合肥学院
HEFEI UNIVERSITY

Optimization Algorithms

4. Random Sampling

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Outline

Introduction

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  1. put each of the numbers from 0 to $n-1$ exactly m times in an integer array of length $m * n$ (so we have a valid point $x_0 \in \mathbb{X}$), then
  2. randomly shuffle the values like a deck of cards (so we get a random valid point $x \in \mathbb{X}$), and
  3. apply the representation mapping γ to get a Gantt chart $y = \gamma(x)$, $y \in \mathbb{Y}$.

Algorithm Concept

Interface for a Function to Sample 1 Point from $\mathbb{X}$

- We already have the interface that we need to implement to do such a thing

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```
package aitoa.structure;

public interface INullarySearchOperator<X> {

  void apply(X dest, Random random);

}
```

Implementation: Create Random Point in $\mathbb{X}$

```
public class JSSPNullaryOperator {
//
// unnecessary stuff omitted here...
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//
}
```

Implementation: Create Random Point in $\mathbb{X}$

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public class JSSPNullaryOperator implements
    INullarySearchOperator<int[]> {
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//
//
  }
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```

Implementation: Create Random Point in $\mathbb{X}$

```java
public class JSSPNullaryOperator implements
    INullarySearchOperator<int[]> {
// unnecessary stuff omitted here...
  public void apply(int[] dest, Random random) {
// fill first part of array with 0, 1, 2, ..., n
//
//
//
//
//
//
//
//
//
//
//
//
//
//
  }
}
```

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    INullarySearchOperator<int[]> {
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  public void apply(int[] dest, Random random) {
// fill first part of array with 0, 1, 2, ..., n
    for (int i = this.n; (--i) >= 0;) {
      dest[i] = i;
    }
//
//
//
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//
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    }
// copy this part m-1 times
//
//
//
//
//
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    for (int i = this.n; (--i) >= 0;) {
      dest[i] = i;
    }
// copy this part m-1 times
    for (int i = dest.length; (i -= this.n) > 0;) {
      System.arraycopy(dest, 0, dest, i, this.n);
    }
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// copy this part m-1 times
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      System.arraycopy(dest, 0, dest, i, this.n);
    }
// randomly shuffle the array: Fisher-Yates shuffle[3,4]
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    }
// copy this part m-1 times
    for (int i = dest.length; (i -= this.n) > 0;) {
      System.arraycopy(dest, 0, dest, i, this.n);
    }
// randomly shuffle the array: Fisher-Yates shuffle[3,4]
    for (int i = dest.length; i > 1;) {
//
//
//
//
    }
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    }
// copy this part m-1 times
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      System.arraycopy(dest, 0, dest, i, this.n);
    }
// randomly shuffle the array: Fisher-Yates shuffle[3,4]
    for (int i = dest.length; i > 1;) {
      int j    = random.nextInt(i--);
      int t    = array[i];
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    }
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    }
// copy this part m-1 times
    for (int i = dest.length; (i -= this.n) > 0;) {
      System.arraycopy(dest, 0, dest, i, this.n);
    }
// randomly shuffle the array: Fisher-Yates shuffle[34]
    for (int i = dest.length; i > 1;) {
      int j     = random.nextInt(i--);
      int t     = array[i];
      array[i] = array[j];
      array[j] = t;
    } // implemented as RandomUtils.shuffle in code repo
  }
}
```

Implementation: Single Random Sampling Algorithm

```java
package aitoa.algorithms;

public class SingleRandomSample<X, Y> {
//
// unnecessary stuff (e.g., constructor) omitted here...
//
//

//

//
//
}
```

Implementation: Single Random Sampling Algorithm

```
package aitoa.algorithms;

public class SingleRandomSample<X, Y> extends
    Metaheuristic0<X, Y> {
// unnecessary stuff (e.g., constructor) omitted here...
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  public void solve(IBlackBoxProcess<X, Y> process) {
    X x = process.getSearchSpace().create(); // allocate

    this.nullary.apply(x, process.getRandom());

    process.evaluate(x); // evaluate
  }
}
```

Experiment and Analysis

So what do we get?

