合肥学院
HEFEI UNIVERSITY

Optimization Algorithms

5. Stochastic Hill Climbing

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Outline

Introduction

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- It does not make any use of the information it “sees” during the optimization process.
- Each search step consists of creating an entirely new, entirely random candidate solution.
- Each search step is thus independent of all prior steps.
- Is this a good idea?
- Probably not.
- In almost all practical scenarios, good solutions are somewhat similar to other good solutions.
- In other words, every good solution we see is some useful information.

Basic Idea

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Basic Idea

- So how we can make use of the information we have seen during the search?
- Instead of generating a completely random new candidate solution in each step...
- ...why can't we try to iteratively improve the best solution we have seen so far?

Algorithm Concept

Stochastic Hill Climbing

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    1. create random initial solution
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    3. if it is better, it becomes the new best-so-far solution (if it is not better, discard it).

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- Simple Concept:
  1. create random initial solution
  2. make a modified copy of best-so-far solution
  3. if it is better, it becomes the new best-so-far solution (if it is not better, discard it).
  4. go back to 2. (until the time is up)

Implementation of the Stochastic Hill Climber

```
package aitoa.algorithms;

public class HillClimber<X, Y> {
// unnecessary stuff omitted here...
//
//
//
//

//
//

//
//
//
//
//
//
//
//
//
}
```

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package aitoa.algorithms;

public class HillClimber<X, Y> extends Metaheuristic1<X, Y> {
// unnecessary stuff omitted here...
  public void solve(IBlackBoxProcess<X, Y> process) {
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      X xCur  = process.getSearchSpace().create();
//
//

//
//

//
//
//
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//
//
//
//
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      X xCur  = process.getSearchSpace().create();
      X xBest = process.getSearchSpace().create();
      Random random = process.getRandom();

//
//

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      Random random = process.getRandom();

      this.nullary.apply(xBest, random);
//

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     Random random = process.getRandom();

     this.nullary.apply(xBest, random);
     double fBest = process.evaluate(xBest);

//
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     this.nullary.apply(xBest, random);
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//
     this.unary.apply(xBest, xCur, random);
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//
      this.unary.apply(xBest, xCur, random);
      double fCur = process.evaluate(xCur);
      if (fCur < fBest) {
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      if (fCur < fBest) {
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      double fBest = process.evaluate(xBest);

//
      this.unary.apply(xBest, xCur, random);
      double fCur = process.evaluate(xCur);
      if (fCur < fBest) {
        fBest = fCur;
        process.getSearchSpace().copy(xCur, xBest);
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      double fBest = process.evaluate(xBest);

      while (!process.shouldTerminate()) {
        this.unary.apply(xBest, xCur, random);
        double fCur = process.evaluate(xCur);
        if (fCur < fBest) {
          fBest = fCur;
          process.getSearchSpace().copy(xCur, xBest);
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- Causality means that small changes in the features of an object (or candidate solution) also lead to small changes in its behavior (or objective value).
- If an optimization problem exhibits causality, then there should be good solutions that are similar to other good solutions.
- The idea is that if we have a good candidate solution, then there may exist similar solutions which are better.
- We hope to find one of them and then continue trying to do the same from there.

Ingredient: Unary Search Operator

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```
package aitoa.structure;

public interface IUnarySearchOperator<X> {

  void apply(X x, X dest, Random random);

}
```

Unary Search Operator

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- How can we implement this for our JSSP scenario?

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- How can we implement this for our JSSP scenario?
- Easy: Just swap two (different) job IDs in the string!
- Since the numbers of occurrences of the IDs will not change, the new strings will be valid.

Example for our `1swap` Operator

$\mathbb{X}$ (2,0,1,0,1,1,2,3,2,3,
2,0,0,1,3,3,2,3,1,0)

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$\downarrow \gamma$

$\mathbb{Y}$

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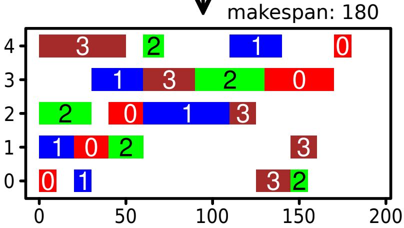
$\downarrow \gamma$

$\mathbb{Y}$



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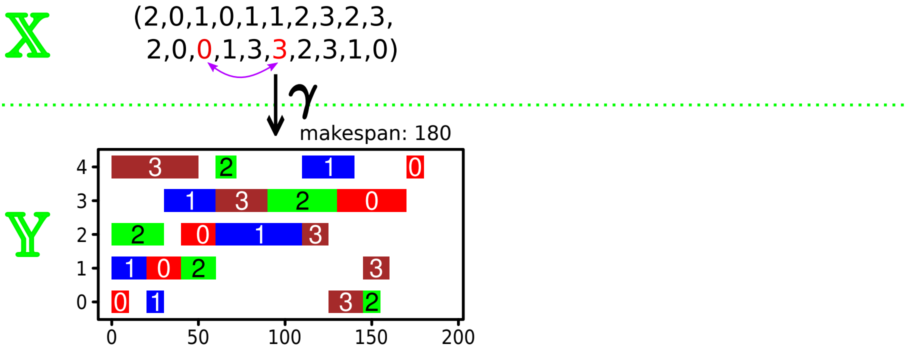

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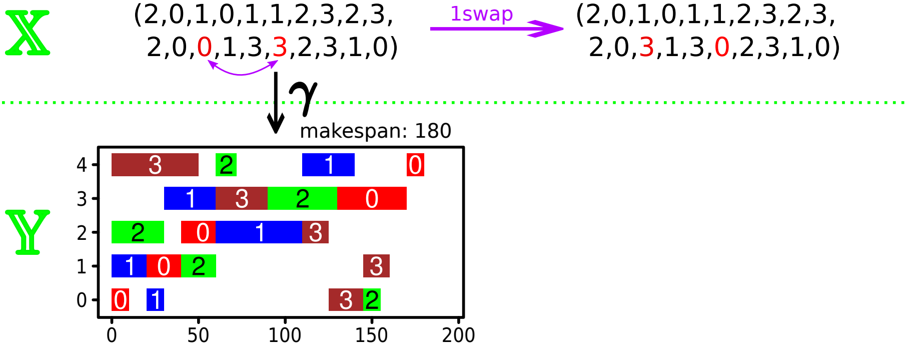

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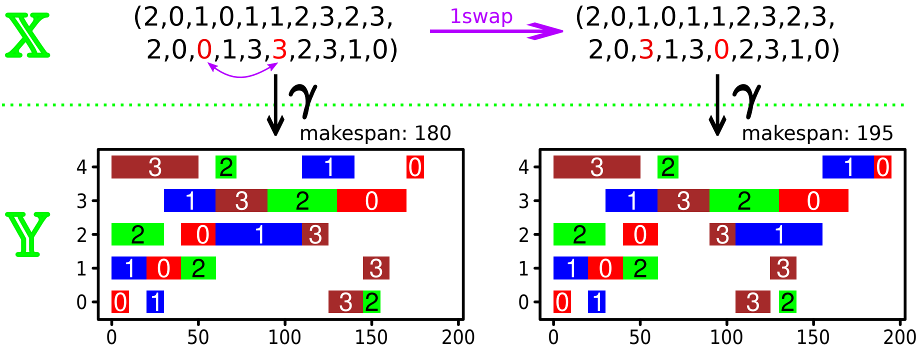

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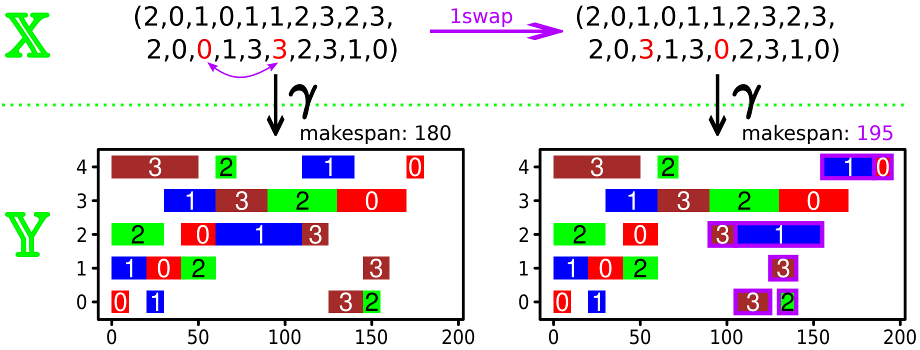

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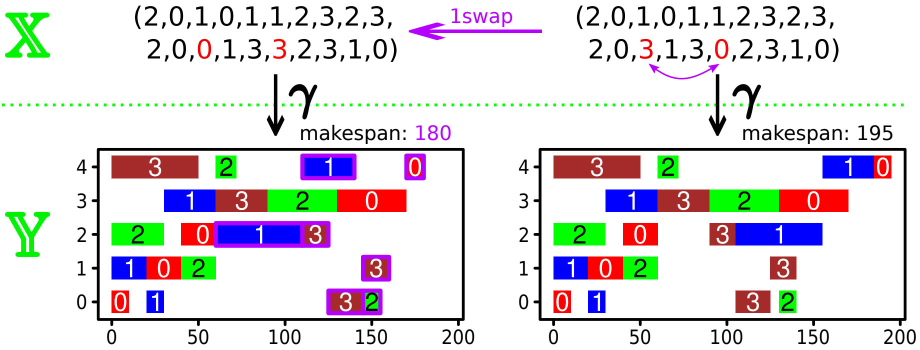

```java
package aitoa.examples.jssp;

public class JSSPUnaryOperator1Swap {
//
// unnecessary stuff omitted here...
//
//
//

//
//
//

//
//
//
//
//
//
//
//
//
//
}
```

```java
package aitoa.examples.jssp;

public class JSSPUnaryOperator1Swap implements
    IUnarySearchOperator<int[]> {
// unnecessary stuff omitted here...
//
//
//

//
//
//

//
//
//
//
//
//
//
//
//
//
}
```

```
package aitoa.examples.jssp;

public class JSSPUnaryOperator1Swap implements
    IUnarySearchOperator<int[]> {
// unnecessary stuff omitted here...
  public void apply(int[] x, int[] dest, Random random) {
//
//

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  public void apply(int[] x, int[] dest, Random random) {
// copy the source point in search space to the dest
    System.arraycopy(x, 0, dest, 0, x.length);

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// copy the source point in search space to the dest
    System.arraycopy(x, 0, dest, 0, x.length);

// choose the index of the first operation to swap
    int i    = random.nextInt(dest.length);
    int jobI = dest[i]; // remember job id

//
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    System.arraycopy(x, 0, dest, 0, x.length);

// choose the index of the first operation to swap
    int i    = random.nextInt(dest.length);
    int jobI = dest[i]; // remember job id

// choose index of second operation to swap
    int j = random.nextInt(dest.length);
    int jobJ = dest[j];
//
//
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//
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// choose index of second operation to swap
    int j = random.nextInt(dest.length);
    int jobJ = dest[j];
//
    dest[i] = jobJ;
    dest[j] = jobI;    // then we swap the values
//
//
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// choose the index of the first operation to swap
    int i    = random.nextInt(dest.length);
    int jobI = dest[i]; // remember job id

// choose index of second operation to swap
    int j = random.nextInt(dest.length);
    int jobJ = dest[j];
    if (jobI != jobJ) { // we found two locations with two
      dest[i] = jobJ;   // different values
      dest[j] = jobI;   // then we swap the values
//
    }
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  }
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public class JSSPUnaryOperator1Swap implements
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  public void apply(int[] x, int[] dest, Random random) {
// copy the source point in search space to the dest
    System.arraycopy(x, 0, dest, 0, x.length);

