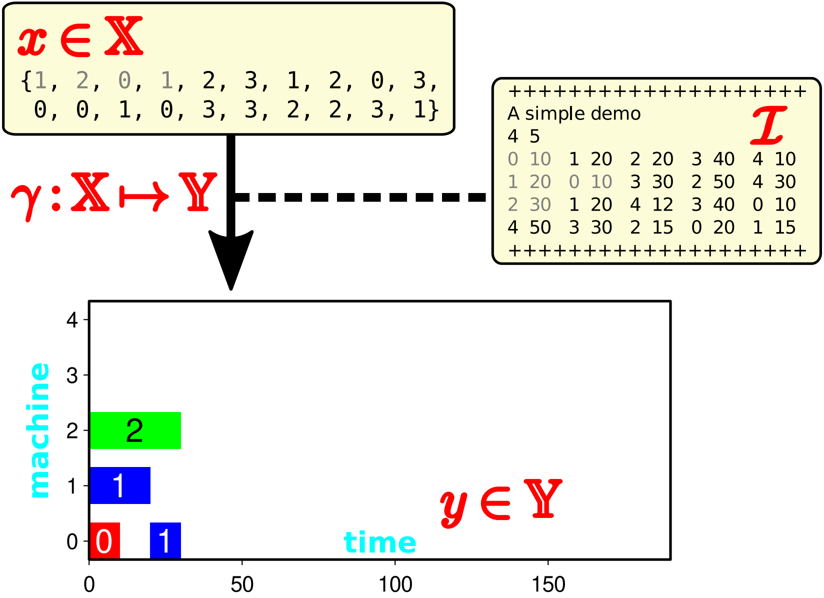

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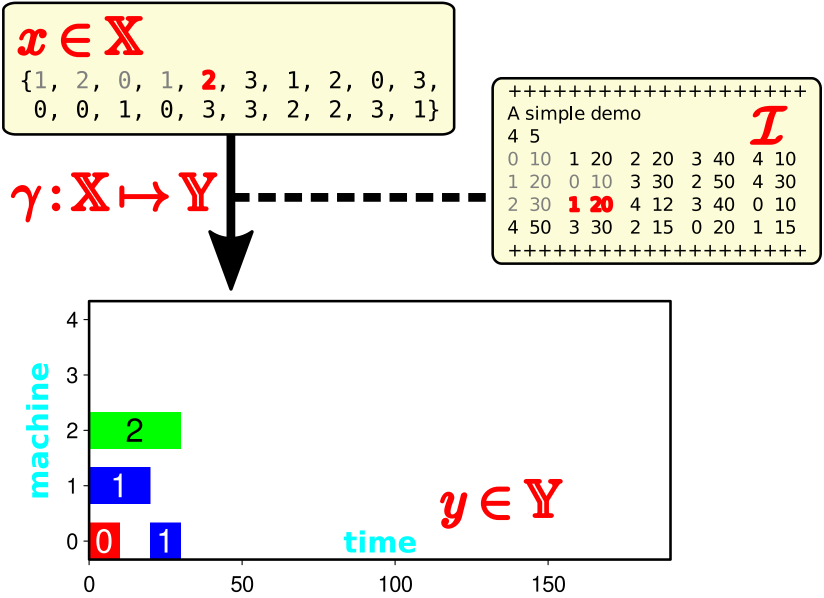

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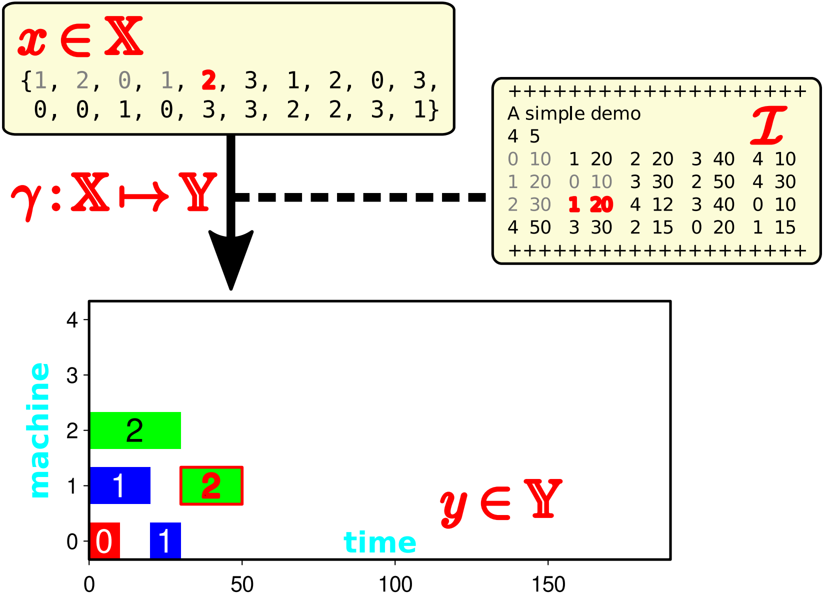

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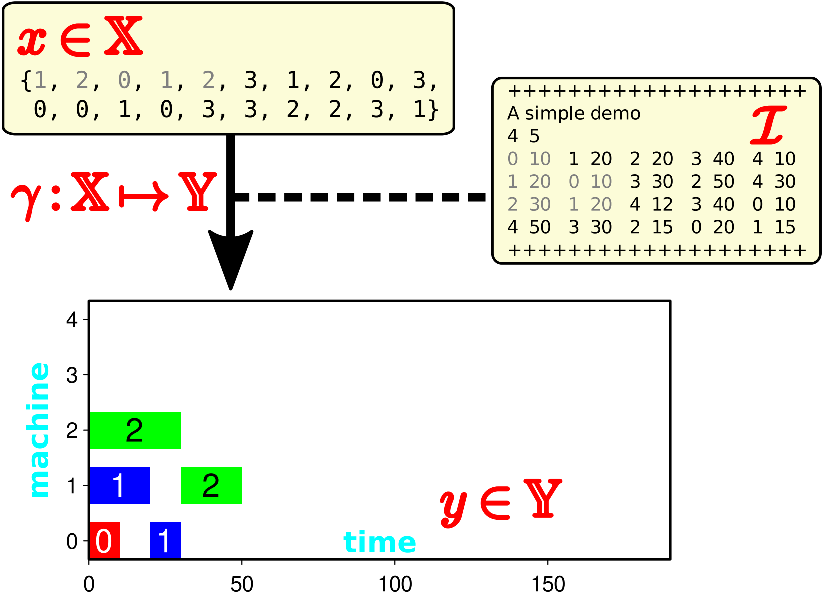

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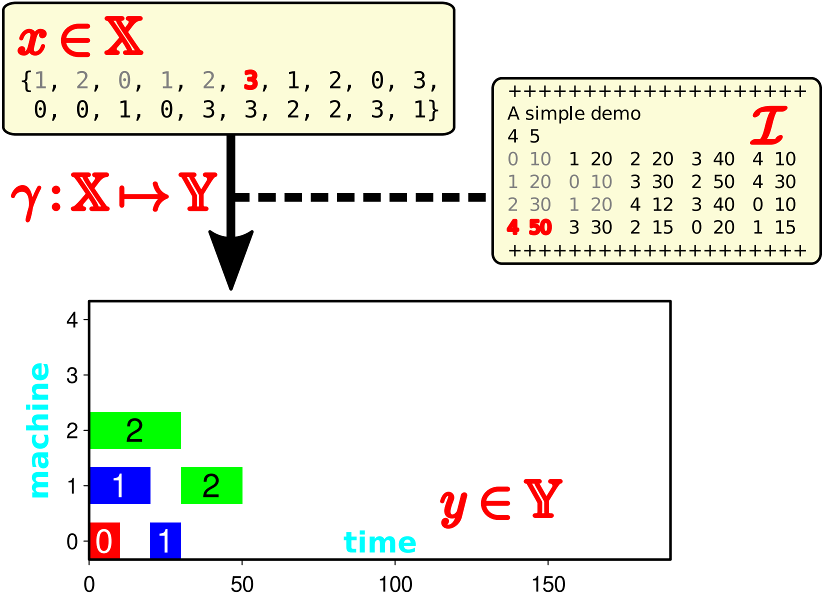

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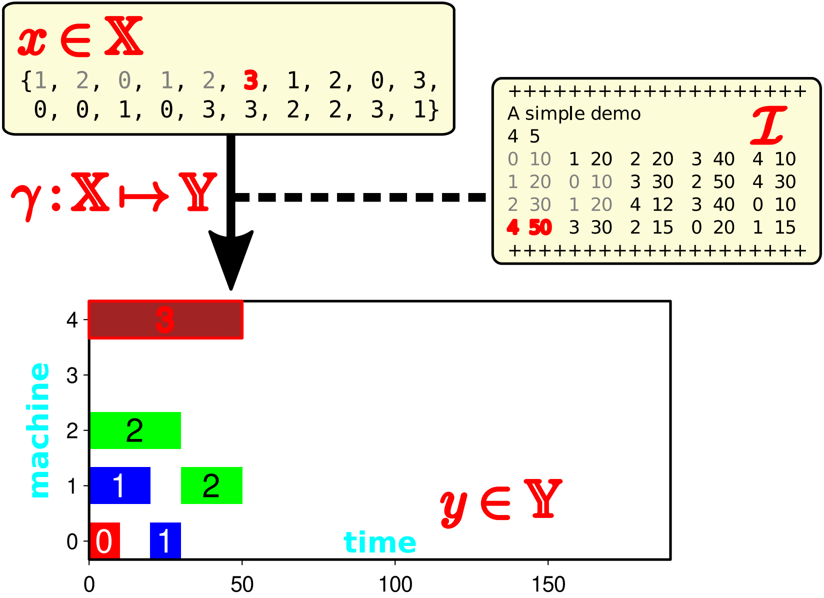

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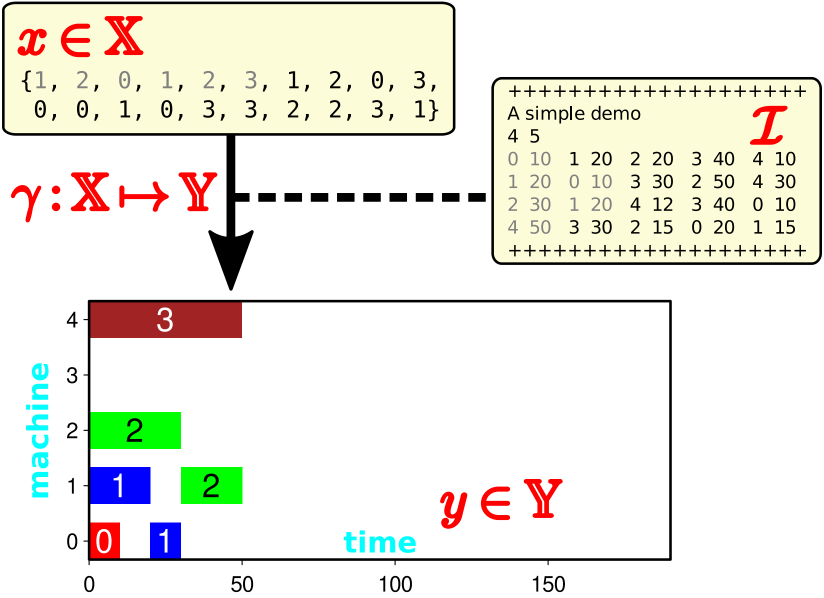