- I execute the program 101 times for each of the instances `abz7`, `la24`, `swv15`, and `yn4`

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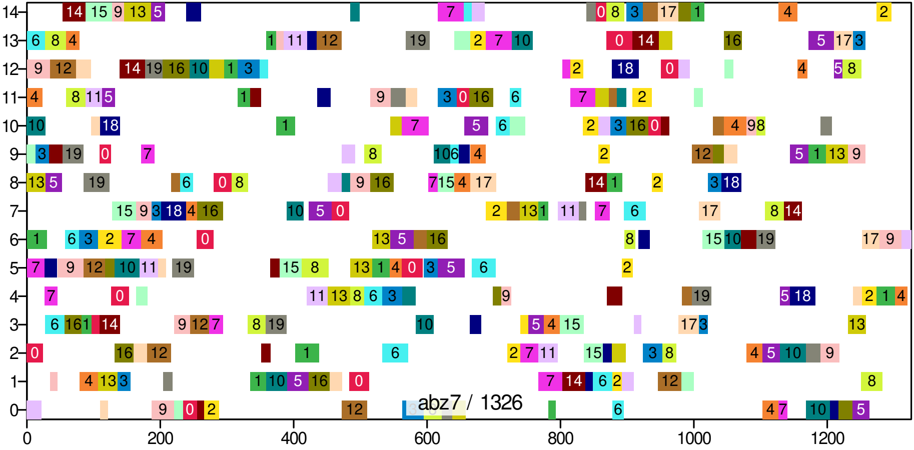
	makespan				last improvement	
$\mathcal{I}$	best	mean	med	sd	med(t)	med(FEs)
`abz7`	1'131	1'334	1'326	106	0s	1
`la24`	1'487	1'842	1'814	165	0s	1
`swv15`	5'935	6'600	6'563	346	0s	1
`yn4`	1'754	2'036	2'039	125	0s	1

So what do we get?

Median solution for abz7



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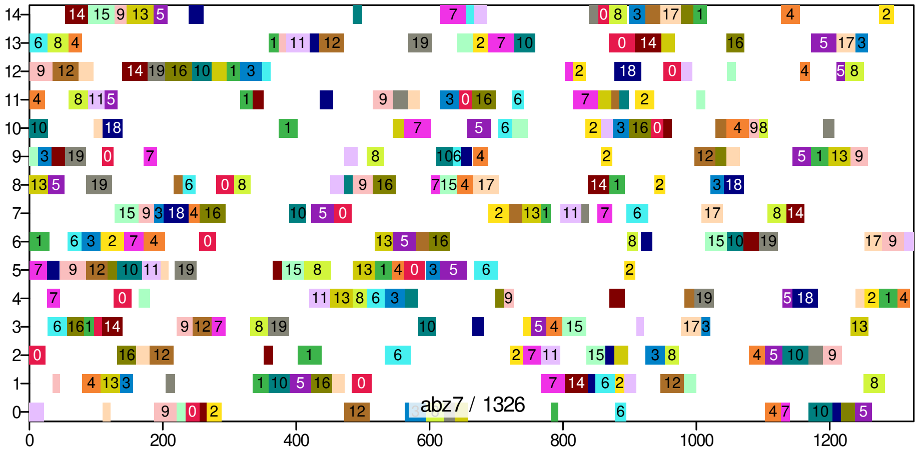

. . . there is lots of white space between the operations. . .

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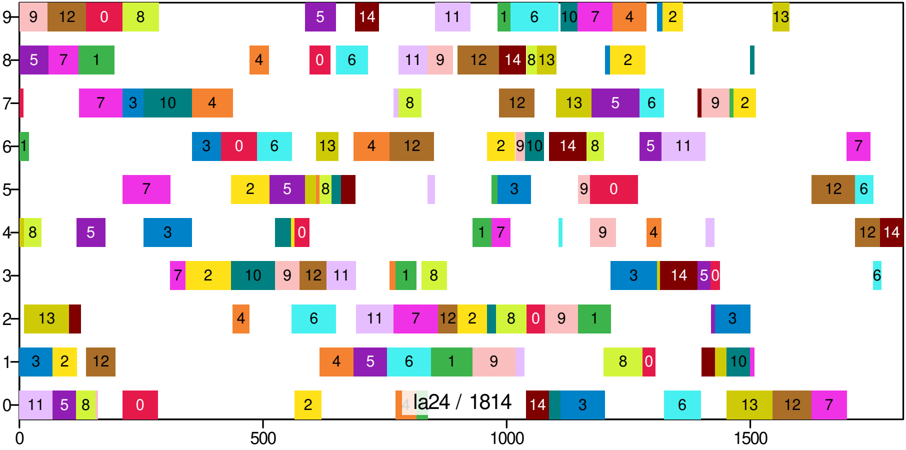
Median solution for `la24`



