合肥学院
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Optimization Algorithms

3. Metaheuristics

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Outline

Introduction

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- But how are they solved?
- Optimization problems are solved by optimization algorithms.
- Optimization algorithms can be divided into exact and heuristic methods.

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- Many exact methods can be halted before completing their run and they can then still provide an approximate solution (without the guarantee that it is optimal).

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- They either do not make any guarantees at all how good it will be or, sometimes, provide some bound guarantee (like: "This solution will not cost more than two times of the optimal cost.")
- Simple heuristics are usually tailor-made for specific problems, like the TSP or JSSP.

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- We explore them by using the Job Shop Scheduling Problem (JSSP)[6–10] as example.

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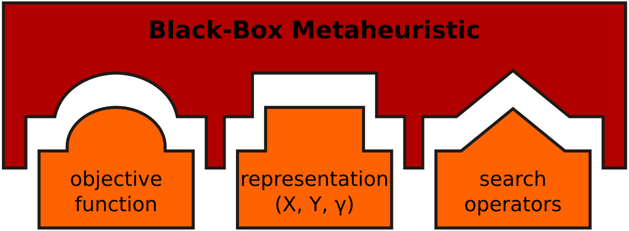

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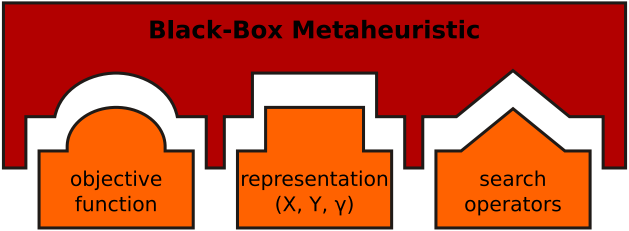

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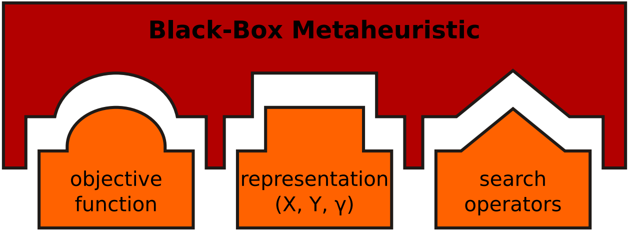

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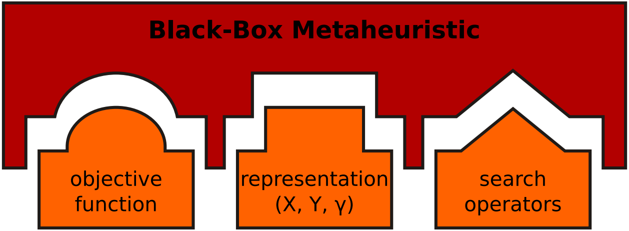

- The metaheuristic does not care (much) how the objective function $f : \mathbb{Y} \mapsto \mathbb{R}$ is shaped.
- It does not care much about the structure of $\mathbb{Y}$, $\mathbb{X}$, and γ either.
- We "plug them in" together with the search operators (about which we will talk later), and the metaheuristic will "work."

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- Matter of fact: Metaheuristics could be used for any of the above tasks!
- The metaheuristics are general algorithms into which a representation fitting any of these tasks can be "plugged."

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- For this, I provide one abstraction: the interface `IBlackBoxProcess`.
- I will not discuss here how exactly it is implemented, but we will take a quick peek on what it can do.

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- represents a termination criterion (e.g., maximum FEs, maximum runtime, reaching a goal objective value), and
- logs the improvements that the algorithm makes to a text file, so that we can use them to make tables and draw diagrams.

IBlackBoxProcess

```
package aitoa.structure;

public interface IBlackBoxProcess<X, Y> extends
    IObjectiveFunction<X>, // evaluate works on x ∈ 𝕏 and performs γ
    ITerminationCriterion, // shouldTerminate() tells when to stop
    Closeable { // when closed, can write log file with trace

  Random getRandom(); // replicable random numbers
// ...
  double getBestF();     // get (current best or end) quality
  void getBestX(X dest); // get (current best or end) x ∈ 𝕏
  void getBestY(Y dest); // get (current best or end) y ∈ 𝕐
// ...
  long getConsumedFEs();       // get number of calls to evaluate
  long getLastImprovementFE(); // get last FE when improved

/** Some stuff that is not relevant here has been omitted.
    You can find it in the full code online. */
}
```

Summary

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- Now we finally have all the components together to implement metaheuristic optimization algorithms!

谢谢

Thank you

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