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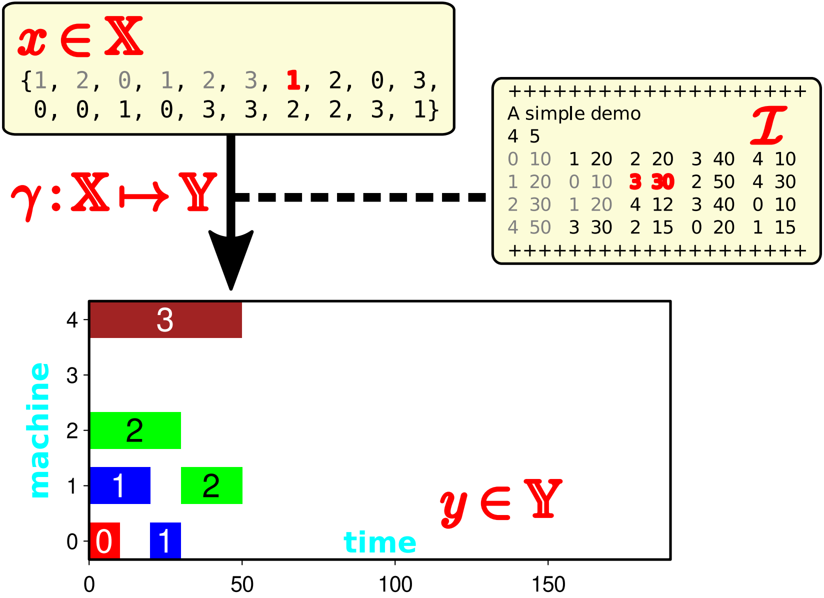

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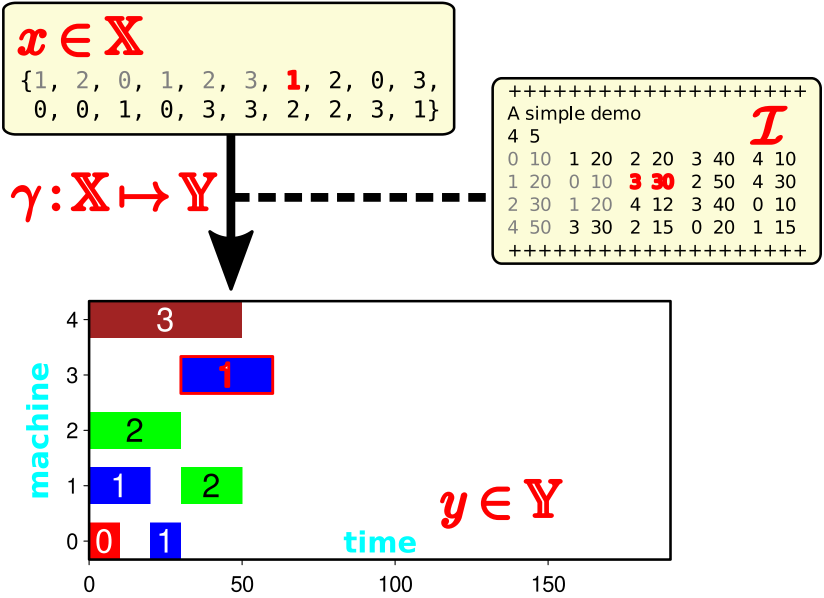

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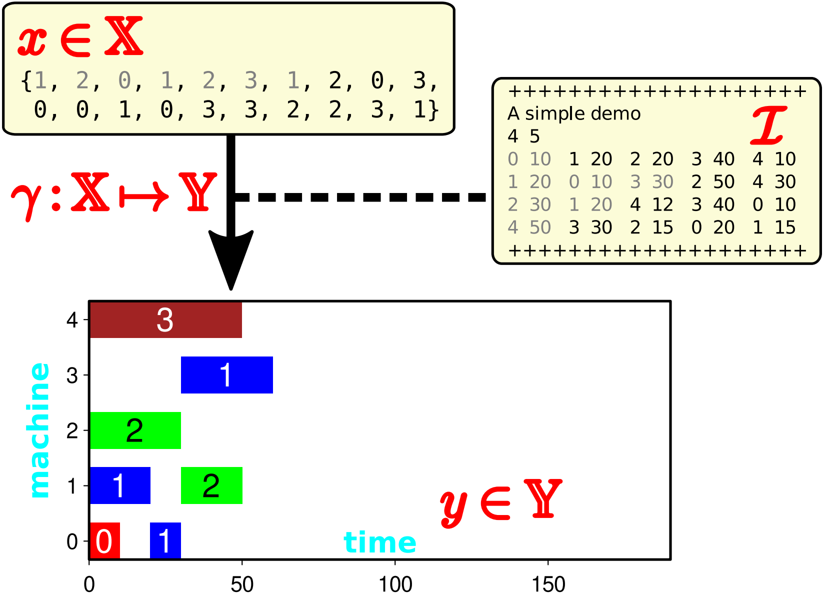

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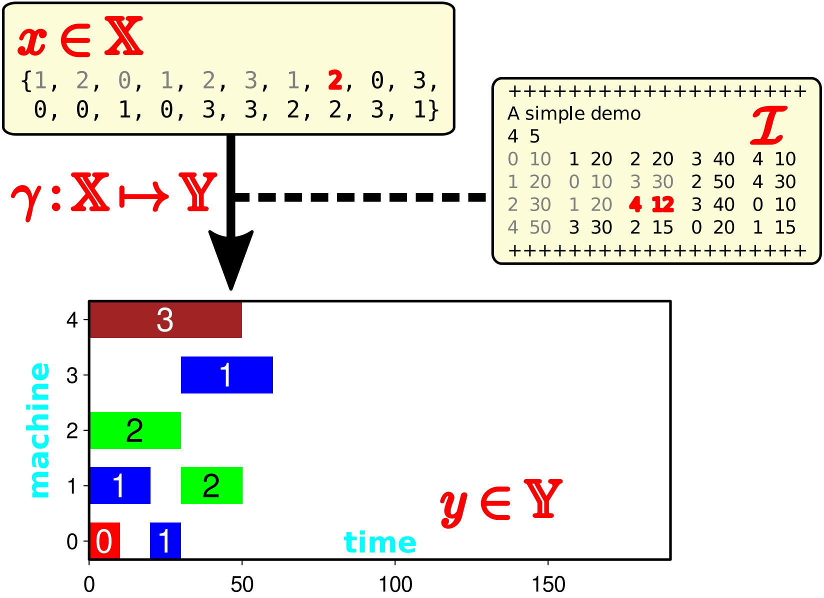

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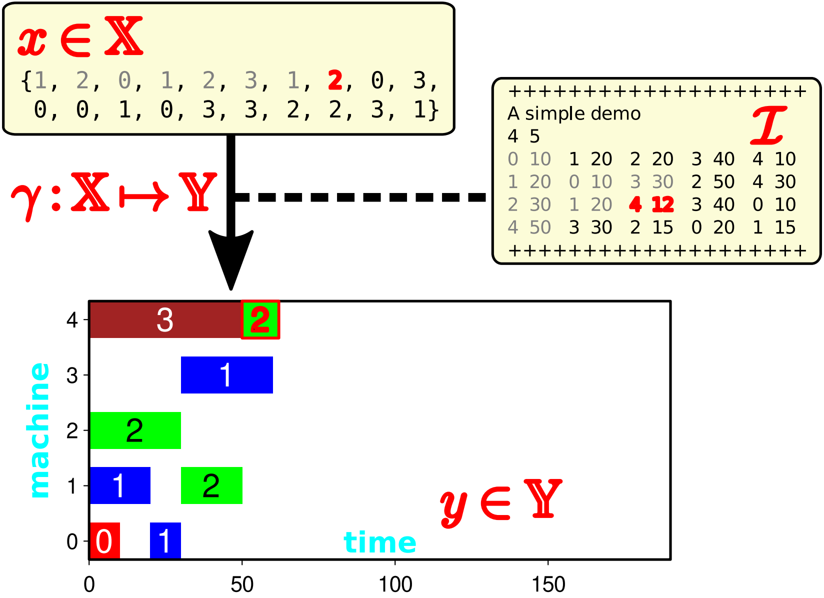

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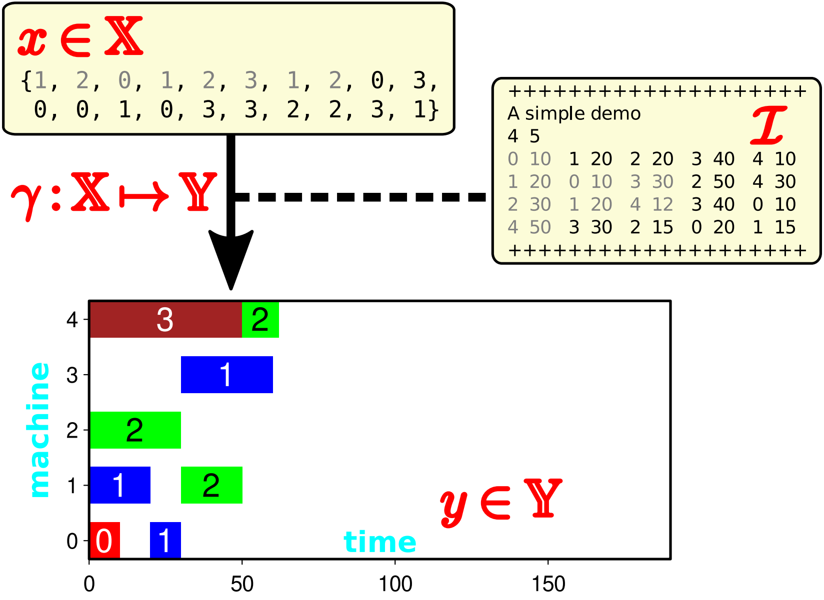