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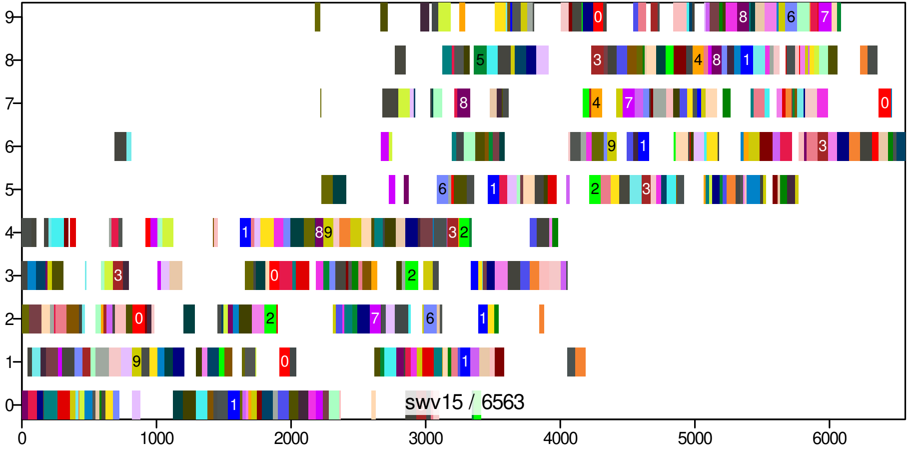
Median solution for `swv15`



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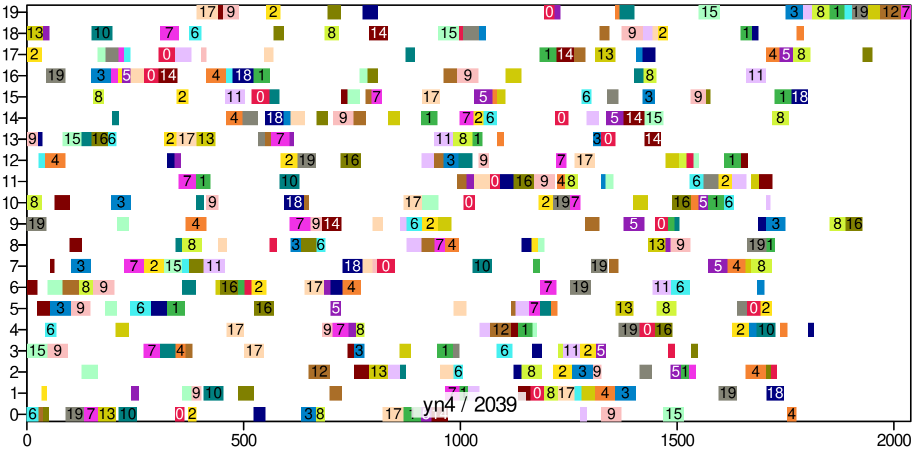
Median solution for `yn4`



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- The results are not good, there is lots of white space $\equiv$ wasted time.

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- The results are not good, there is lots of white space $\equiv$ wasted time. That was expected: Our solutions are random.
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- Notice 2. There is a high variance in the results due to randomness.

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Improved Algorithm Concept

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    1. create new random candidate solution y (via random sampling from the search space)

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- It works as follows:
  1. create new random candidate solution y (via random sampling from the search space)
  2. remember best solution ever encountered
  3. repeat until 3 min are exhausted