// choose the index of the first sub-job to swap
    int i    = random.nextInt(dest.length);
    int jobI = dest[i]; // remember job id

    for (;;) { // try to find a location j with a different job
      int j = random.nextInt(dest.length);
      int jobJ = dest[j];
      if (jobI != jobJ) { // we found two locations with two
        dest[i] = jobJ;   // different values
        dest[j] = jobI;   // then we swap the values
        return;           // and are done
      }
    }
  }
}
```

Experiment and Analysis

So what do we get?

- I execute the program 101 times for each of the instances `abz7`, `la24`, `swv15`, and `yn4`

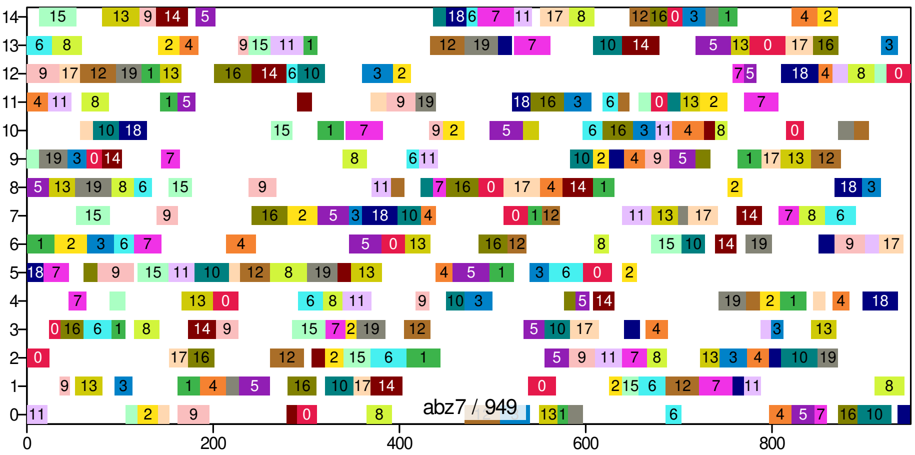
So what do we get?

- I execute the program 101 times for each of the instances `abz7`, `la24`, `swv15`, and `yn4`

		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
`abz7`	`rs`	895	947	949	**12**	85s	6'512'505
	`hc_1swap`	**717**	**800**	**798**	28	0s	16'978
`la24`	`rs`	1153	1206	1208	**15**	82s	15'902'911
	`hc_1swap`	**999**	**1095**	**1086**	56	0s	6'612
`swv15`	`rs`	4988	5166	5172	**50**	87s	5'559'124
	`hc_1swap`	**3837**	**4108**	**4108**	137	1s	104'598
`yn4`	`rs`	1460	1498	1499	**15**	76s	4'814'914
	`hc_1swap`	**1109**	**1222**	**1220**	48	0s	31'789

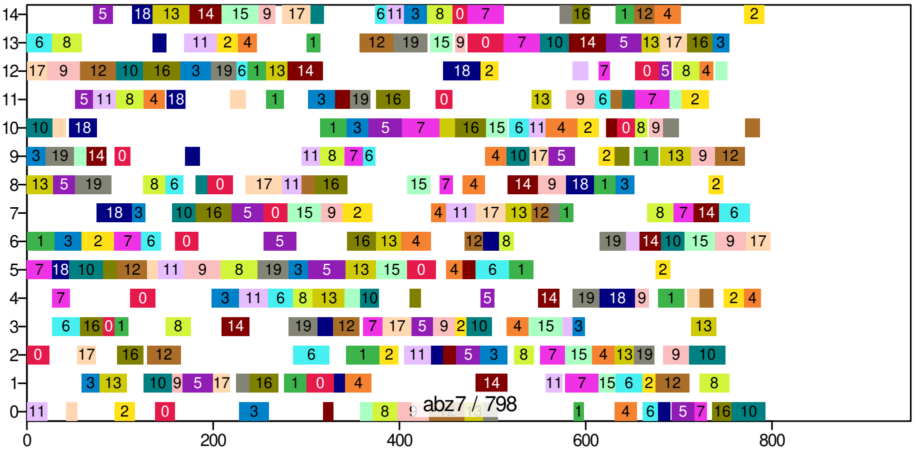
So what do we get?

`rs`: median result of 3 min of random sampling



So what do we get?

hc_1swap: median result of 3 min of hill climber



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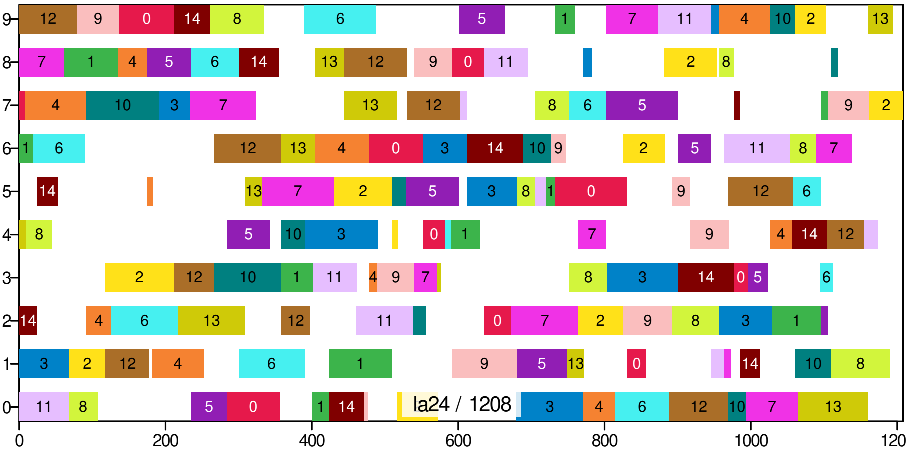
`rs`: median result of 3 min of random sampling



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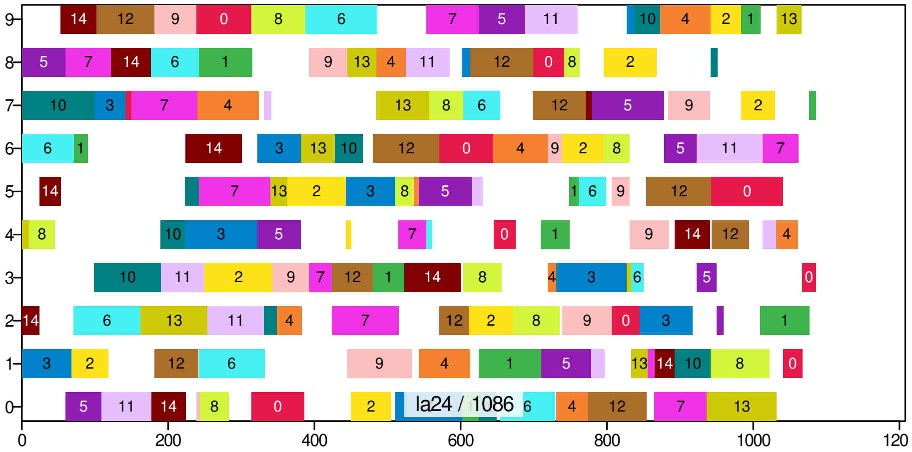

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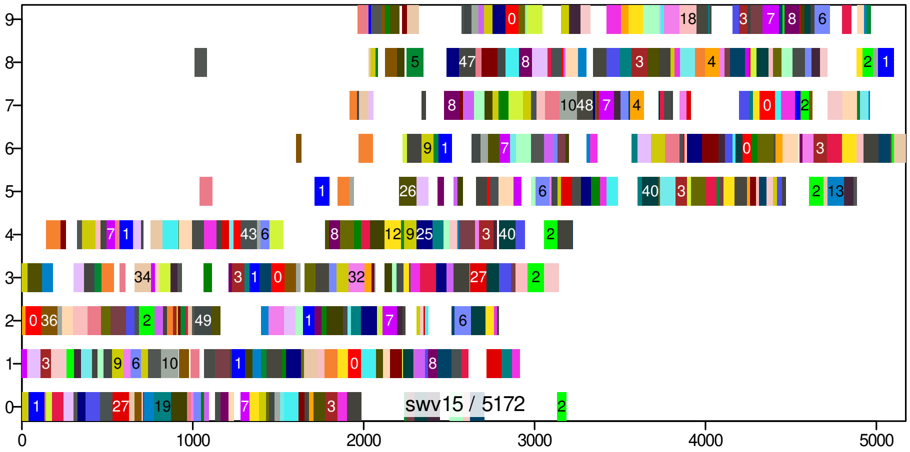

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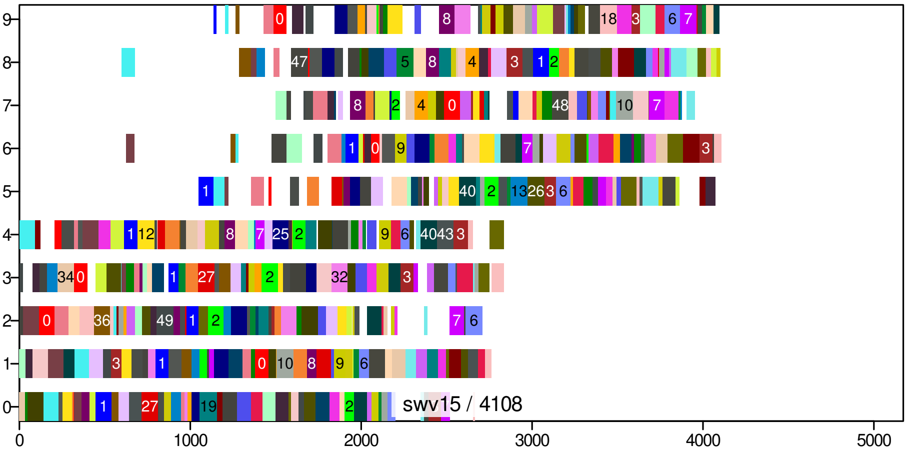

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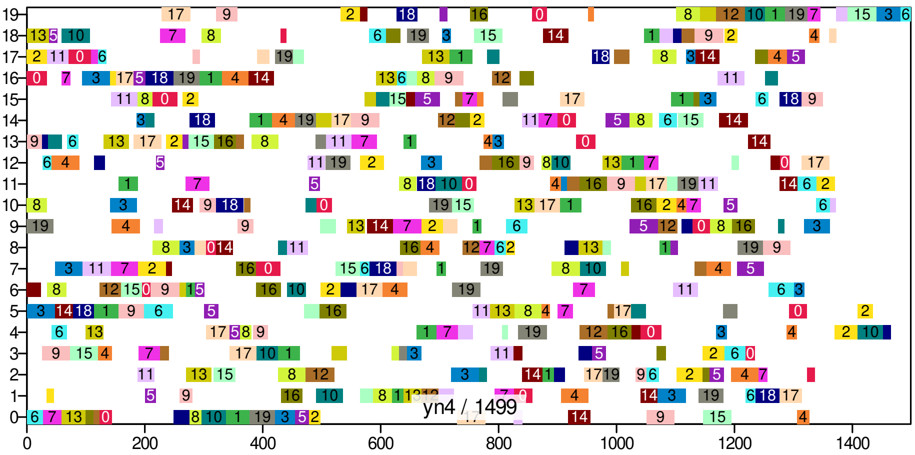

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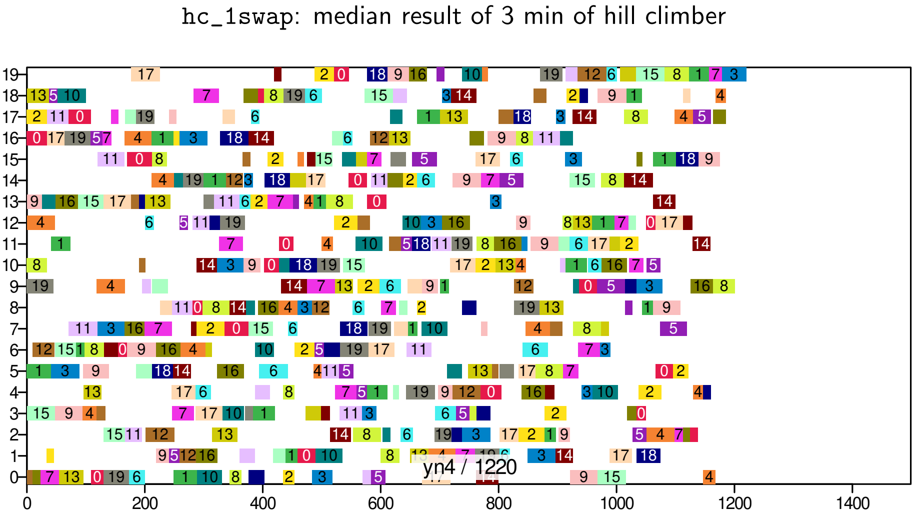

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What progress does the algorithm make over time?

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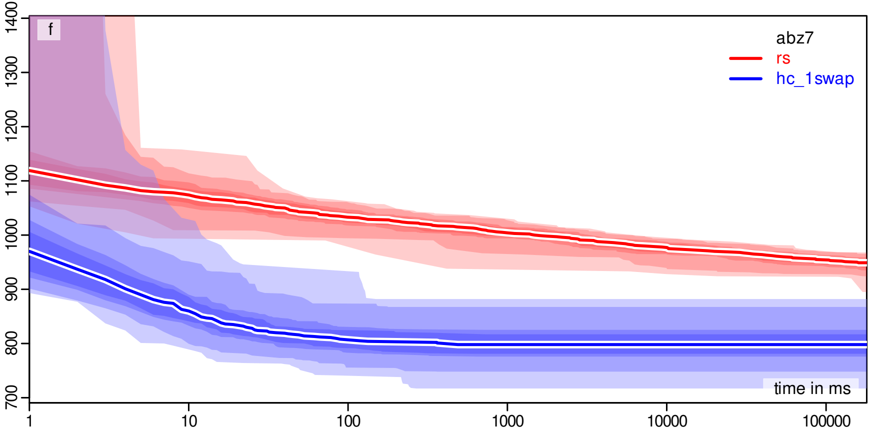