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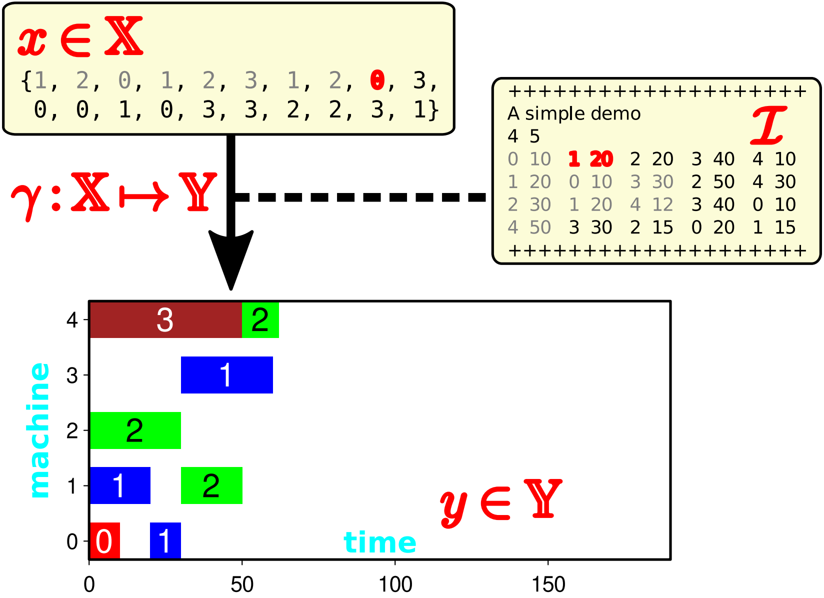

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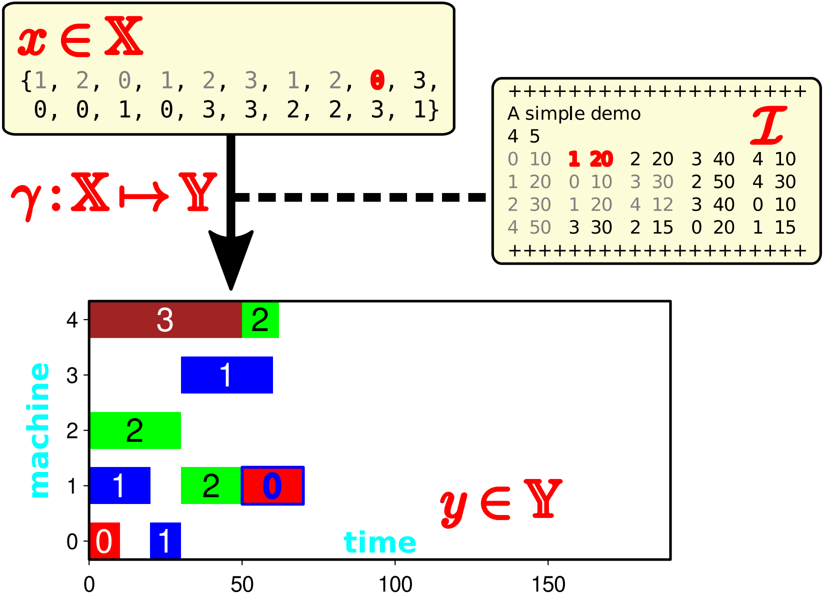

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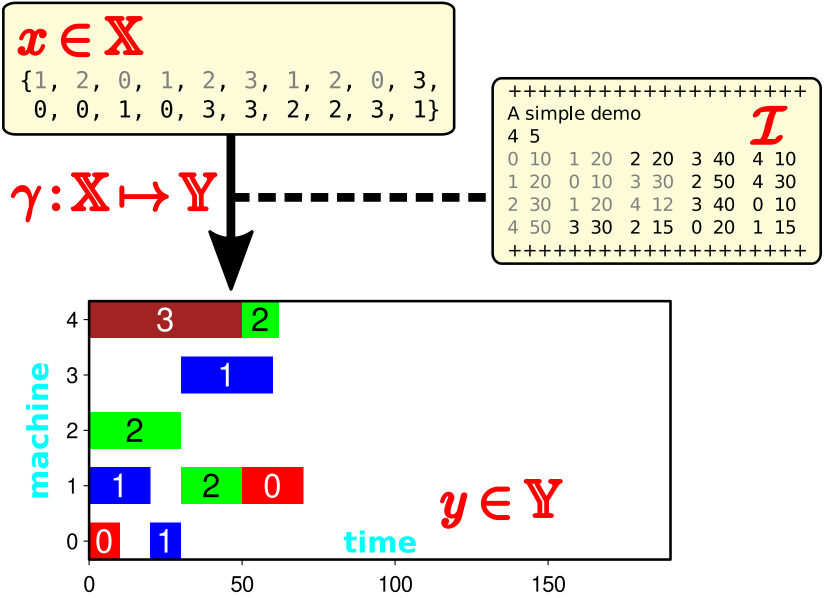

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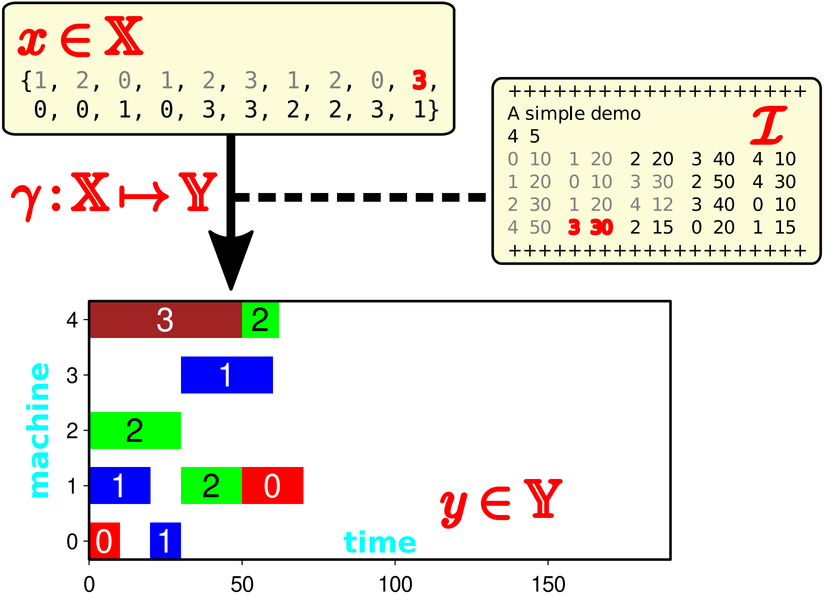

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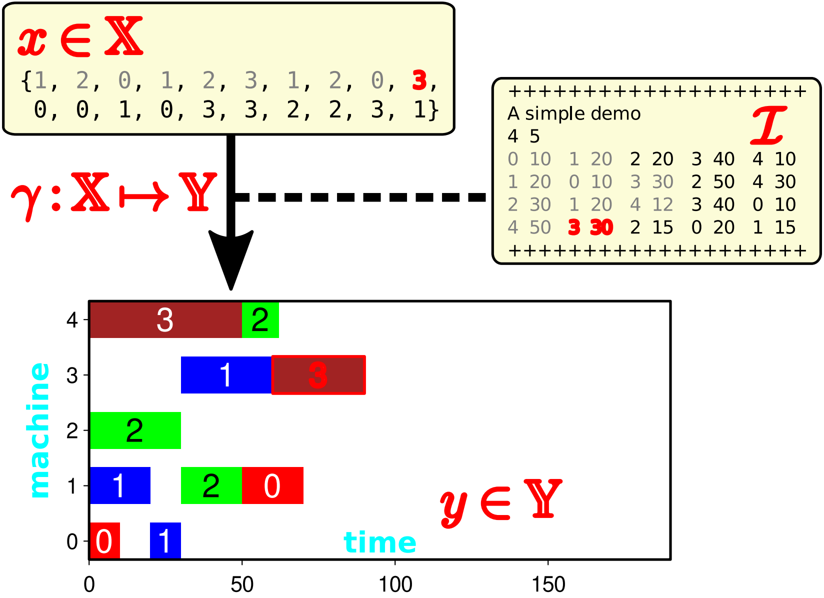

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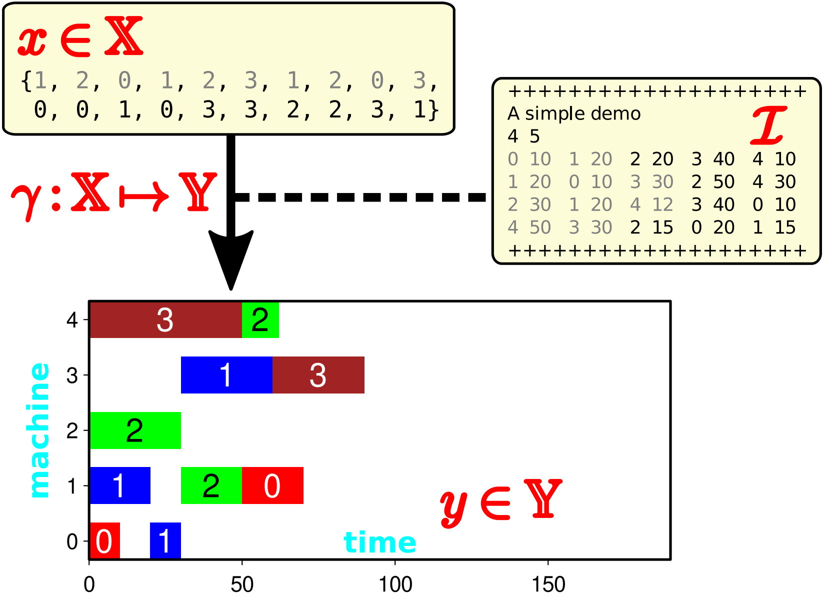

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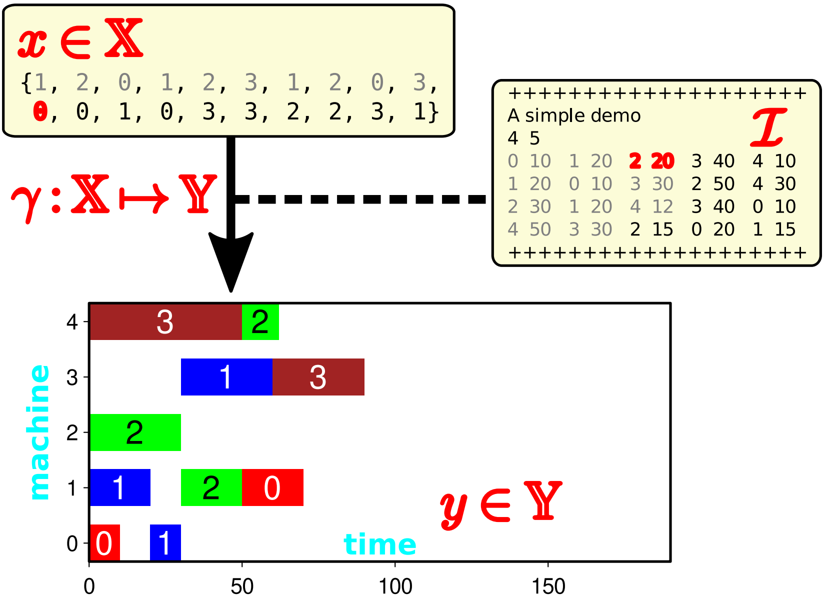