Random Sampling Algorithm

```
package aitoa.algorithms;

public class RandomSampling<X, Y> {
//
// unnecessary stuff (e.g., constructor) omitted here...
//
//

//

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    do {
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    } while (!process.shouldTerminate());
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Experiment and Analysis 2

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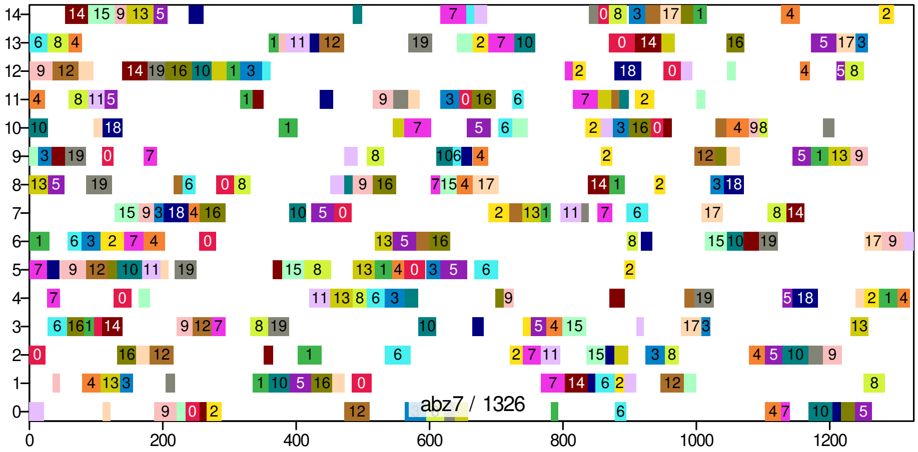
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
`abz7`	`1rs`	1131	1334	1326	106	0s	1
	`rs`	**895**	**947**	**949**	**12**	85s	6'512'505
`la24`	`1rs`	1487	1842	1814	165	0s	1