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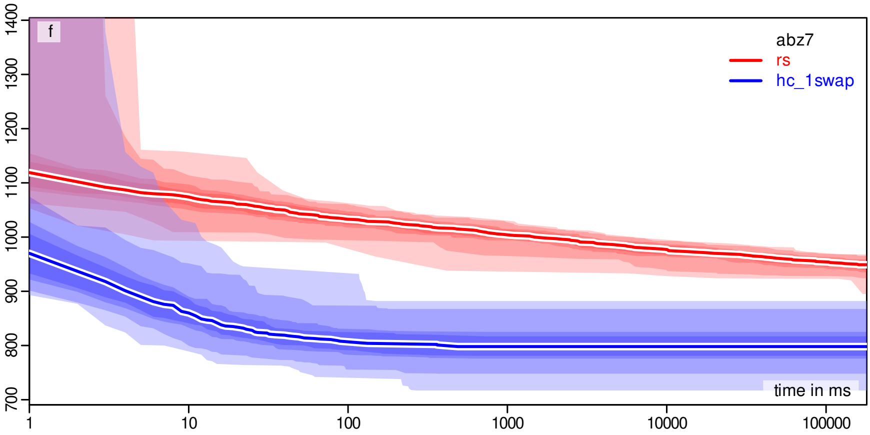

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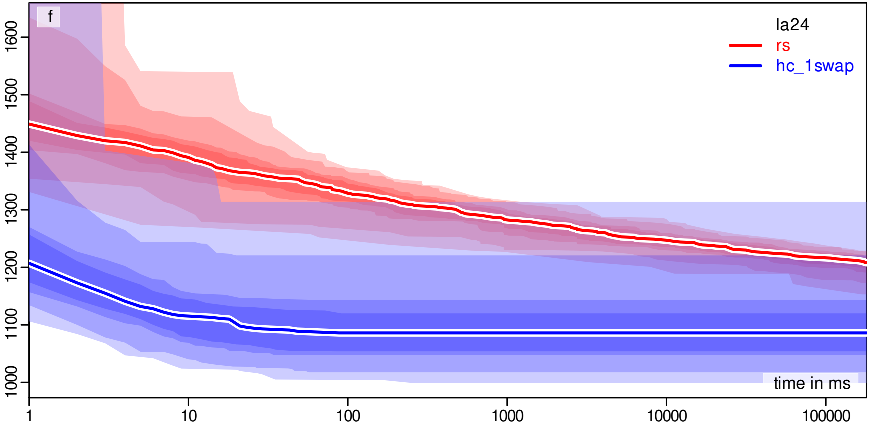

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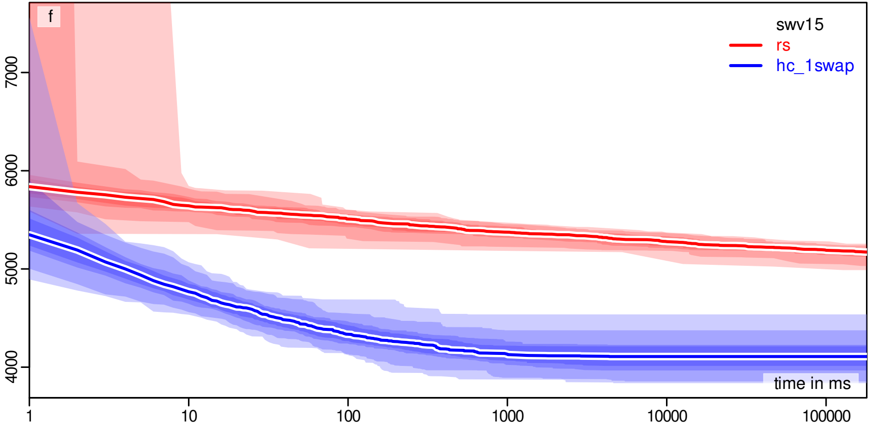

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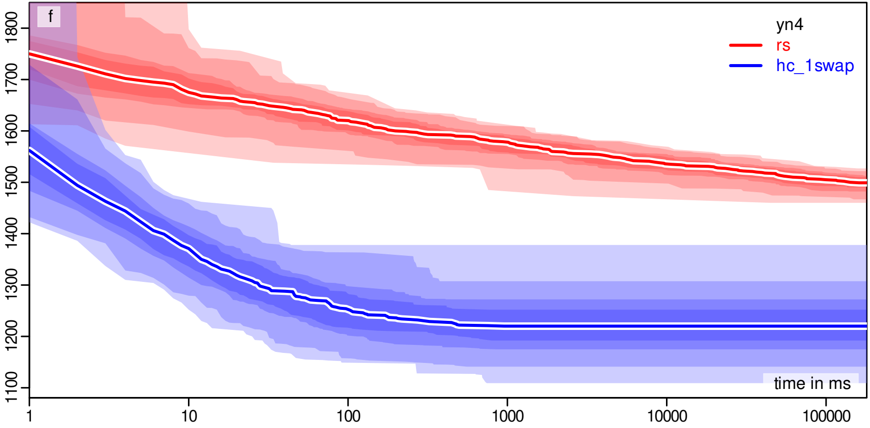

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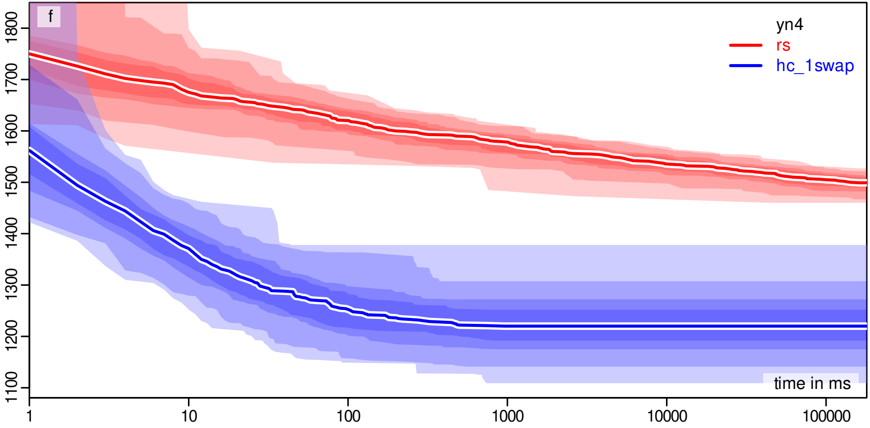

First we have much progress. . .

. . . but then the hill climber stagnates!

But we waste time. . .

What if we look at this without log-scaling the time axis?

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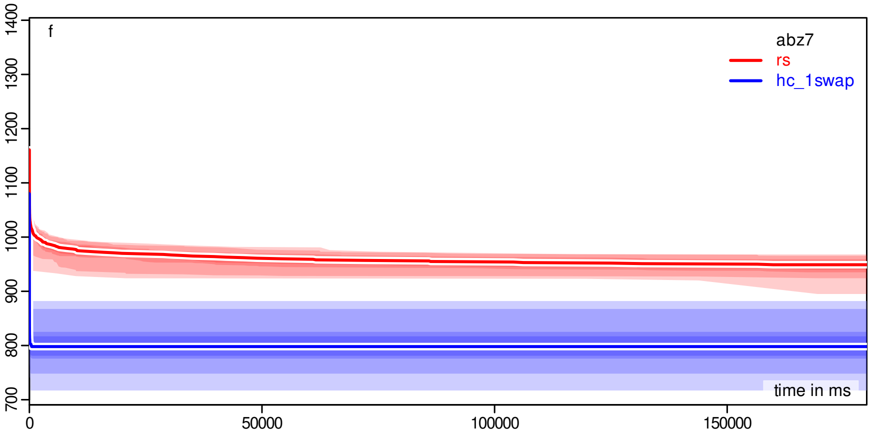

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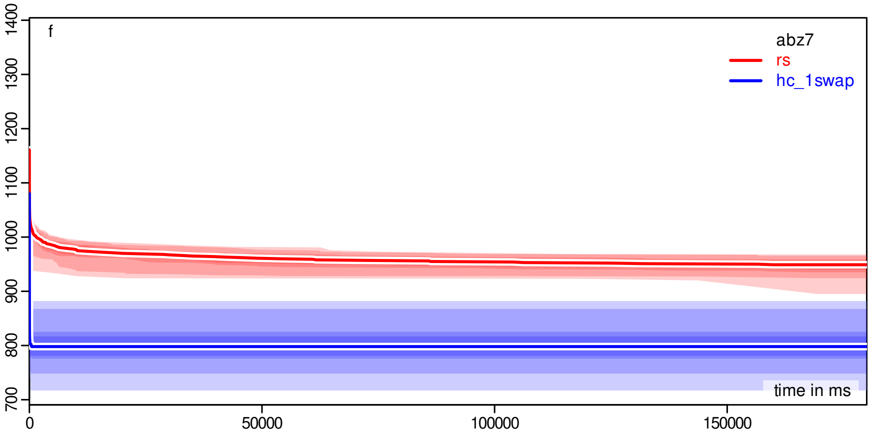

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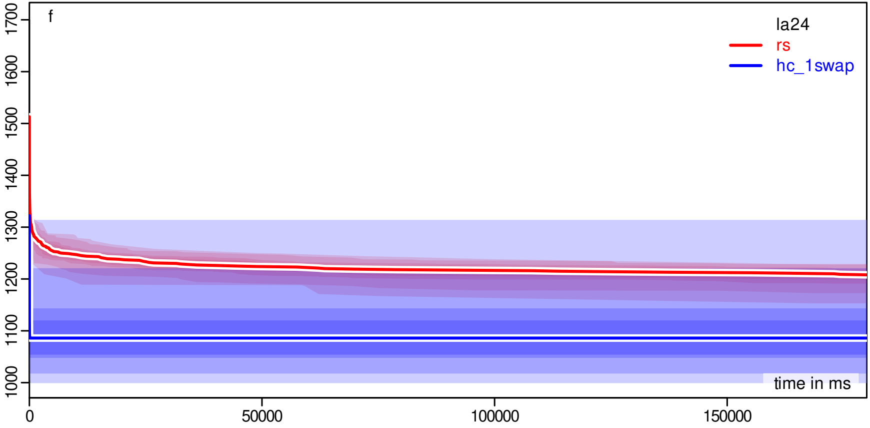

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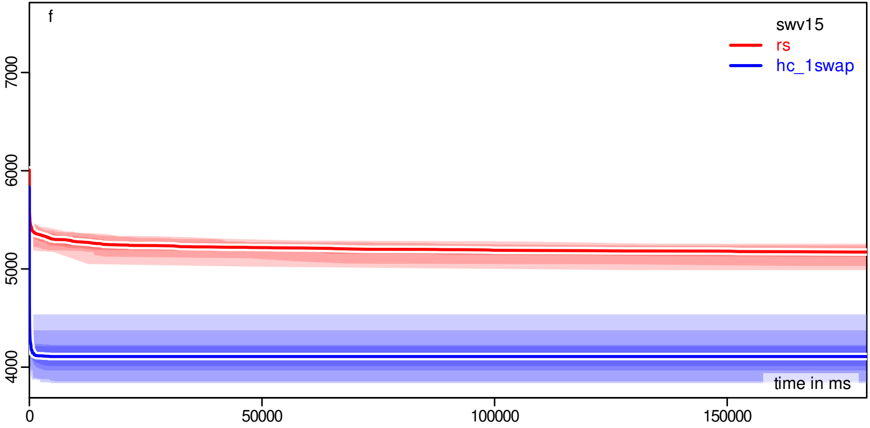

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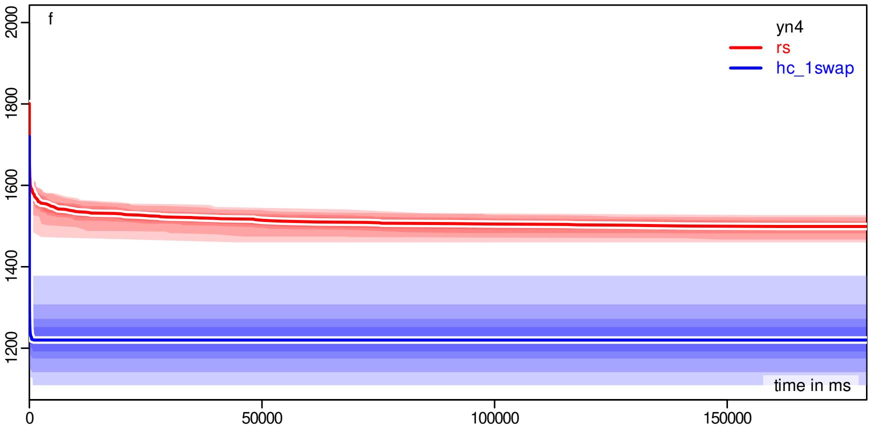

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Indeed, we waste time!

		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	rs	895	947	949	**12**	85s	6'512'505
	hc_1swap	**717**	**800**	**798**	28	0s	16'978
la24	rs	1153	1206	1208	**15**	82s	15'902'911
	hc_1swap	**999**	**1095**	**1086**	56	0s	6'612
swv15	rs	4988	5166	5172	**50**	87s	5'559'124
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- We have three minutes but after about 1 second, our `hc_1swap` algorithm stops improving!

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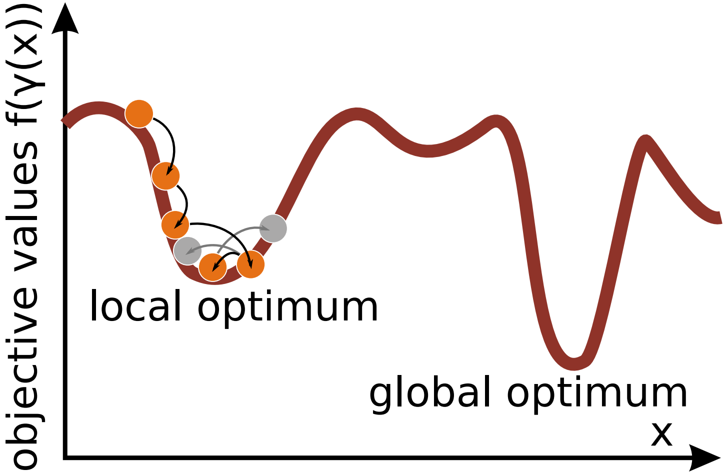
- This is called Premature Convergence.[8] [9]

Premature Convergence



Improved Algorithm Concept 1

Stochastic Hill Climber with Restarts

- Idea: We have seen that the results of the hill climber exhibit a relatively high standard deviation.

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- Let us exploit this variance!
- Idea: If we did not make any progress for a number L of algorithm steps, we simply restart at a new random solution.
- Of course, we will always remember the overall best solution we ever had (in another variable).

Stochastic Hill Climbing Algorithm with Restarts