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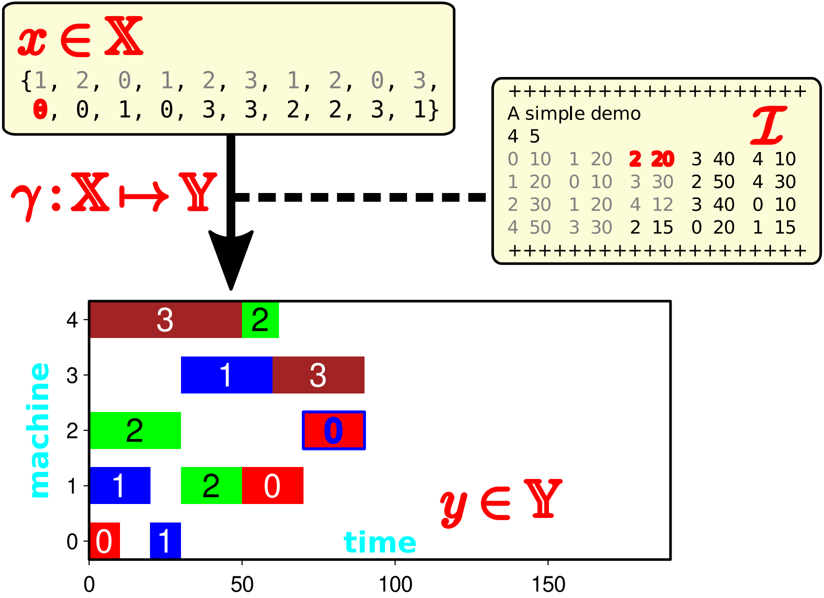

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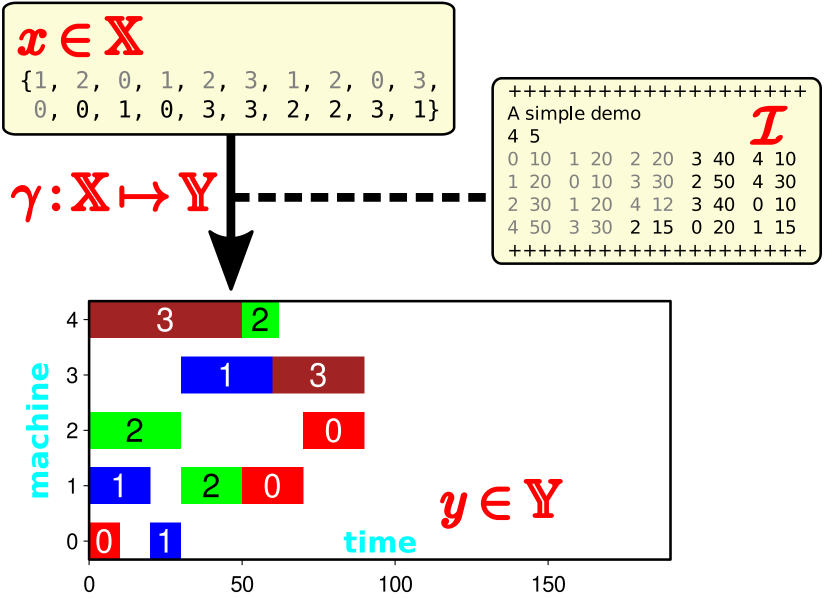

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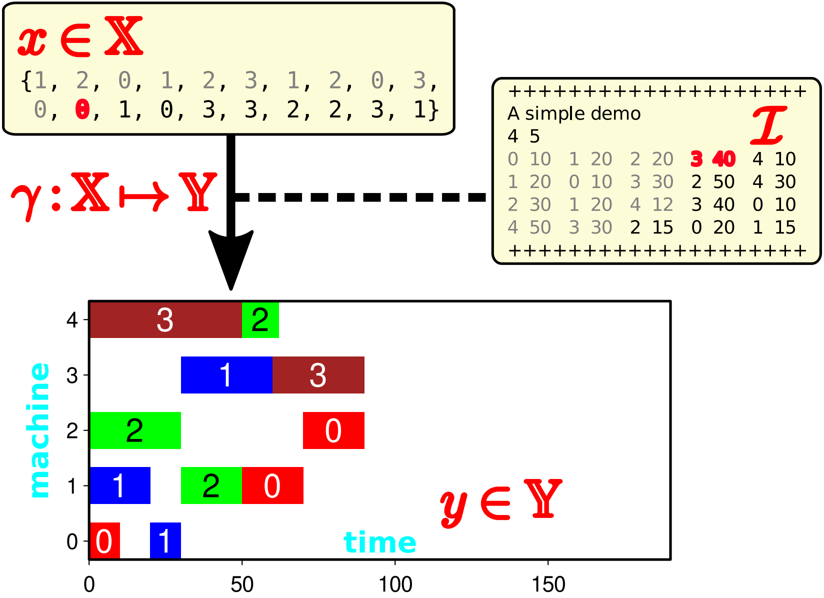

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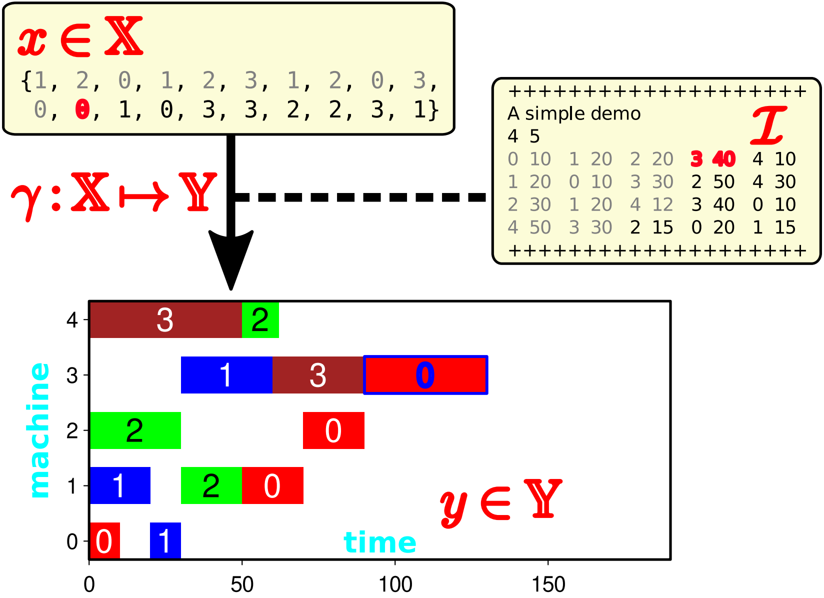

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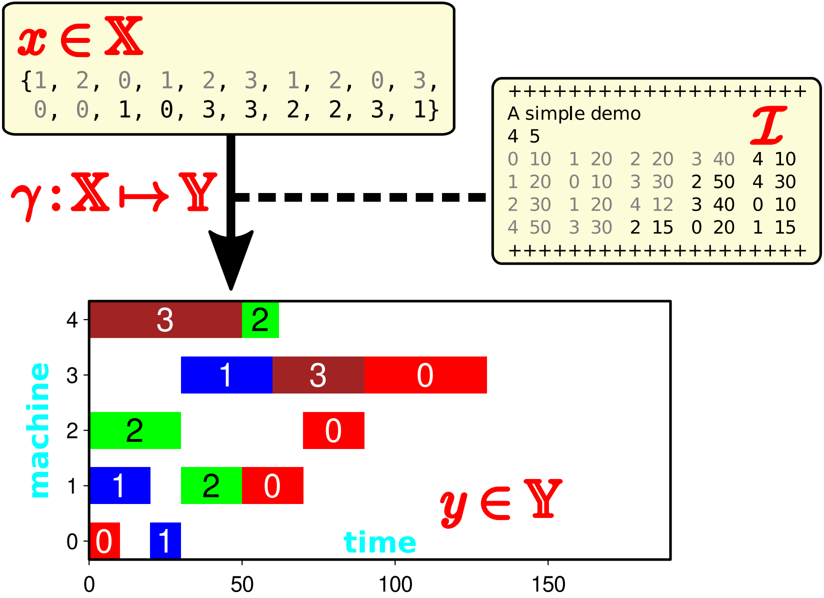

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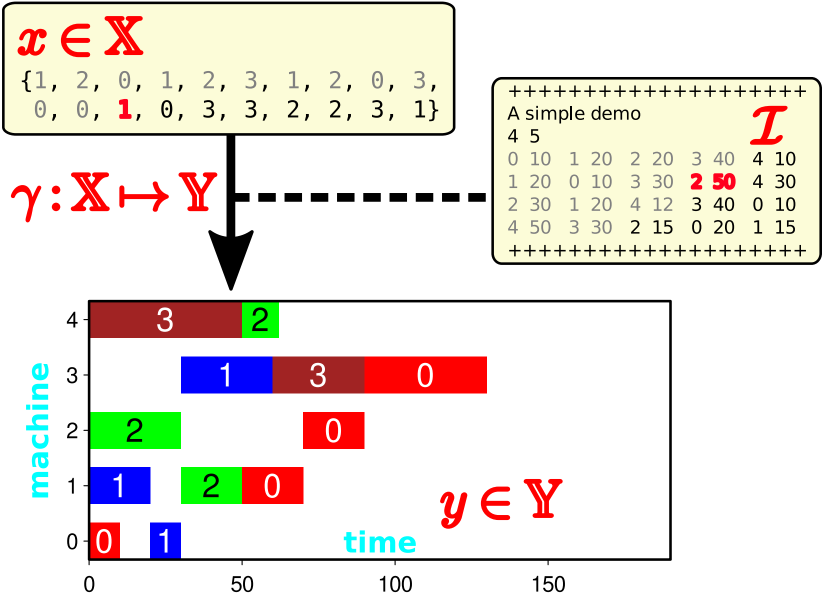

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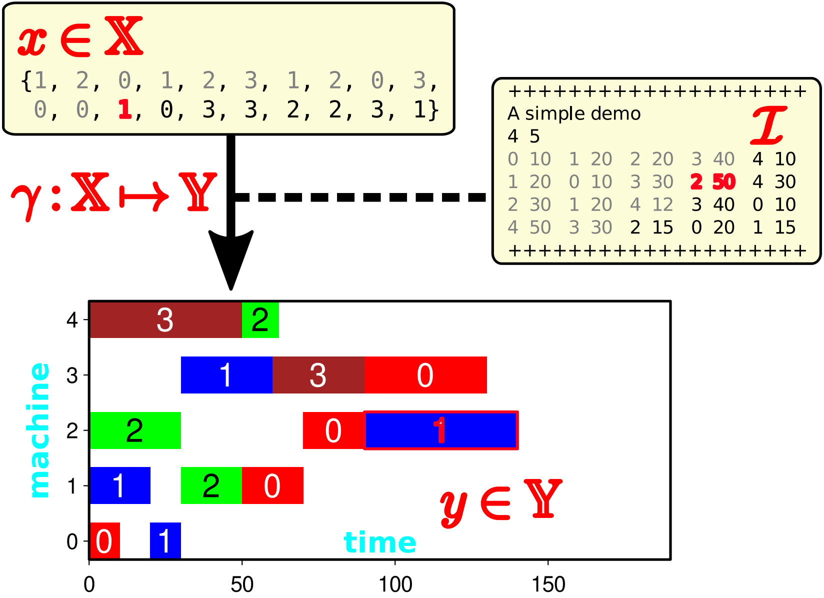