	`rs`	**1153**	**1206**	**1208**	**15**	82s	15'902'911
`swv15`	`1rs`	5935	6600	6563	346	0s	1
	`rs`	**4988**	**5166**	**5172**	**50**	87s	5'559'124
`yn4`	`1rs`	1754	2036	2039	125	0s	1
	`rs`	**1460**	**1498**	**1499**	**15**	76s	4'814'914

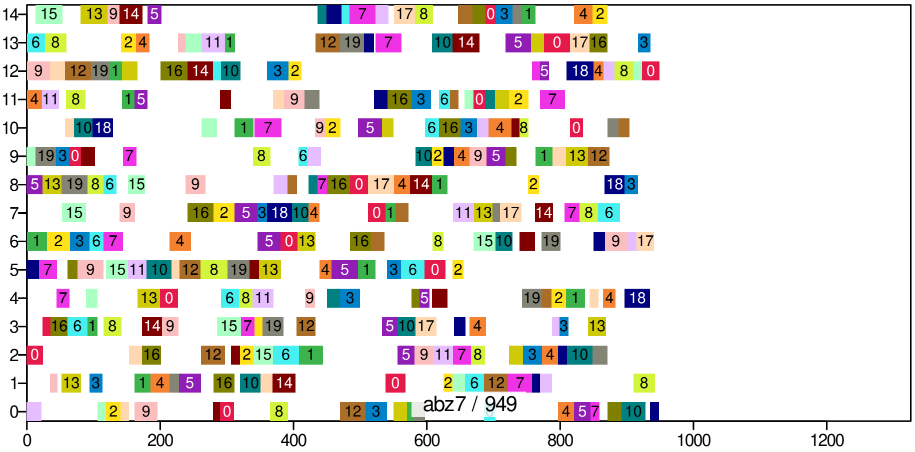
So what do we get?

`1rs`: median result of single random sample algorithm



So what do we get?

rs: median result of 3 min of random sampling algorithm



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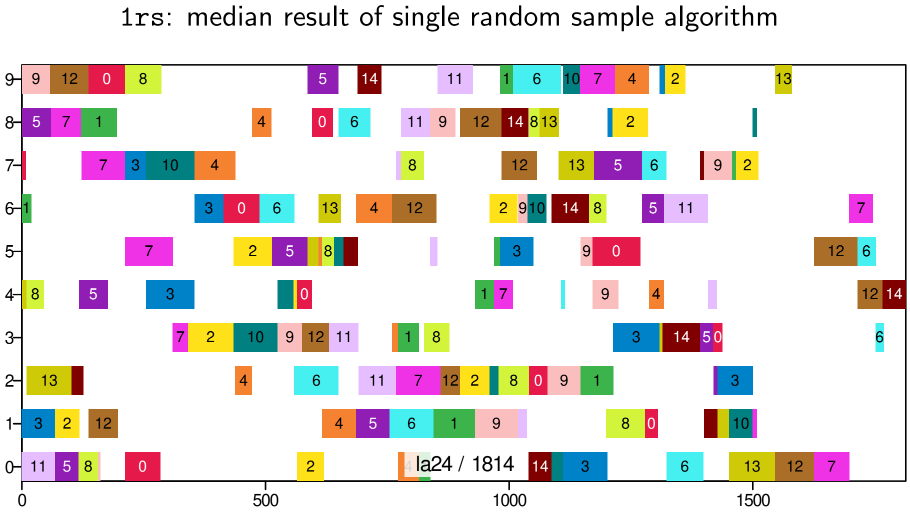

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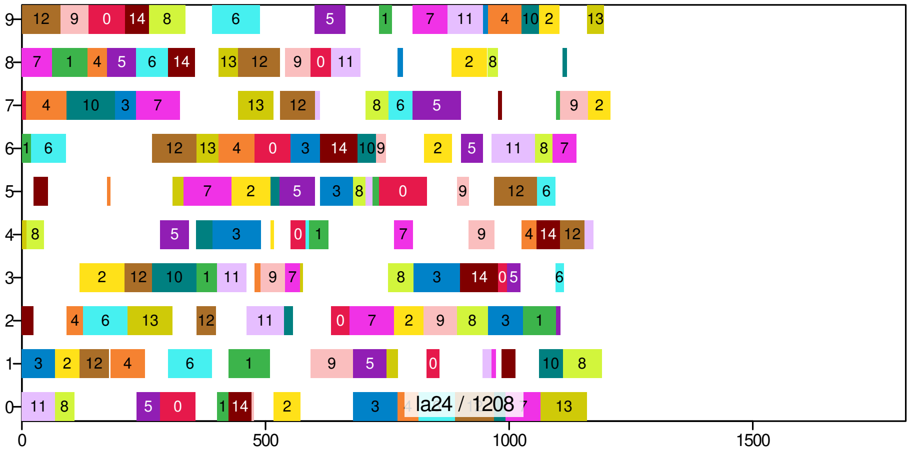

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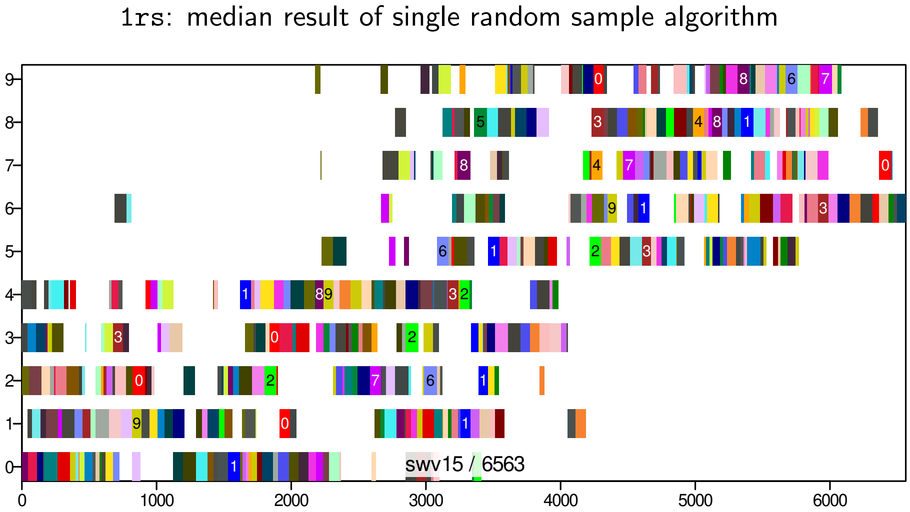