```
package aitoa.algorithms;

public class HillClimberWithRestarts<X, Y> {
// unnecessary stuff omitted here...
//
//
//
//

//
//
//
//

//
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//

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//
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//

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//

//
    this.unary.apply(xBest, xCur, random); // try to improve
    double fCur = process.evaluate(xCur); // evaluate
    if (fCur < fBest) { // we found a better solution
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    this.unary.apply(xBest, xCur, random); // try to improve
    double fCur = process.evaluate(xCur); // evaluate
    if (fCur < fBest) { // we found a better solution
      fBest = fCur; // remember best quality
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//
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    double fCur = process.evaluate(xCur); // evaluate
    if (fCur < fBest) { // we found a better solution
      fBest = fCur; // remember best quality
      process.getSearchSpace().copy(xCur, xBest); // copy
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//

    while (!(process.shouldTerminate())) { // inner loop
      this.unary.apply(xBest, xCur, random); // try to improve
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Stochastic Hill Climbing Algorithm with Restarts

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          if ((++failCounter) >= this.failsBeforeRestart) {
            break; // jump back to outer loop for restart
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Experiment and Analysis

Configuring the Algorithm: Parameter L

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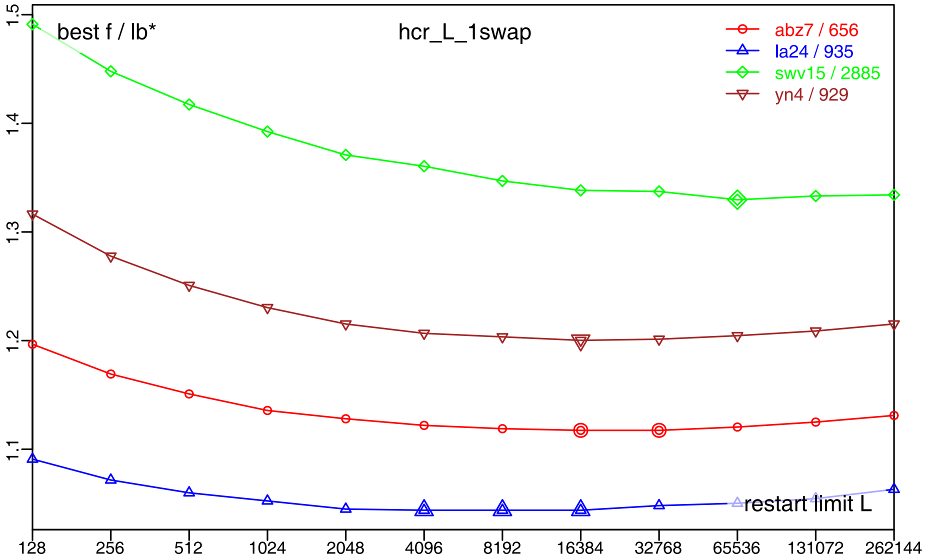

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- $L = 2^{14} = 16'384$ seems to be a reasonable choice.

So what do we get?

- I execute the program 101 times for each of the instances `abz7`, `la24`, `swv15`, and `yn4`

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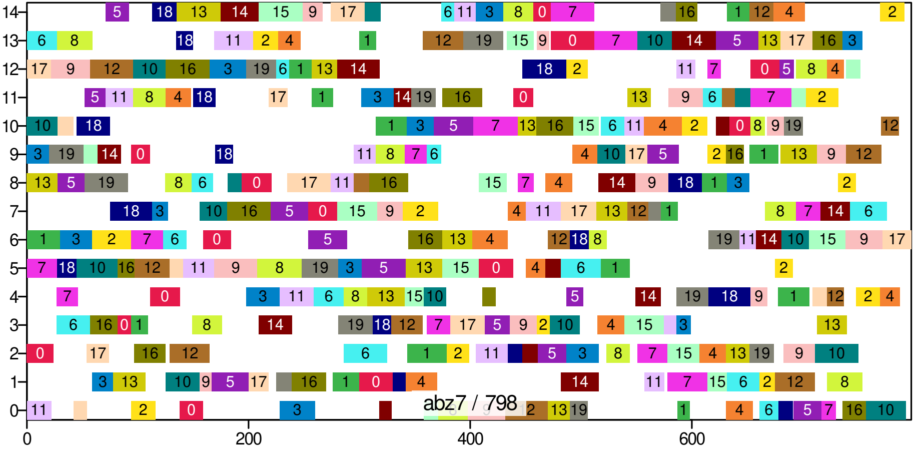
		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	rs	895	947	949	12	85s	6'512'505
	hc_1swap	717	800	798	28	0s	16'978
	hcr_16384_1swap	**714**	**732**	**733**	**6**	91s	18'423'530
la24	rs	1153	1206	1208	15	82s	15'902'911
	hc_1swap	999	1095	1086	56	0s	6'612
	hcr_16384_1swap	**953**	**976**	**976**	**7**	80s	34'437'999
swv15	rs	4988	5166	5172	50	87s	5'559'124
	hc_1swap	3837	4108	4108	137	1s	104'598
	hcr_16384_1swap	**3752**	**3859**	**3861**	**42**	92s	11'756'497
yn4	rs	1460	1498	1499	15	76s	4'814'914
	hc_1swap	1109	1222	1220	48	0s	31'789
	hcr_16384_1swap	**1081**	**1115**	**1115**	**11**	91s	14'804'358

So what do we get?

hc_1swap: median result of 3 min of hill climber



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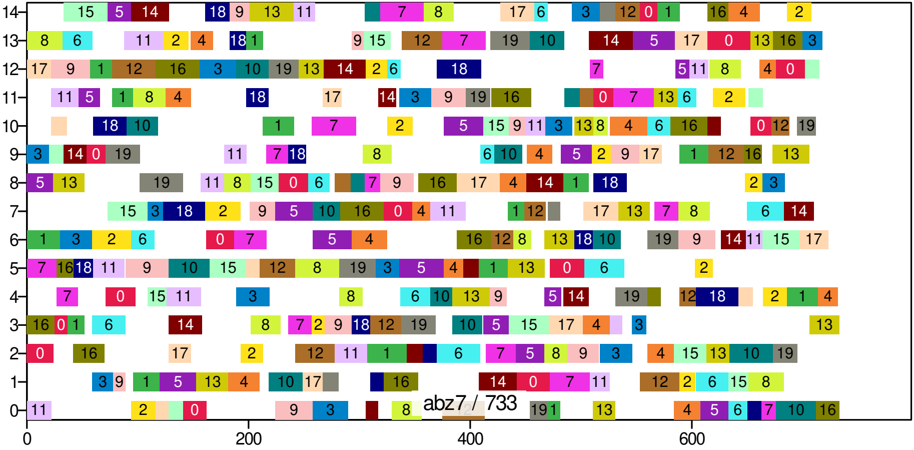

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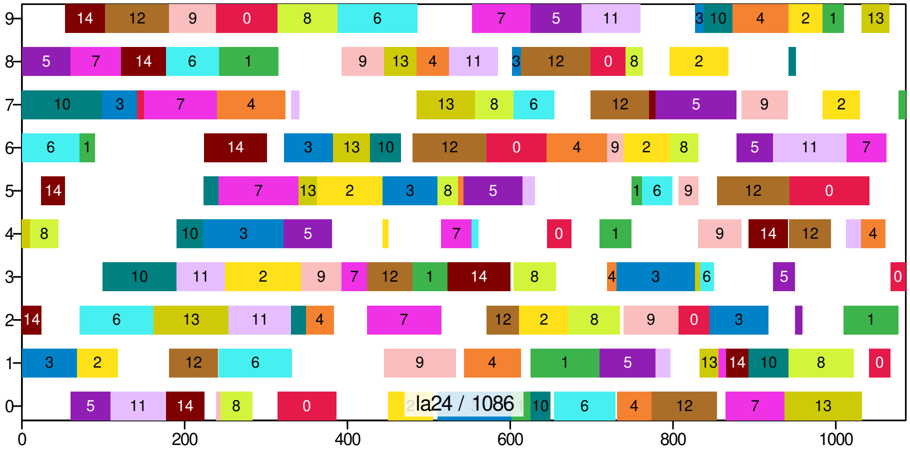

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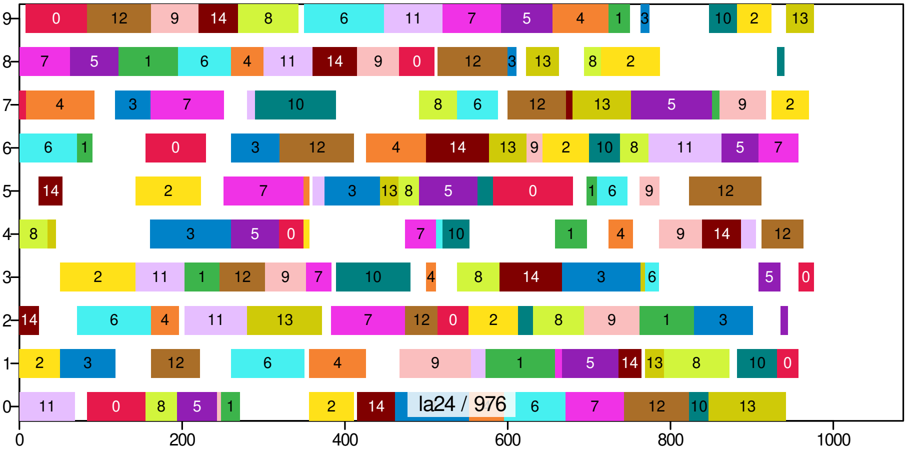