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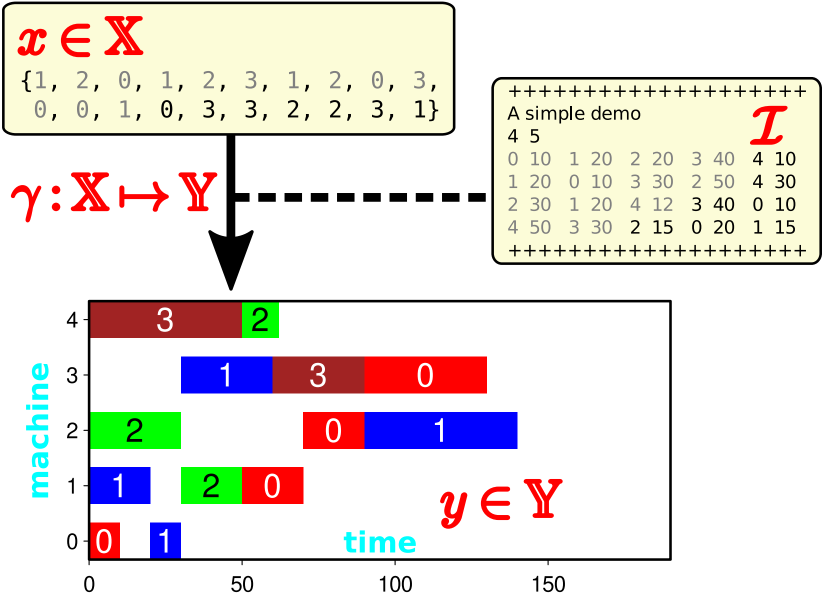

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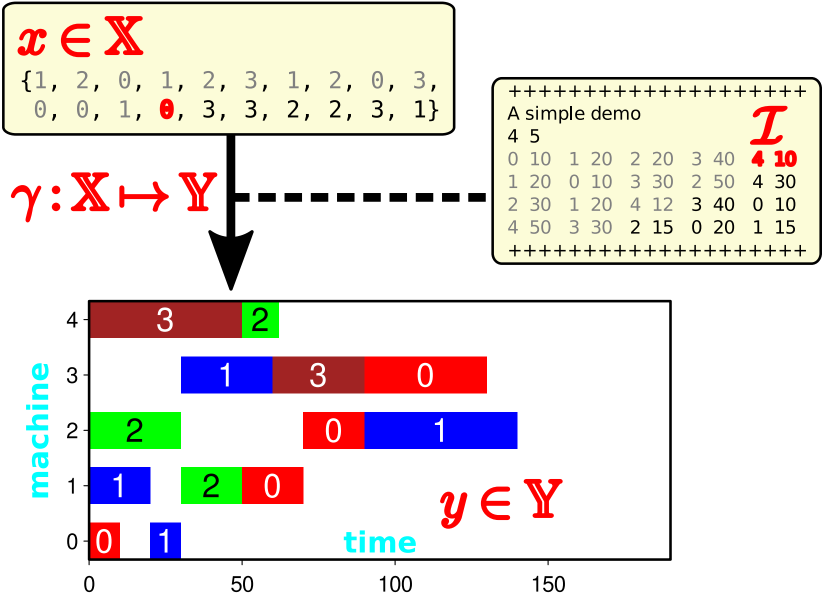

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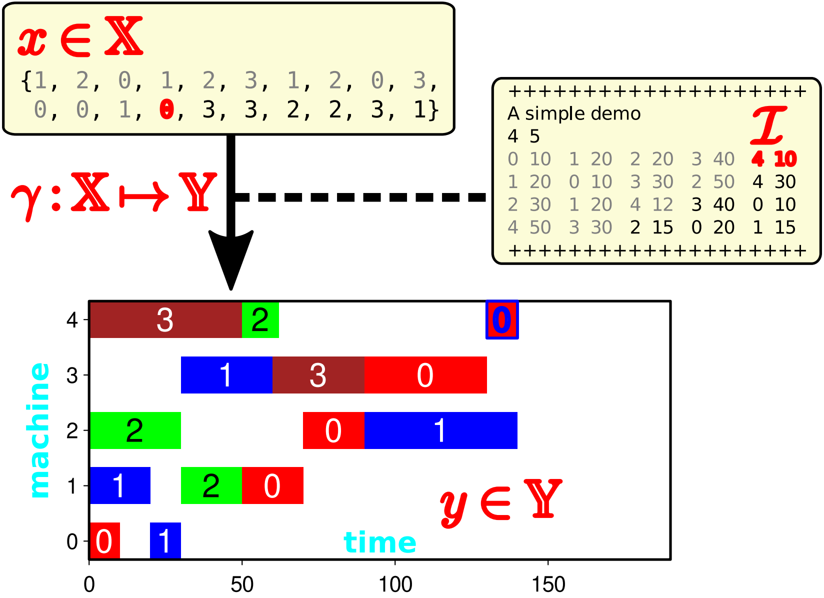

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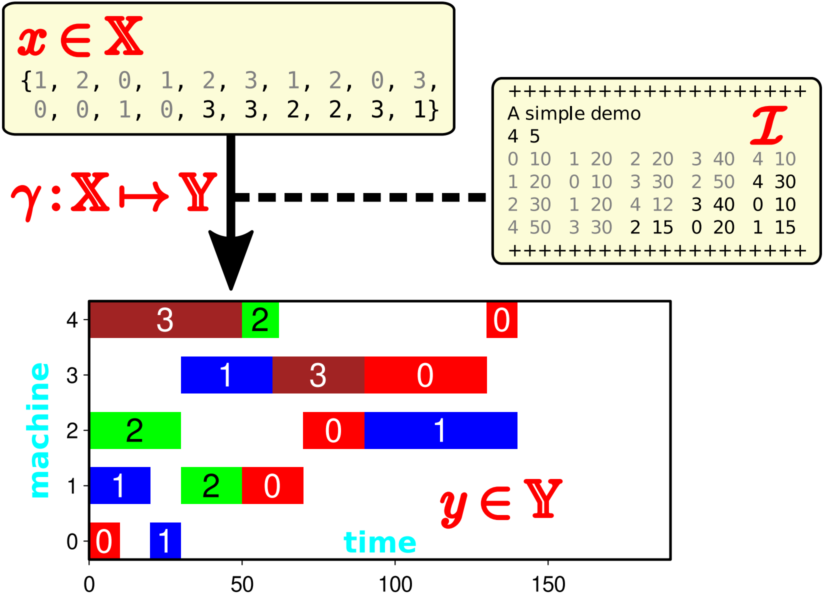

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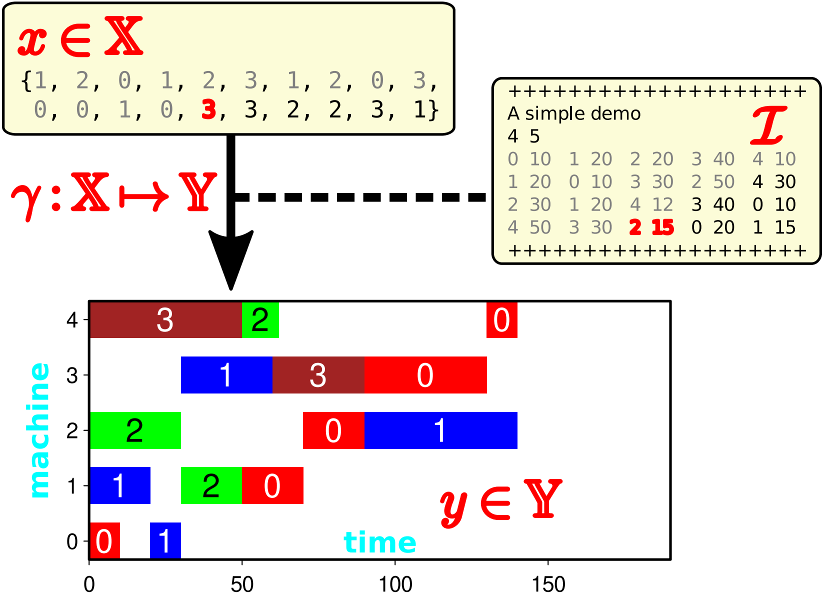

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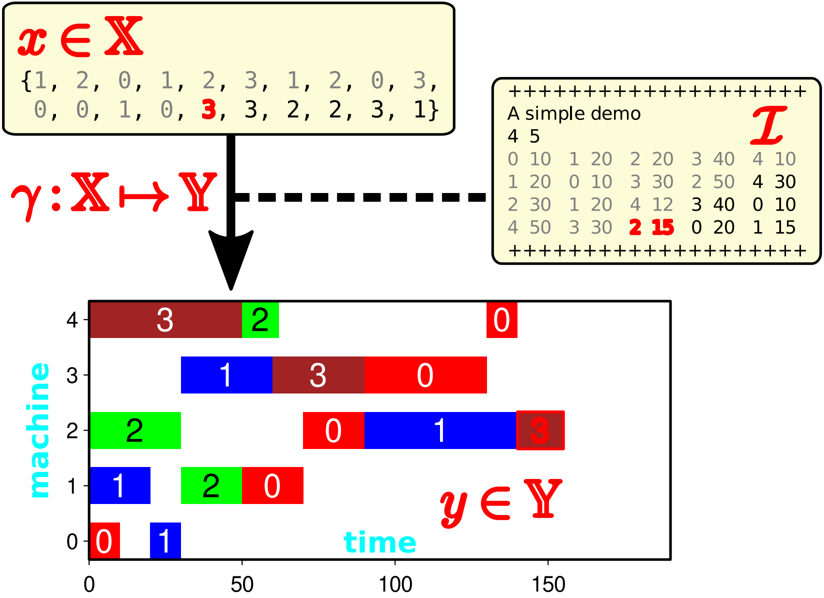

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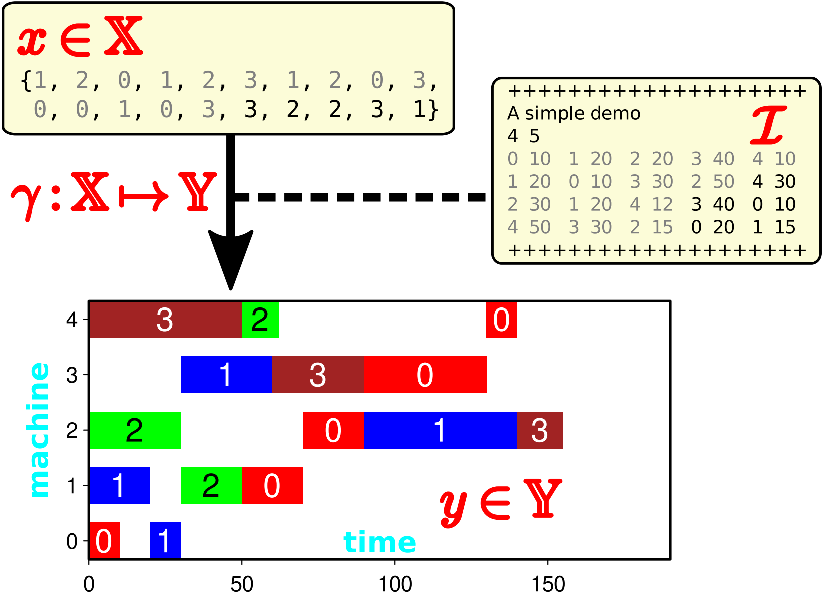