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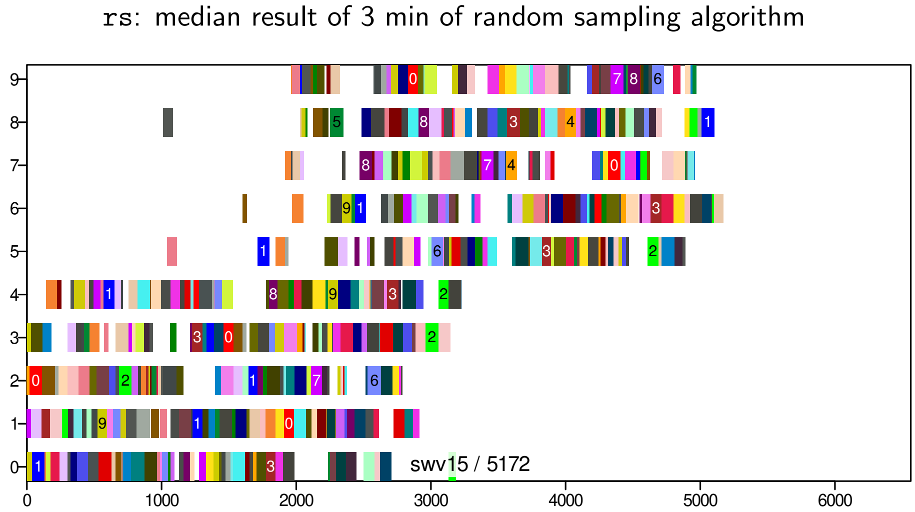
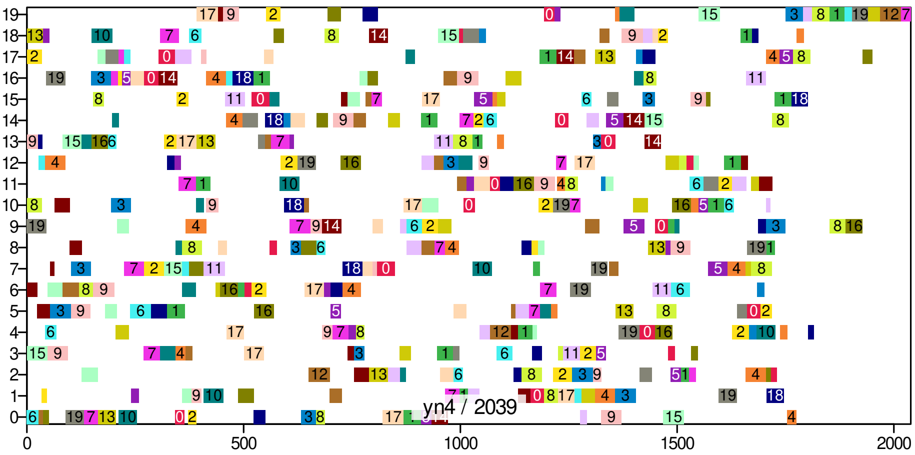


So what do we get?

`1rs`: median result of single random sample algorithm



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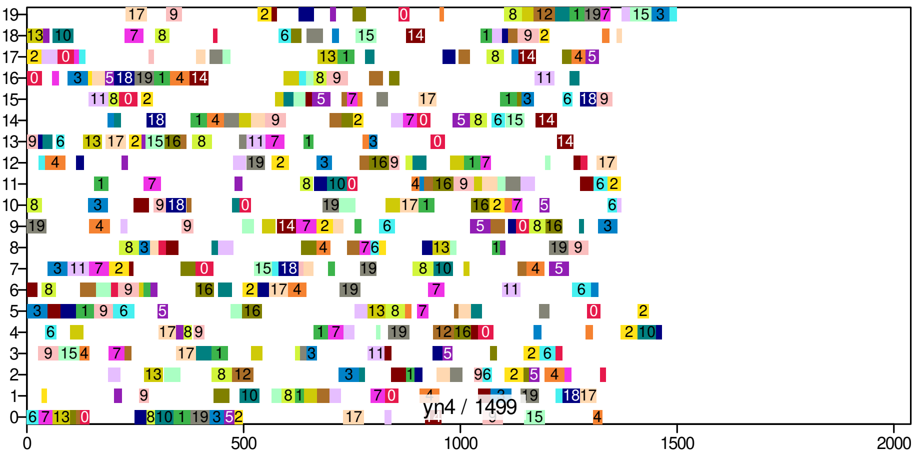
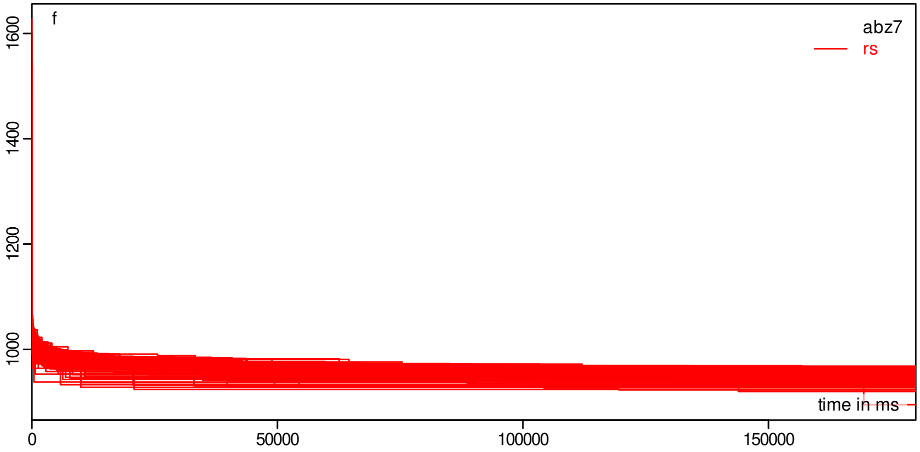
`rs`: median result of 3 min of random sampling algorithm



Progress over Time

What progress does the algorithm make over time?

Progress over Time



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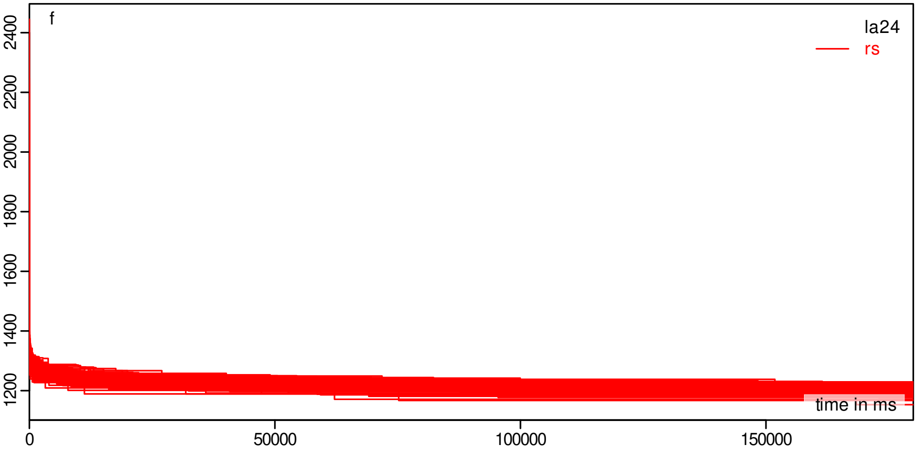