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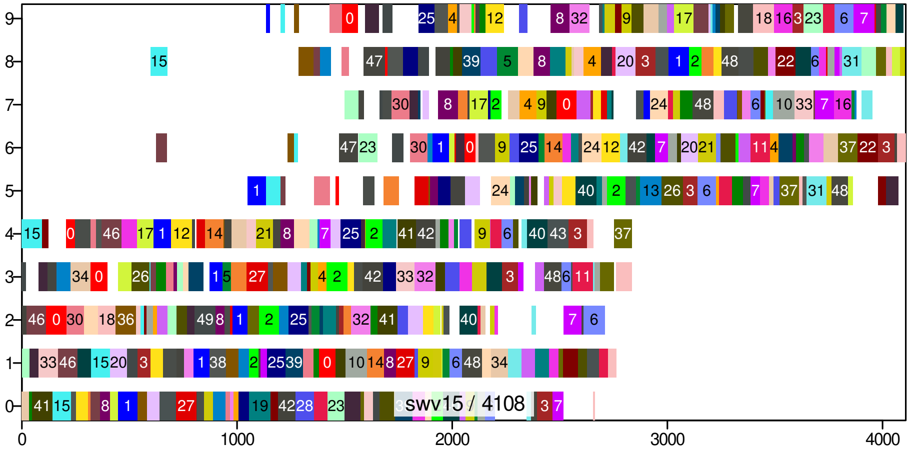

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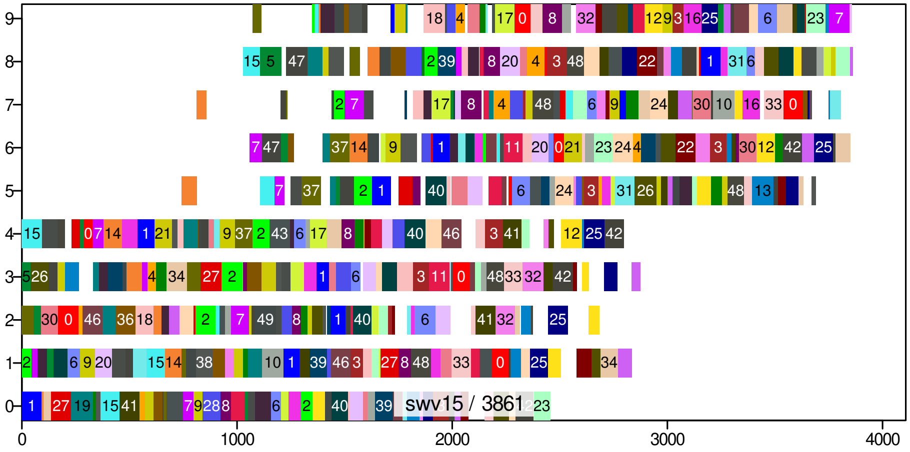

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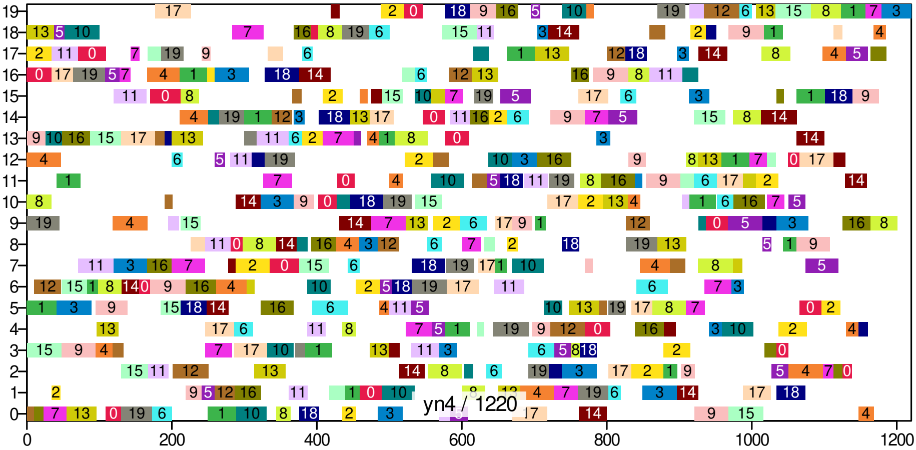

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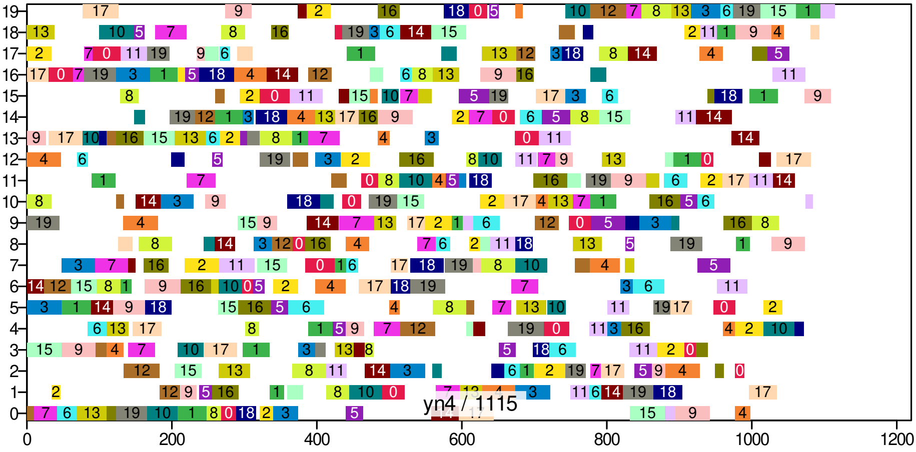

Progress over Time

What progress does the algorithm make over time?

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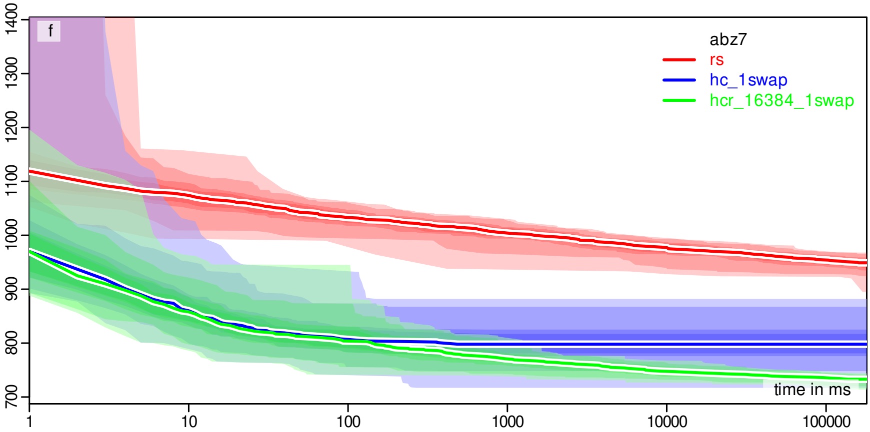

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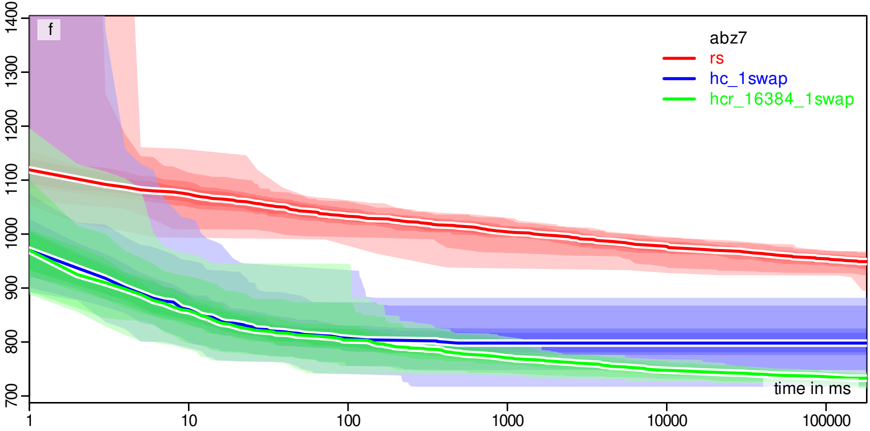

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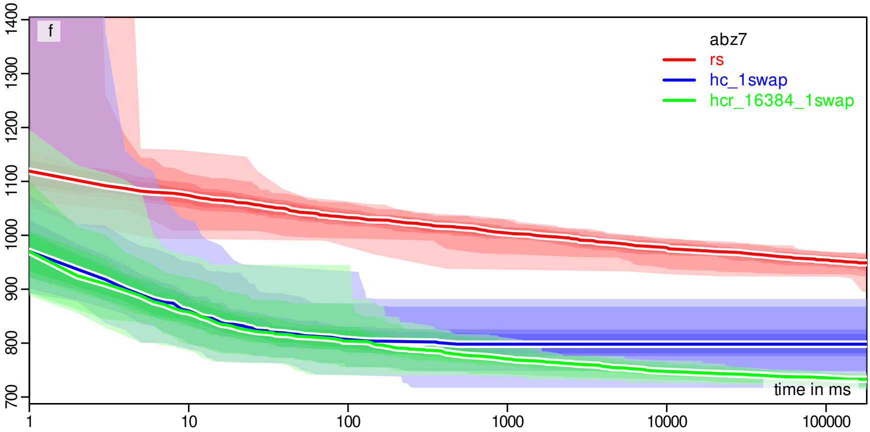

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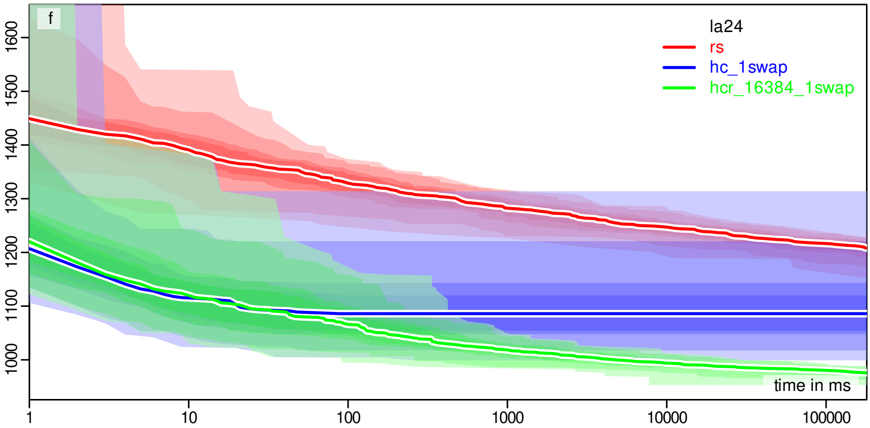

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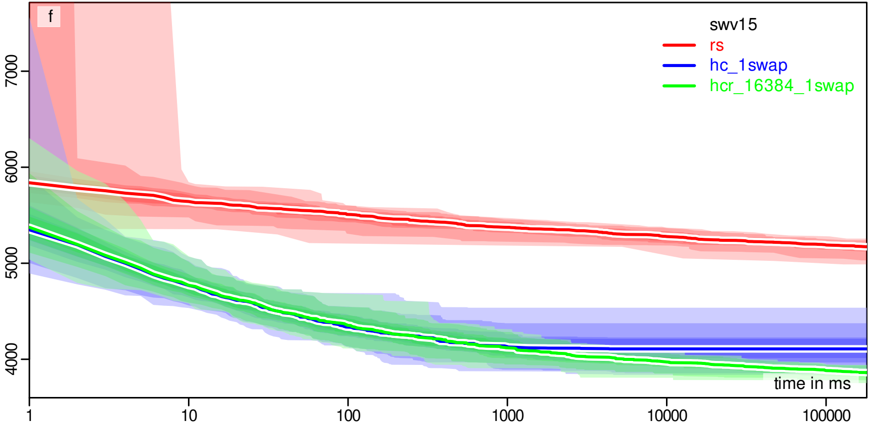

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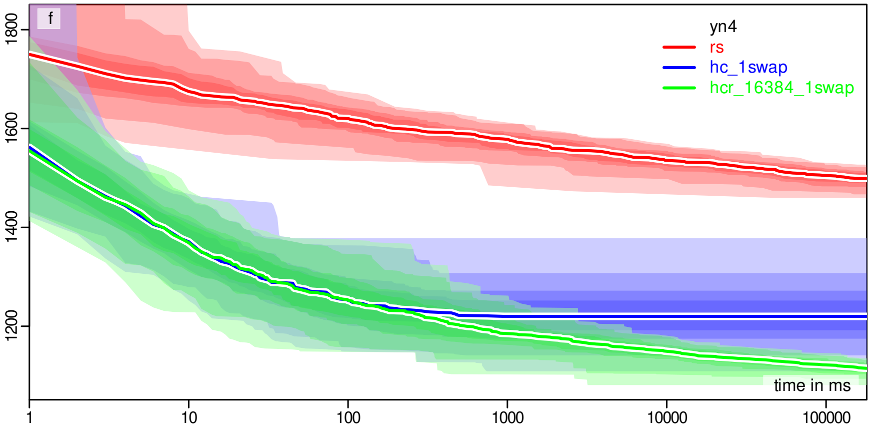