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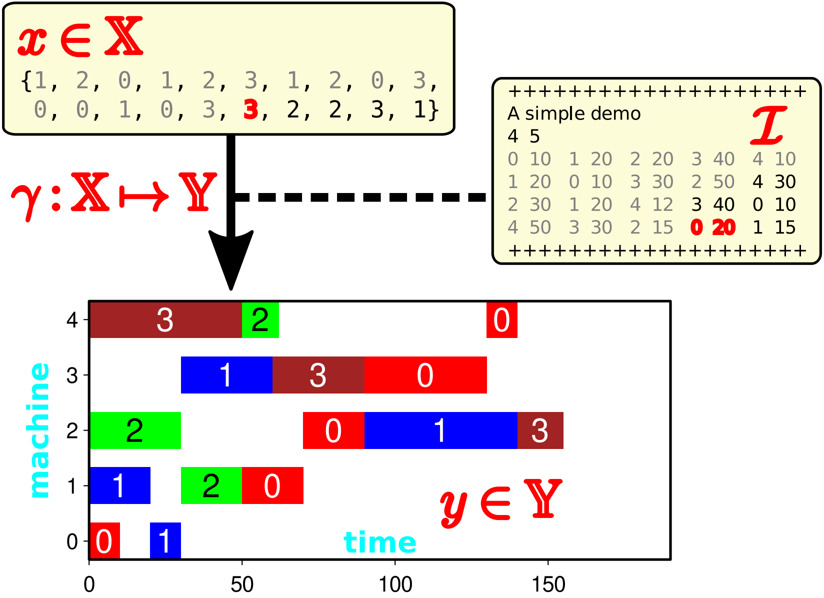

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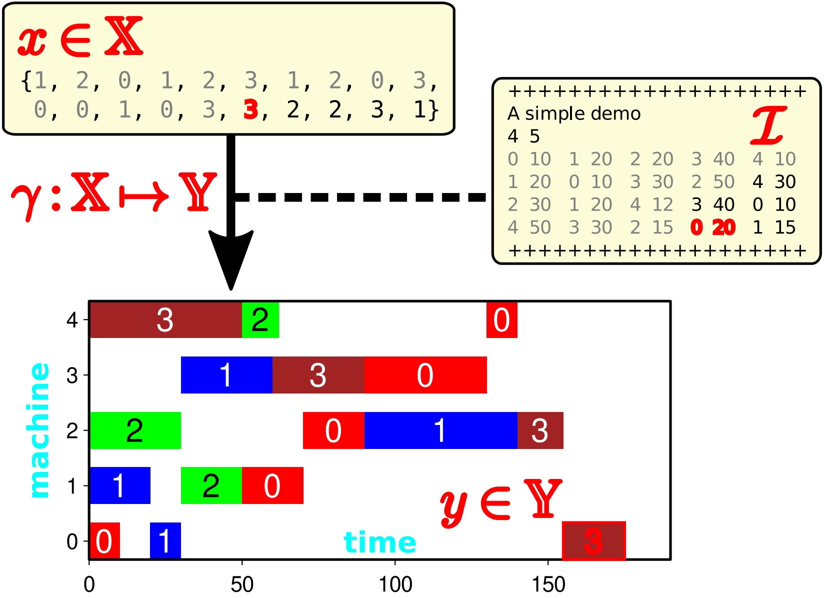

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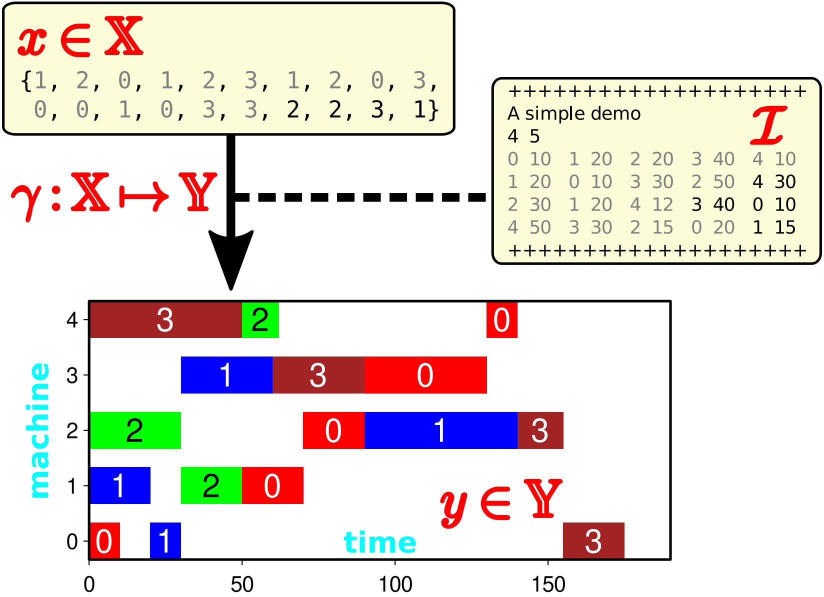

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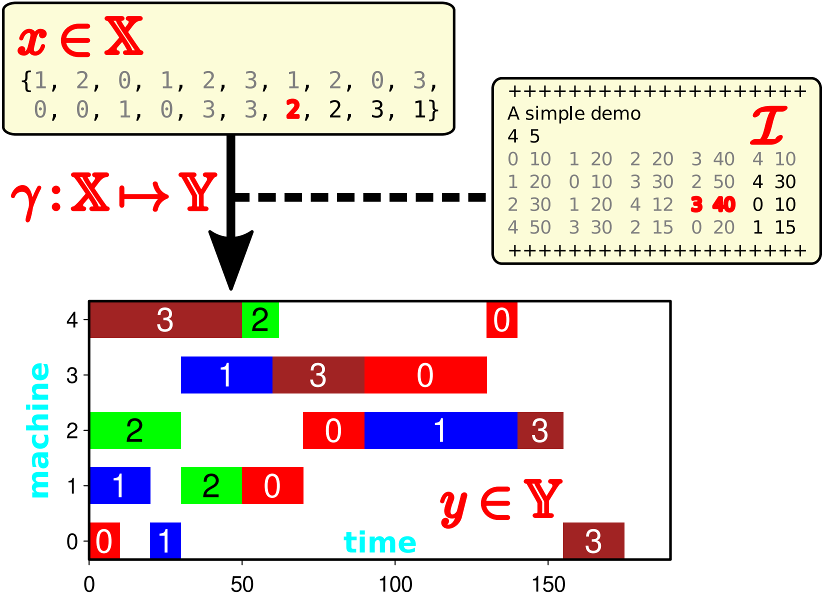

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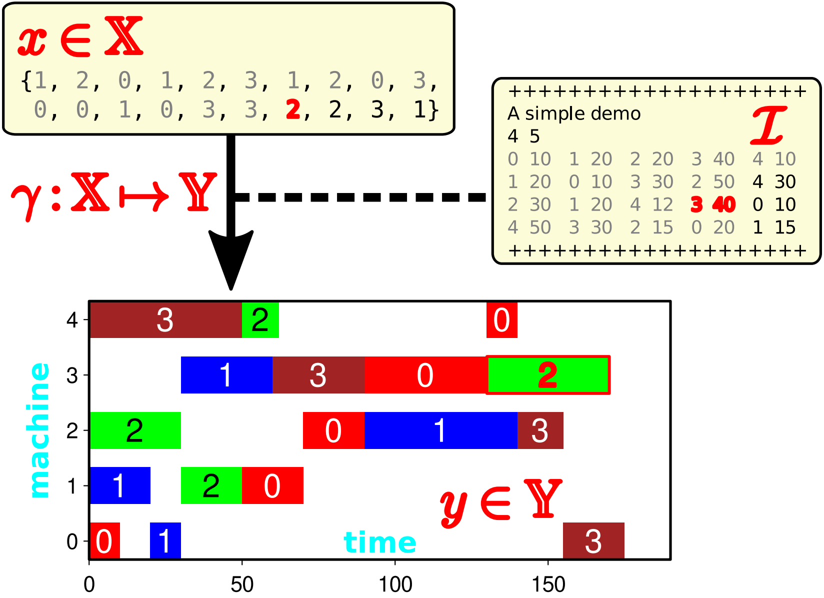

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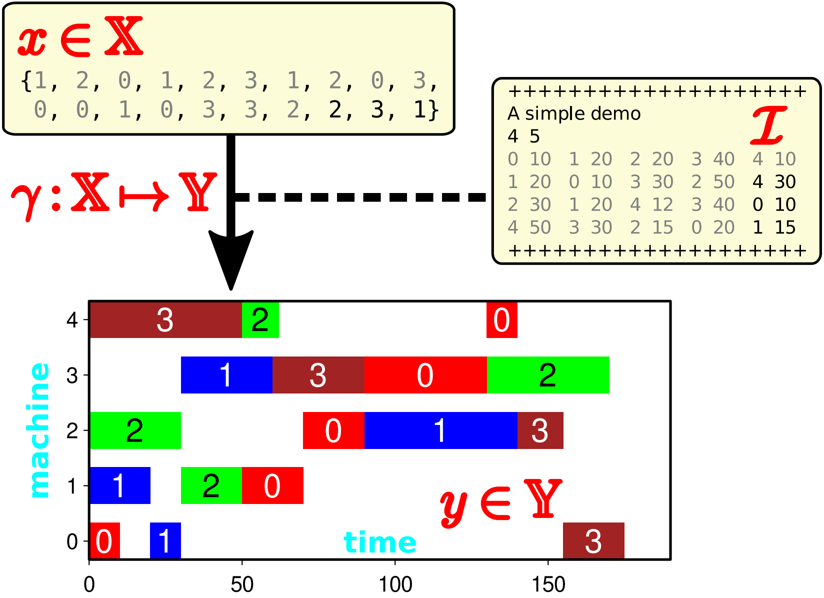