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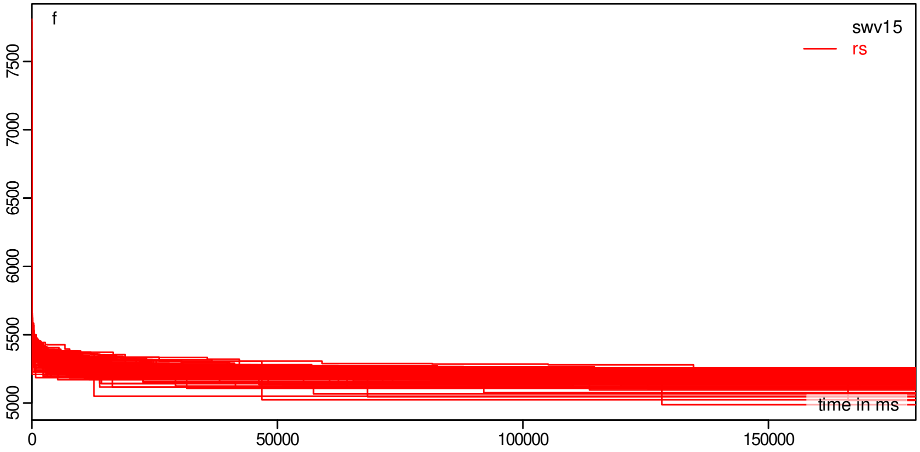

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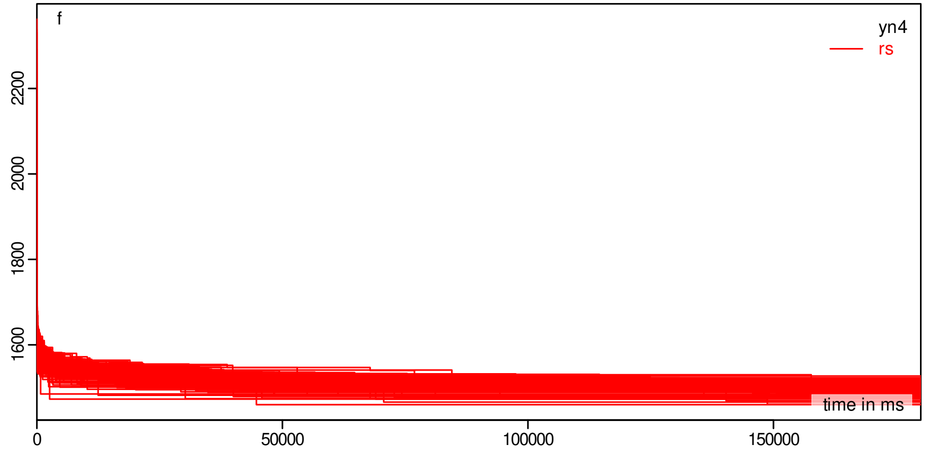

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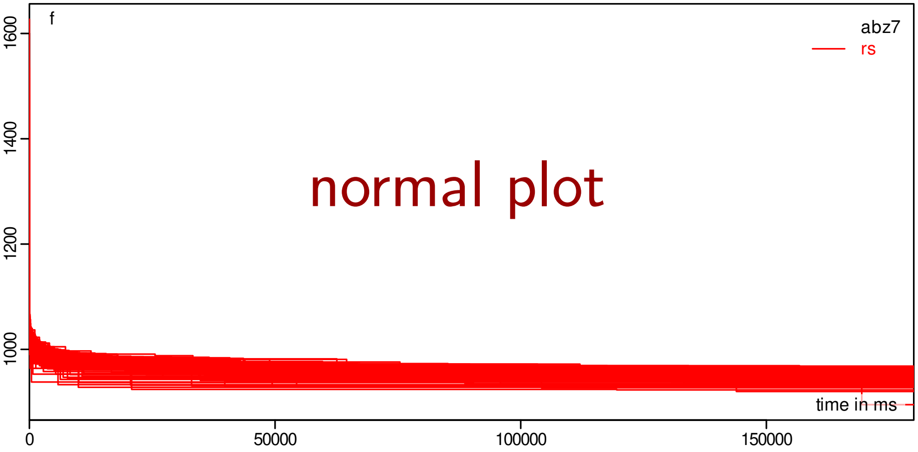

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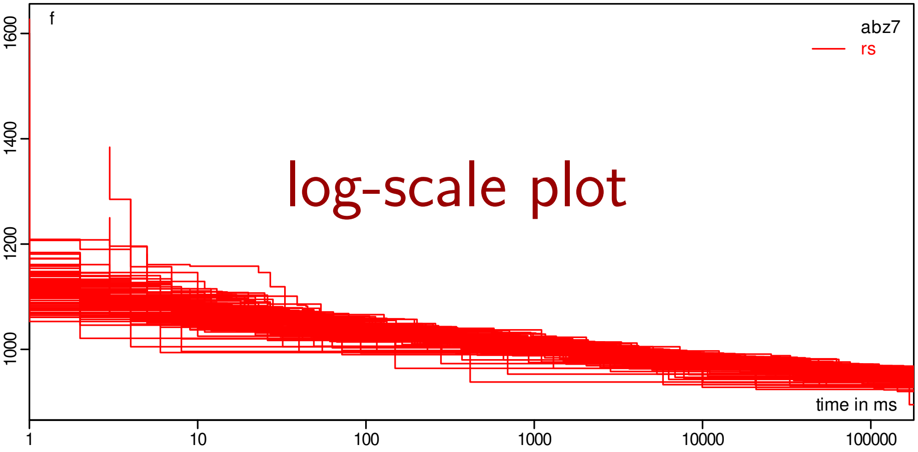

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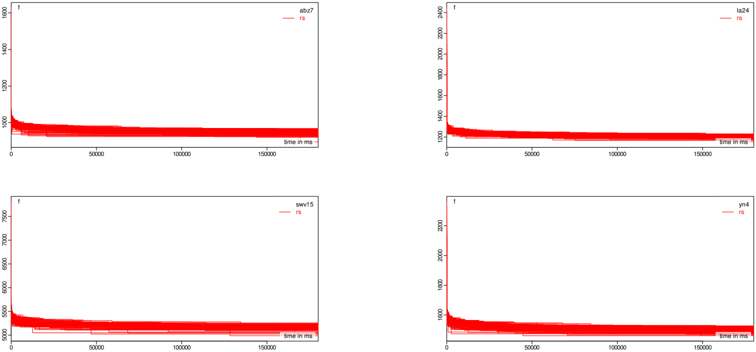

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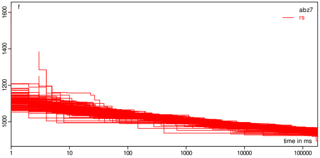

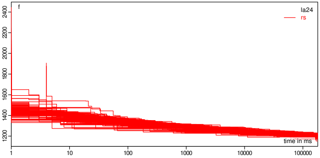

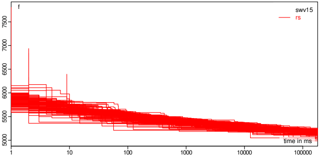

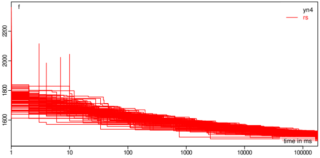

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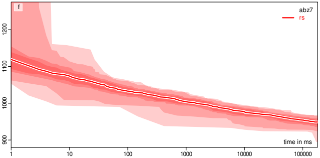
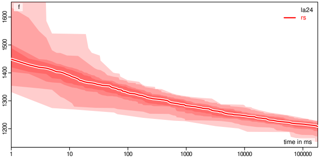
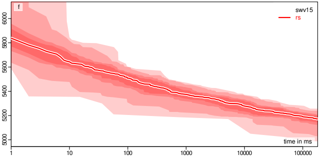
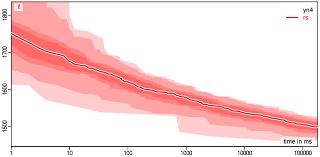
- This holds for runtime, but also for improvements of algorithms.









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    1. a first algorithm for solving optimization: random sampling.
    2. a tool to improve algorithm performance: restarts.
    3. an inherent nature of optimization processes: much progress early, fewer and smaller improvements later.

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谢谢

Thank you

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