Optimization Algorithms

3. Metaheuristics

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2. Black-Box Characteristic
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Introduction
• OK, by now we know already something about optimization problems and how we can give them a “structure.”
Optimization Algorithms

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- But how are they solved?
- Optimization problems are solved by optimization algorithms.
- Optimization algorithms can be divided into exact and heuristic methods.
Exact Algorithms

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• Many exact methods can be halted before completing their run and they can then still provide an approximate solution (without the guarantee that it is optimal).
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• Simple heuristics are usually tailor-made for specific problems, like the TSP or JSSP.
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• We explore them by using the Job Shop Scheduling Problem (JSSP)\textsuperscript{6–10} as example.
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Black-Box Optimization

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We “plug them in” together with the search operators (about which we will talk later), and the metaheuristic will “work.”
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- Metaheuristics are nothing like that.
- Matter of fact: Metaheuristics could be used for any of the above tasks!
- The metaheuristics are general algorithms into which a representation fitting any of these tasks can be “plugged.”
Implementing the Black Box Idea

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• For this, I provide one abstraction: the interface \texttt{IBlackBoxProcess}.
• I will not discuss here how exactly it is implemented, but we will take a quick peek on what it can do.
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• logs the improvements that the algorithm makes to a text file, so that we can use them to make tables and draw diagrams.
package aitoa.structure;

public interface IBlackBoxProcess<X, Y> extends IObjectiveFunction<X>, ITerminationCriterion, Closeable {
    // when closed, can write log file with trace

    Random getRandom(); // replicable random numbers

    double getBestF(); // get (current best or end) quality
    void getBestX(X dest); // get (current best or end) \( x \in X \)
    void getBestY(Y dest); // get (current best or end) \( y \in Y \)

    long getConsumedFEs(); // get number of calls to evaluate
    long getLastImprovementFE(); // get last FE when improved

    /** Some stuff that is not relevant here has been omitted. 
     * You can find it in the full code online. */
}
Summary
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• Now we finally have all the components together to implement metaheuristic optimization algorithms!
Thank you
References