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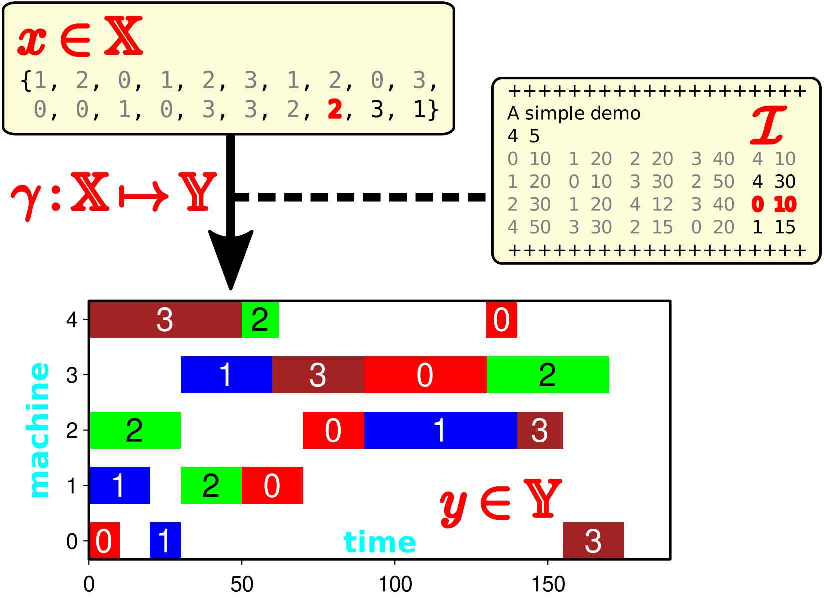

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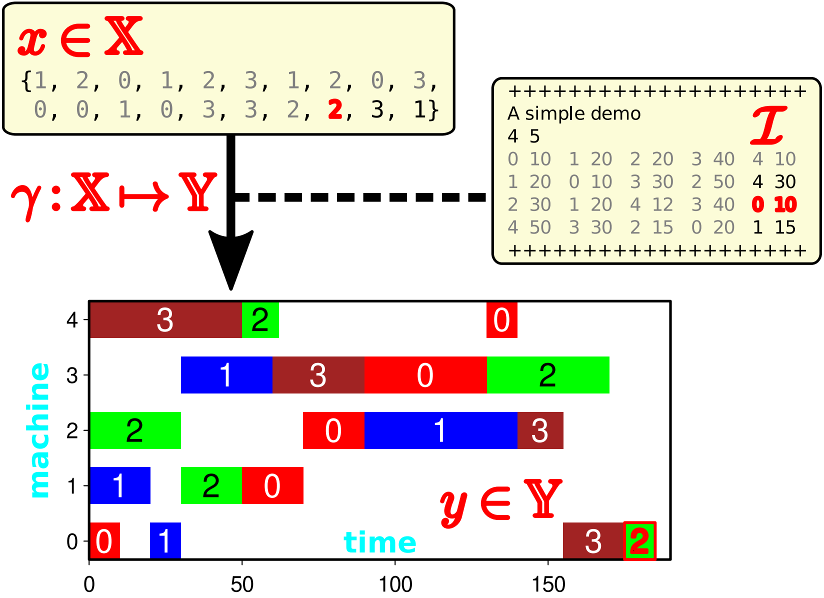

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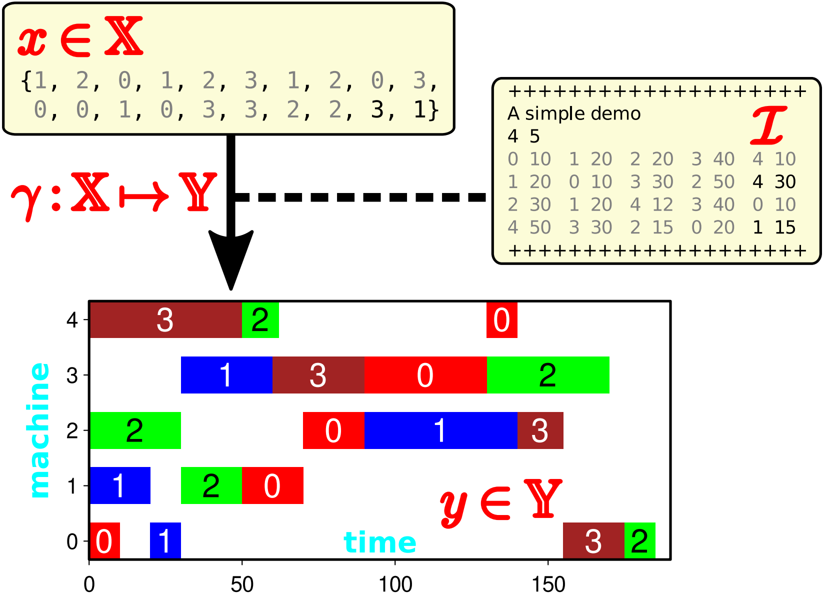

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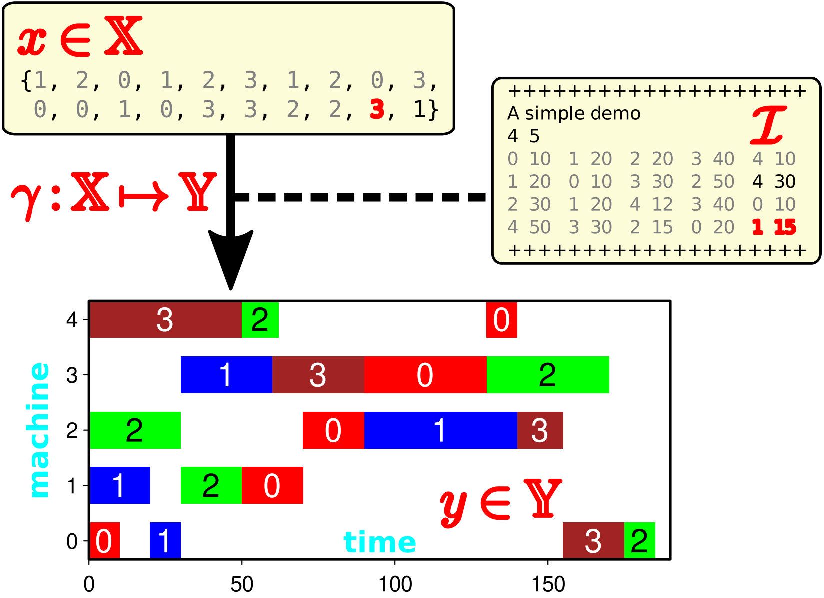

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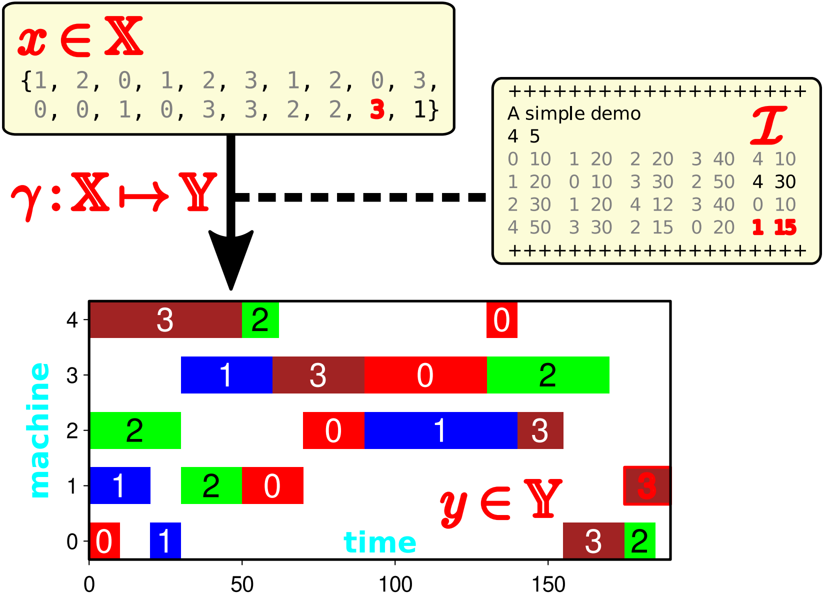

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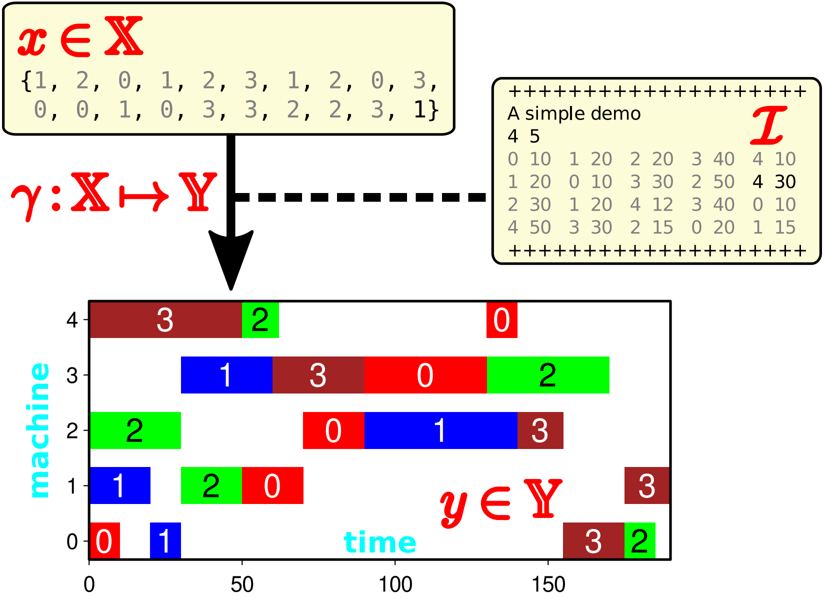

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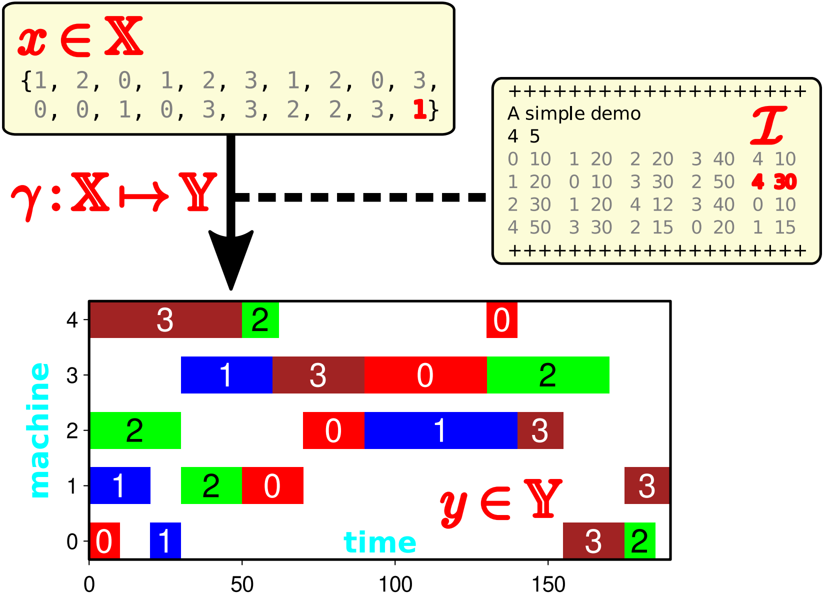