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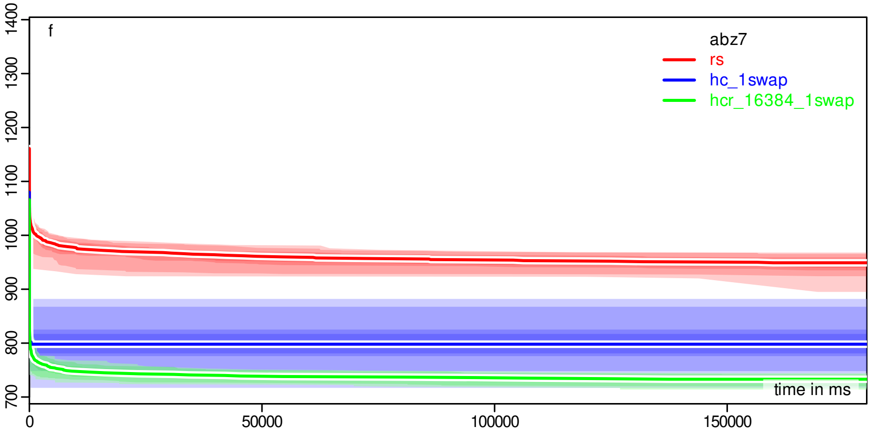

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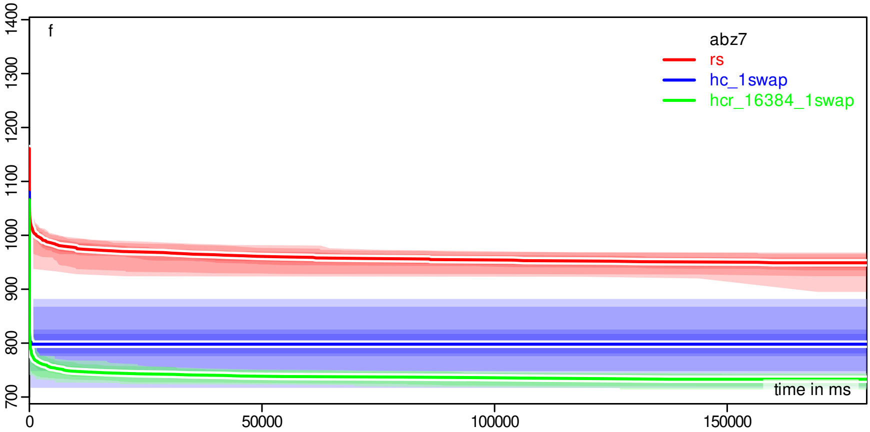

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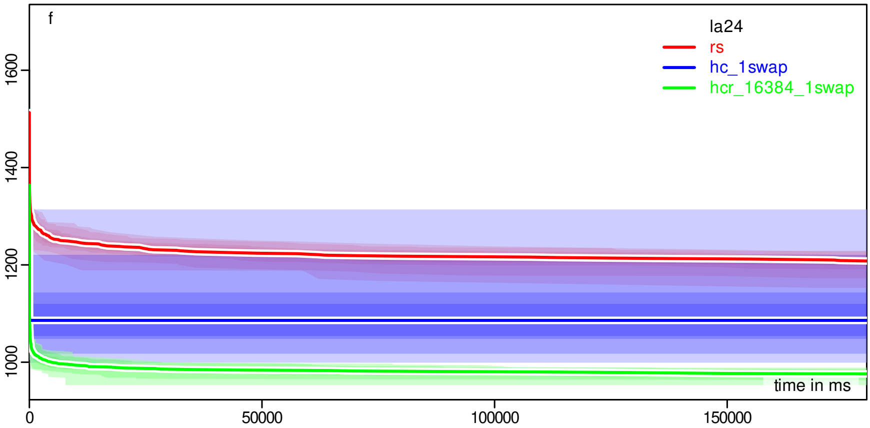

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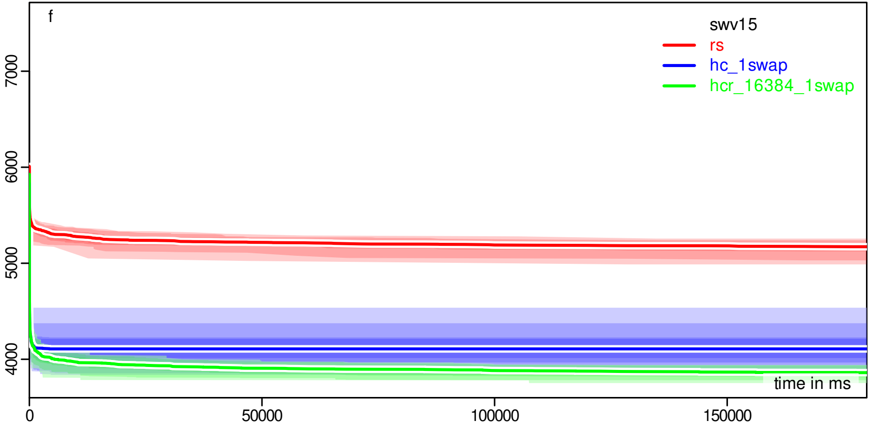

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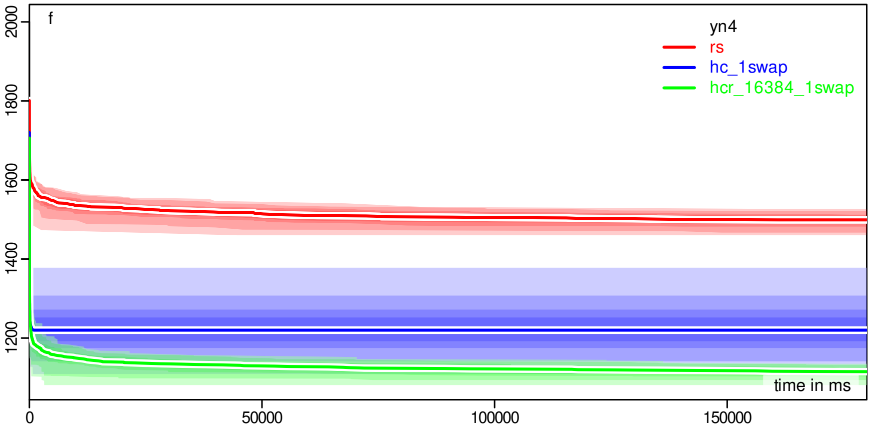

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Improved Algorithm Concept 2

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- Restarts are therefore also wasteful.

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- Notice: Whether or not a point x is a local optimum, is determined entirely by the unary search operator!
- If we had a different operator with a bigger neighborhood, then maybe $x^{\times}$ would no longer be a local optimum and we could still improve the results after reaching it...

Making the neighborhood bigger

- Two solutions x_1 and x_2 are "neighbors" if I can reach x_2 by applying the search operator one time to x_1.

Making the neighborhood bigger

- Two solutions x_1 and x_2 are “neighbors” if I can reach x_2 by applying the search operator one time to x_1.
- The search operator determines which solutions are “neighbors”.

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- On the other end of the spectrum, we could simply swap all jobs in our points x randomly.

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- On one hand, we already have `1swap`, which swaps two jobs. This is the smallest step I can imagine.
- On the other end of the spectrum, we could simply swap all jobs in our points x randomly. Is this a good idea?Probably not: It would turn our algorithm into random sampling!
- We should respect the causality: small changes to the solution cause small changes in the objective value – big changes will lead to unpredictable results.

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs, but sometimes 3

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs, but sometimes 3, less often 4

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs, but sometimes 3, less often 4, from time to time 5

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs, but sometimes 3, less often 4, from time to time 5, rarely 6

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs, but sometimes 3, less often 4, from time to time 5, rarely 6, hardly ever 7

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs, but sometimes 3, less often 4, from time to time 5, rarely 6, hardly ever 7, ...

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs, but sometimes 3, less often 4, from time to time 5, rarely 6, hardly ever 7, ...
- `nswap` operator idea

Making the neighborhood bigger

- Idea: Let's most often swap 2 jobs, but sometimes 3, less often 4, from time to time 5, rarely 6, hardly ever 7, ...
- `nswap` operator idea:
    1. flip a coin: if it is heads (50% probability), we will swap 2 job ids (and stop).

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    1. flip a coin: if it is heads (50% probability), we will swap 2 job ids (and stop).
    2. otherwise (it was tail), we again flip a coin. if it is heads (50% probability, now 25% in total), we will swap 3 job ids (and stop).

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  4. otherwise (it was tail), we again flip a coin. if it is heads (50% probability, now 6.25% in total), we will swap 5 job ids (and stop).

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- We most often make small moves, but sometimes bigger ones.

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- Theoretically, we could always escape from any local optimum.

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    6. and so on.
- We most often make small moves, but sometimes bigger ones.
- Theoretically, we could always escape from any local optimum, but the probability may sometimes be very very small.

Implementation of the `nswap` Operator

```
package aitoa.examples.jssp;

public class JSSPUnaryOperatorNSwap {
// unnecessary stuff omitted here...
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//
//

//
//
}
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Implementation of the nswap Operator

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package aitoa.examples.jssp;

public class JSSPUnaryOperatorNSwap implements IUnarySearchOperator<int[]> {
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    if (first != jobJ) {
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      dest[i] = jobJ; // overwrite job at index i with jobJ
      i       = j; // remember index j: we will overwrite it next
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    int i     = random.nextInt(dest.length);   // index of first job to swap
    int first = dest[i];
//
//
//
//
    inner: for (;;) { // find a location with a different job
      int j    = random.nextInt(dest.length);
      int jobJ = dest[j];
      if (first != jobJ) {
//
        dest[i] = jobJ; // overwrite job at index i with jobJ
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        break inner;
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    int i     = random.nextInt(dest.length);   // index of first job to swap
    int first = dest[i];
//
    boolean hasNext;
    do { // we repeat a geometrically distributed number of times
      hasNext = random.nextBoolean();
      inner: for (;;) { // find a location with a different job
        int j    = random.nextInt(dest.length);
        int jobJ = dest[j];
        if (first != jobJ) {
//
          dest[i] = jobJ; // overwrite job at index i with jobJ
          i       = j; // remember index j: we will overwrite it next
//
          break inner;
        }
      }
    } while (hasNext); // Bernoulli process

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    int last  = first;   // last stores the job id to "swap in"
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          last    = jobJ; // but not with the same value jobJ...
          break inner;
        }
      }
    } while (hasNext); // Bernoulli process

    dest[i] = first; // write back first id to last copied index
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```

Implementation of the `nswap` Operator

```java
package aitoa.examples.jssp;

public class JSSPUnaryOperatorNSwap implements IUnarySearchOperator<int[]> {
// unnecessary stuff omitted here...
  public void apply(int[] x, int[] dest, Random random) {
    System.arraycopy(x, 0, dest, 0, x.length); // copy x to dest
    int i     = random.nextInt(dest.length);   // index of first job to swap
    int first = dest[i];
    int last  = first;   // last stores the job id to "swap in"
    boolean hasNext;
    do { // we repeat a geometrically distributed number of times
      hasNext = random.nextBoolean();
      inner: for (;;) { // find a location with a different job
        int j    = random.nextInt(dest.length);
        int jobJ = dest[j];
        if ((last != jobJ) && // don't swap job with itself
            (           (first != jobJ))) { // also not at end
          dest[i] = jobJ; // overwrite job at index i with jobJ
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	hcr_16384_1swap	953	**976**	**976**	**7**	80s	34'437'999
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	hcr_16384_1swap	3752	**3859**	**3861**	**42**	92s	11'756'497
	hc_nswap	**3602**	3880	3872	112	70s	8'351'112
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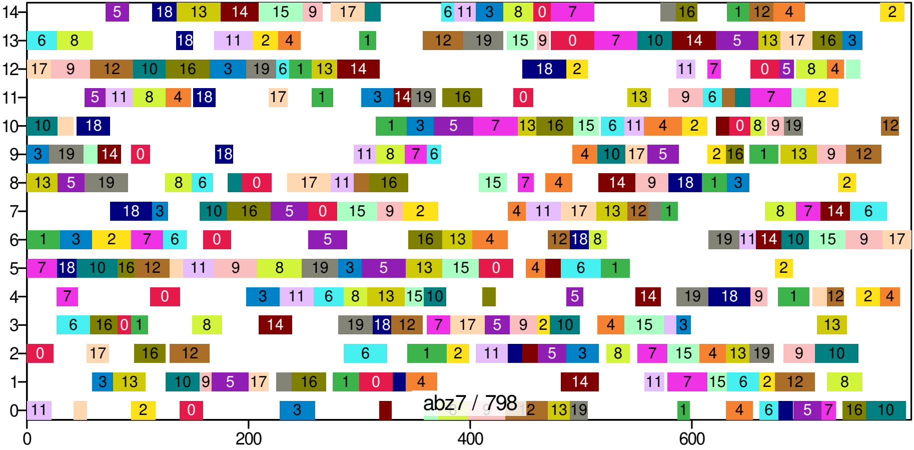