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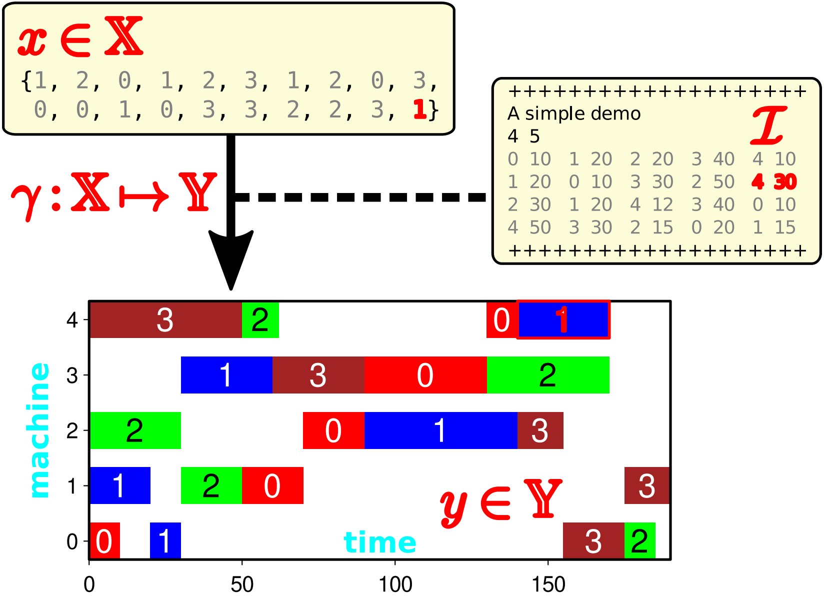

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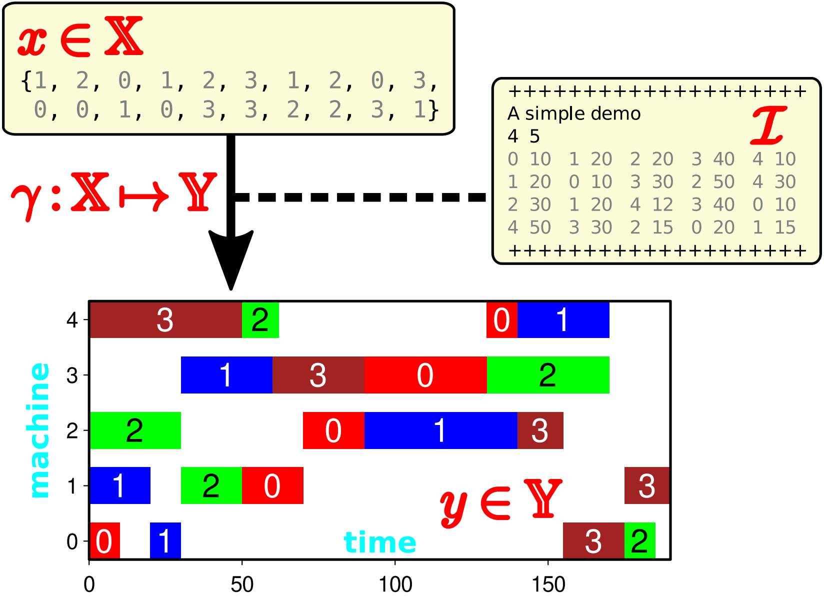

The Search Space $\mathbb{X}$

- We now have search space $\mathbb{X}$ with which we can easily represent all reasonable Gantt charts.

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- We call this the representation.
- If necessary, we could also easily add more constraints, such as job-order specific machine setup times, or job/machine specific transport times – they would all go into the mapping γ.

An Interface for Representation Mappings in Java

```java
package aitoa.structure;

public interface IRepresentationMapping<X, Y> {

  void map(X x, Y y);

}
```

The JSSP Representation Mapping in Java

```java
package aitoa.examples.jssp;

public class JSSPRepresentationMapping {
//
// omitted useless stuff, like member variable "instance"
//
//
//
//
//

//
//
//
//

//
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//

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} // end abridged class
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    int[] jobState     = new int[this.instance.n]; // filled with 0. Then we avoid
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// omitted useless stuff, like member variable "instance"
  public void map(int[] x, JSSPCandidateSolution y) {
    int[] machineState = new int[this.instance.m]; // These variables can be member
    int[] machineTime  = new int[this.instance.m]; // variables that only need to be
    int[] jobState     = new int[this.instance.n]; // filled with 0. Then we avoid
    int[] jobTime      = new int[this.instance.n]; // allocating them each time.

    for (int nextJob : x) { // iterate over job IDs in x
      int[] jobSteps = this.instance.jobs[nextJob]; // get the operations of the job
      int   jobStep  = (jobState[nextJob]++) << 1;  // 2*(increased job step index)
      int   machine  = jobSteps[jobStep];           // get the machine to use

      int   start      = Math.max(machineTime[machine], jobTime[nextJob]);
      int   end        = start + jobSteps[jobStep + 1]; // begin + operation time
      jobTime[nextJob] = machineTime[machine] = end;

      int[] schedule = y.schedule[machine]; // get list of tasks for machine
      schedule[machineState[machine]++] = nextJob; // store job
      schedule[machineState[machine]++] = start;   // store start time
      schedule[machineState[machine]++] = end;     // store end time
    } // end iteration over job IDs
  } // end function
} // end abridged class
```

Number of Possible Solutions

Number of Solutions: Size of $\mathbb{Y}$

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- For three machines, we are at $(n!)^3$.

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- For m machines, we are at $(n!)^m$ possible solutions.
- But some may be wrong, i.e., contain deadlocks!

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name	n	m	$\min(\#\text{feasible})$	$\lvert\mathbb{Y}\rvert$
	2	2	3	4

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name	n	m	$\min(\#\text{feasible})$	$\lvert\mathbb{Y}\rvert$
	2	2	3	4
	2	3	4	8

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name	n	m	$\min(\#\text{feasible})$	$\lvert\mathbb{Y}\rvert$
	2	2	3	4
	2	3	4	8
	2	4	5	16

Number of Solutions: Size of $\mathbb{Y}$

name	n	m	$\min(\#\text{feasible})$	$|\mathbb{Y}|$
	2	2	3	4
	2	3	4	8
	2	4	5	16
	2	5	6	32

Number of Solutions: Size of $\mathbb{Y}$

name	n	m	$\min(\#\text{feasible})$	$\lvert\mathbb{Y}\rvert$
	2	2	3	4
	2	3	4	8
	2	4	5	16
	2	5	6	32
	3	2	22	36
	3	3	63	216
	3	4	147	1'296
	3	5	317	7'776
	4	2	244	576
	4	3	1'630	13'824
	4	4	7'451	331'776

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name	n	m	$\lvert\mathbb{Y}\rvert$	$\lvert\mathbb{X}\rvert$
	3	2	36	90
	3	3	216	1'680
	3	4	1'296	34'650
	3	5	7'776	756'756
	4	2	576	2'520
	4	3	13'824	369'600
	4	4	331'776	63'063'000
	5	2	14'400	113'400
	5	3	1'728'000	168'168'000
	5	4	207'360'000	305'540'235'000
	5	5	24'883'200'000	623'360'743'125'120
`demo`	4	5	7'962'624	11'732'745'024
`la24`	15	10	$\approx 1.462{*}10^{121}$	$\approx 2.293{*}10^{164}$
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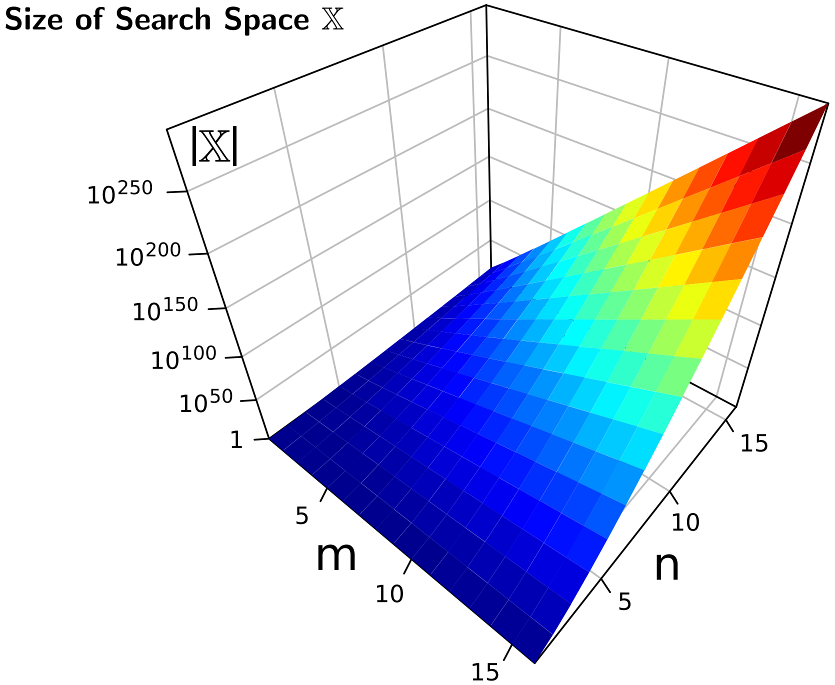
`swv15`	50	10	$\approx 6.772{*}10^{644}$	$\approx 1.254{*}10^{806}$



Size of Search Space $\mathbb{X}$
$|\mathbb{X}|$
10^{250}
10^{200}
10^{150}
10^{100}
10^{50}
1
5
m
10
15
15
10
n
5

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- $\mathbb{X}$ is bigger, we pay with size for the simplicity and the avoidance of infeasible solutions.

Search Operators

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```
package aitoa.structure;

public interface IUnarySearchOperator<X> {

  void apply(X x, X dest, Random random);

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- We will discuss concrete implementations of the operators later.

Termination

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- This is called the termination criterion.

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- Obviously, in other scenarios, there might be vastly different criteria...

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  5. Understand when we need to stop the search!
- If we have this, we can directly use most of the algorithms in the rest of the lecture (almost) as-is.

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- Let us now try to solve the JSSP using metaheuristics that search inside $\mathbb{X}$ (and thus can find solutions in $\mathbb{Y}$).

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- Many other problems are similar and can be represented in a similar way.
- The key is often to translate the complicated task to work with a complex data structure $\mathbb{Y}$ (e.g., Gantt diagram with many constraints) to a simpler scenario where I only need to deal with a basic data structure $\mathbb{X}$ (like a list of integer numbers with few constraints) by putting the "complicated" rules into a mapping $\gamma : \mathbb{X} \mapsto \mathbb{Y}$.
- If I can do that, then from now on I do not need to worry about $\mathbb{Y}$ and its rules any more – I only need to work with $\mathbb{X}$, which is easier to understand and to program.
- Let us now try to solve the JSSP using metaheuristics that search inside $\mathbb{X}$ (and thus can find solutions in $\mathbb{Y}$ within 3 minutes).

谢谢

Thank you

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