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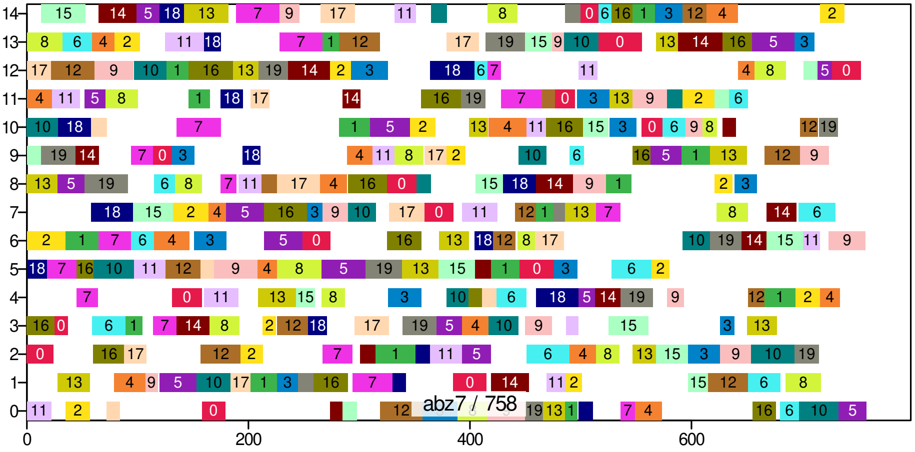

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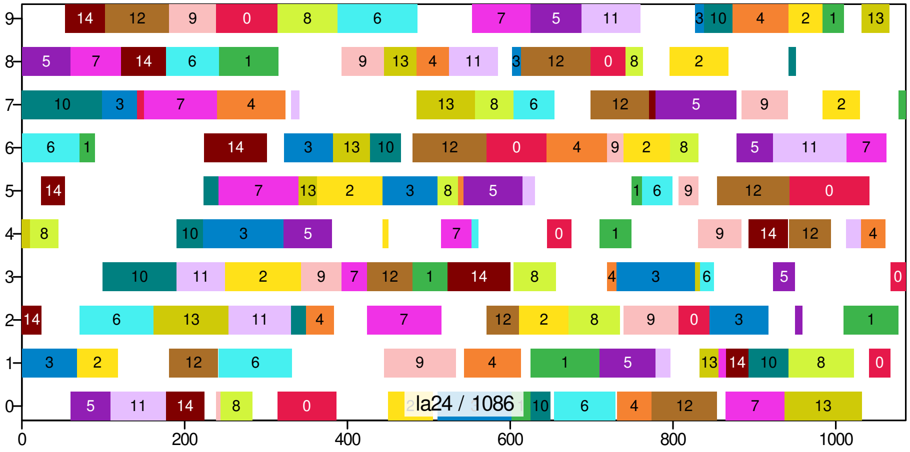

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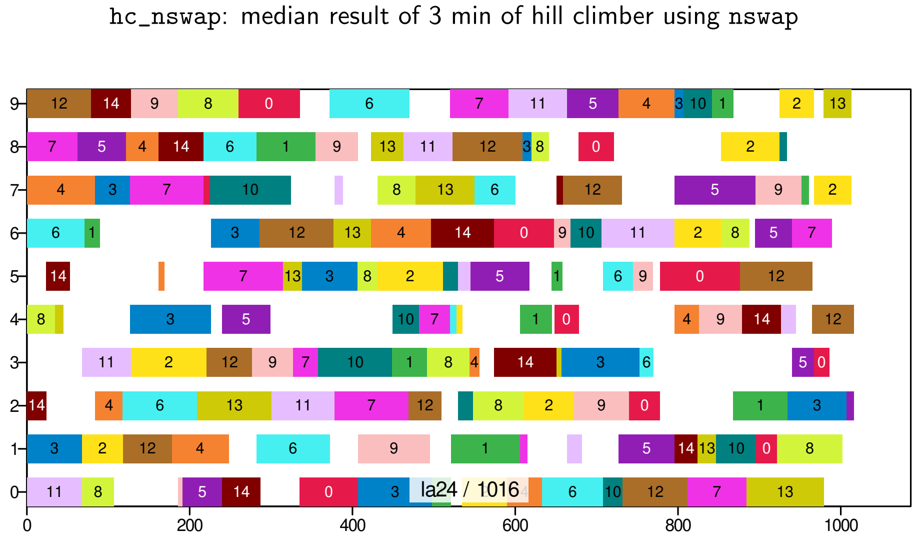

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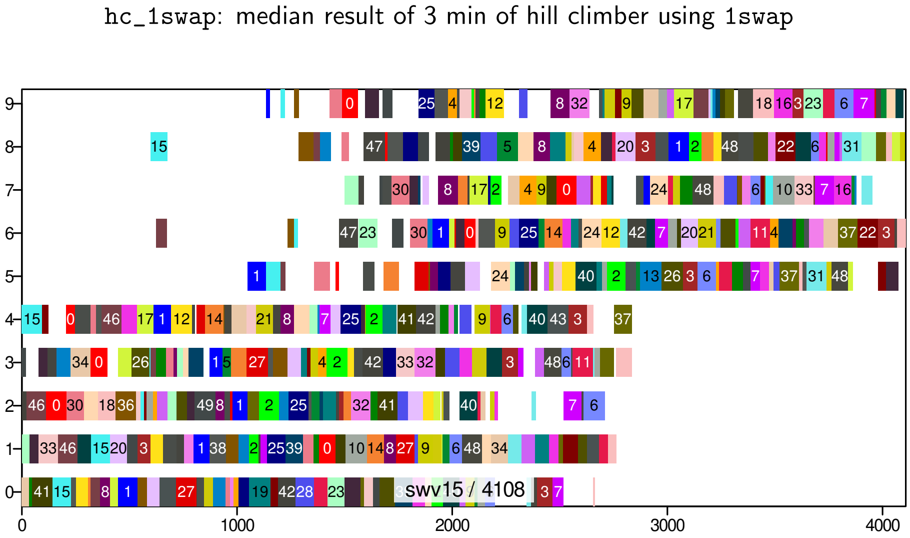

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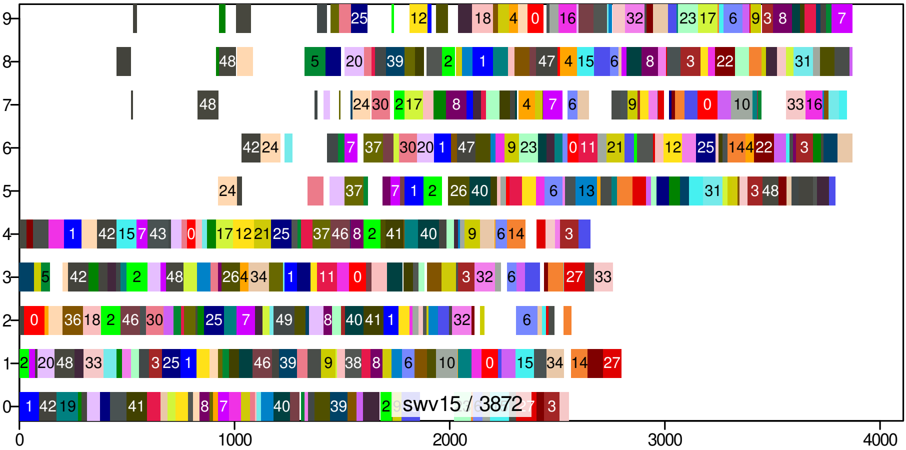

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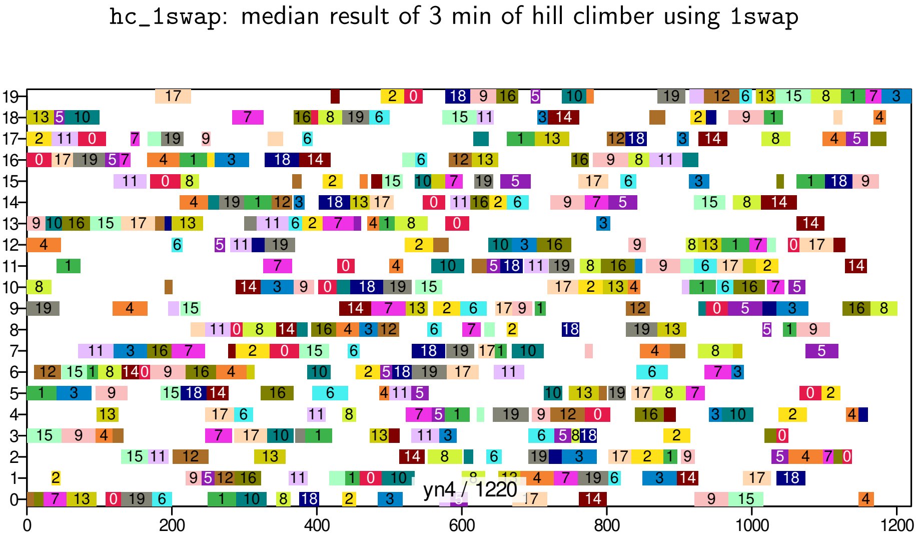

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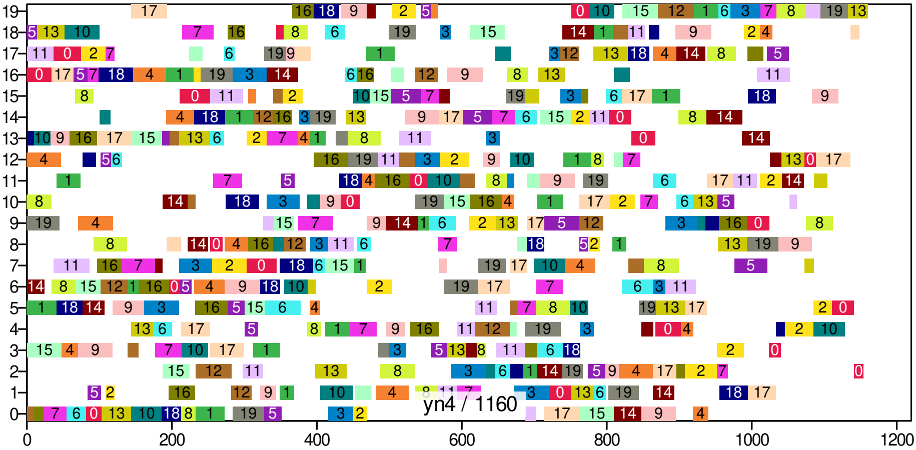

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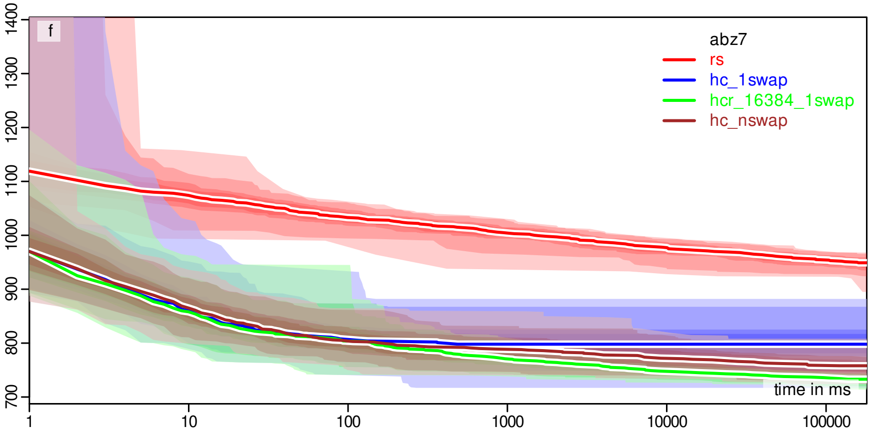

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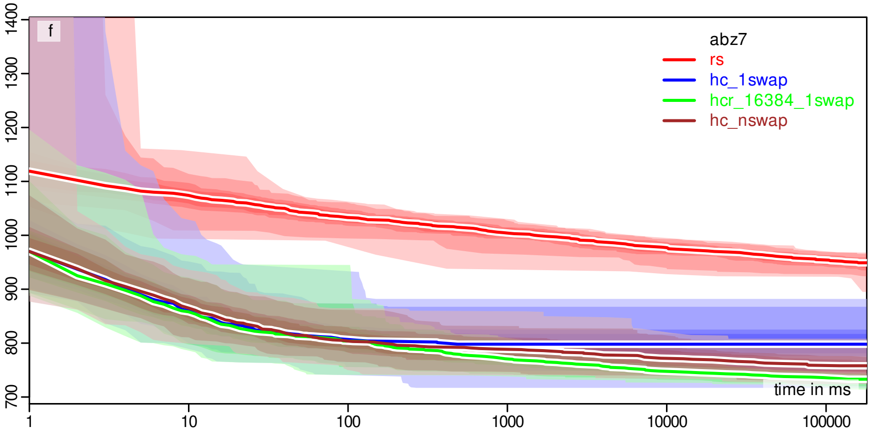

What progress does the algorithm make over time?

- `hc_nswap` first behaves like `hc_1swap`, because most of the `nswap` moves are the same as `1swap` moves.

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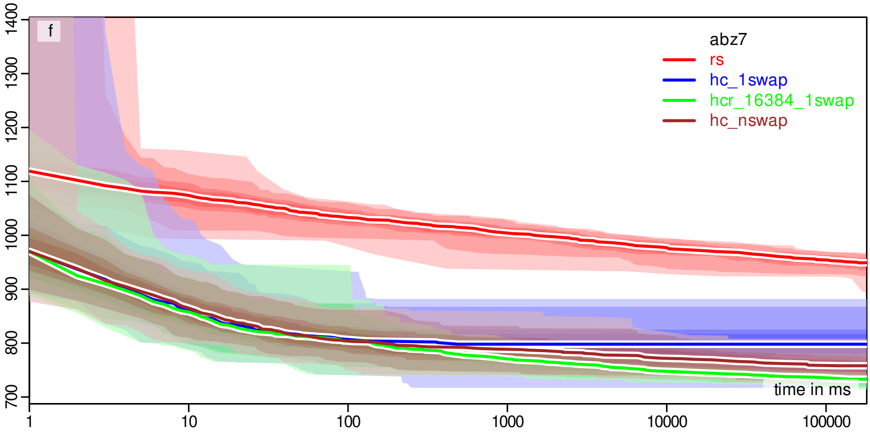

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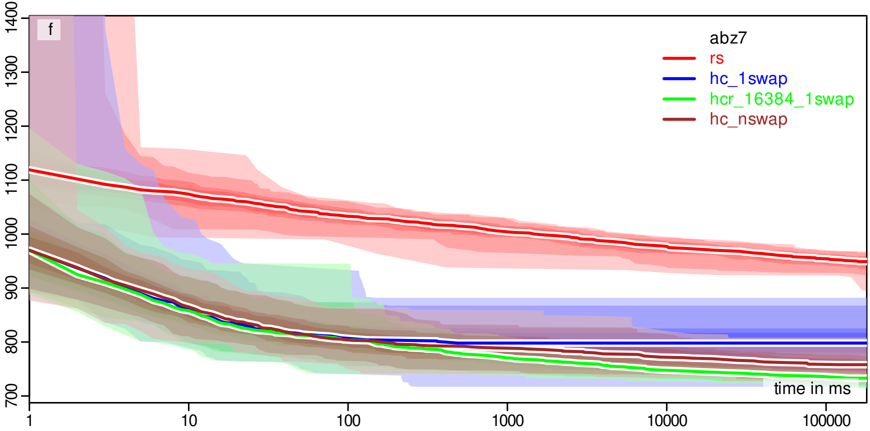

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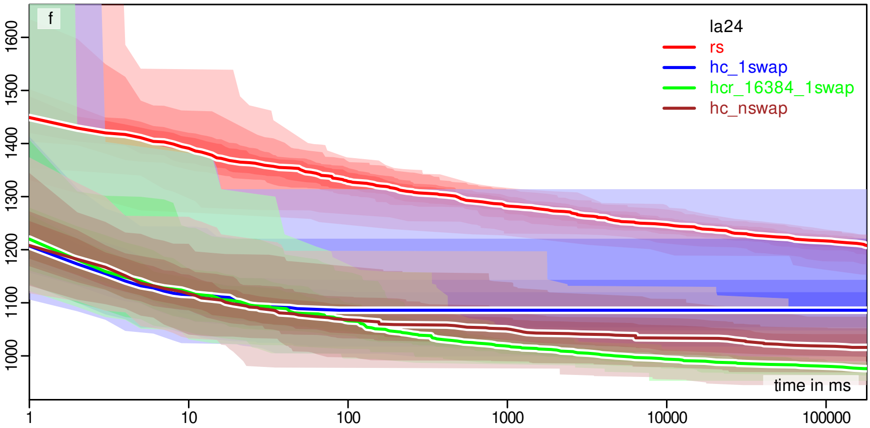

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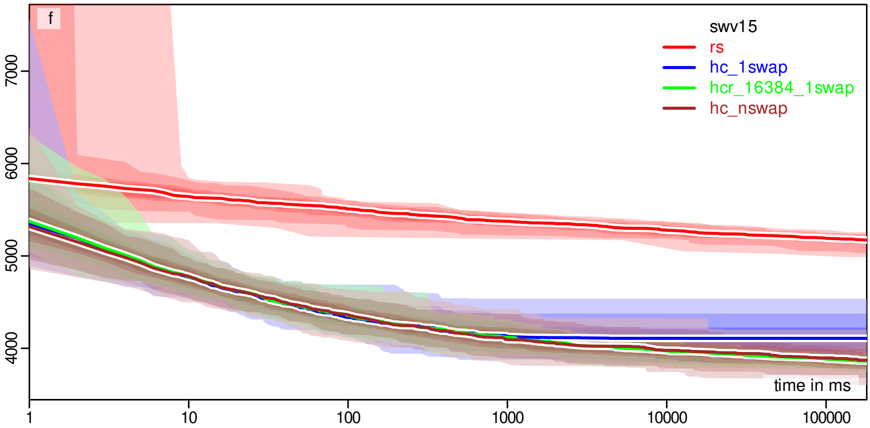

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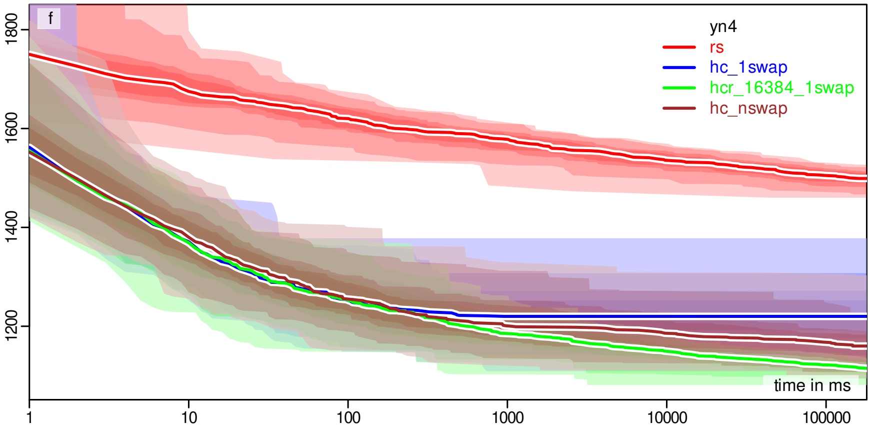

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Improved Algorithm Concept 3

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- Now we know two ways to improve the performance of our hill climber:
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  2. we can use a unary operator with larger neighborhood that still mostly makes small steps.
- It is only natural to try to combine these two improvements.

Configuring the Algorithm

		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	hc_1swap	717	800	798	28	0s	16'978
	hc_nswap	724	758	758	17	35s	7'781'762
la24	hc_1swap	999	1095	1086	56	0s	6'612
	hc_nswap	945	1018	1016	29	25s	9'072'935
swv15	hc_1swap	3837	4108	4108	137	1s	104'598
	hc_nswap	3602	3880	3872	112	70s	8'351'112
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- The `hc_nswap` improves longer than `hc_1swap`
- We can expect that the number L of unsuccessful steps before a restart should be higher now.

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abz7	hc_1swap	717	800	798	28	0s	16'978
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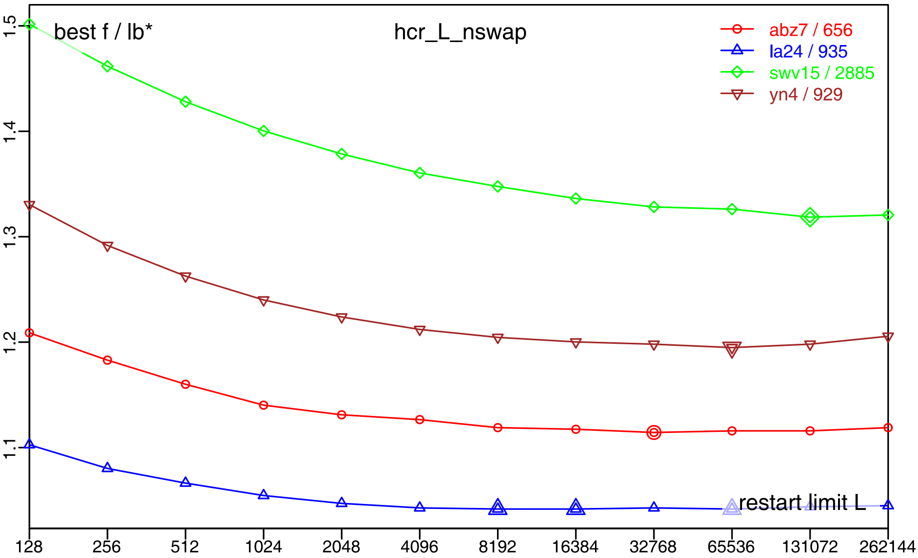

Configuring the Algorithm

- The hc_nswap improves longer than hc_1swap
- We can expect that the number L of unsuccessful steps before a restart should be higher now.
- Let's choose $L = 65'536$, i.e., hcr_65536_nswap.

		makespan				**last improvement**	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	hc_1swap	717	800	798	28	0s	16'978
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	hc_nswap	724	758	758	17	35s	7'781'762
	hcr_65536_nswap	**712**	**731**	**732**	6	96s	21'189'358
la24	hcr_16384_1swap	953	976	976	**7**	80s	34'437'999
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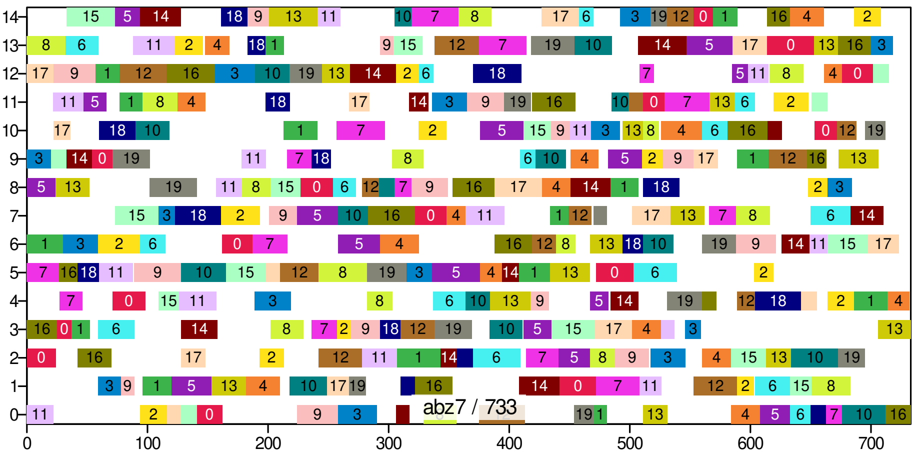
`hcr_16384_1swap`: median result of 3 min of `hcr_16384_1swap`



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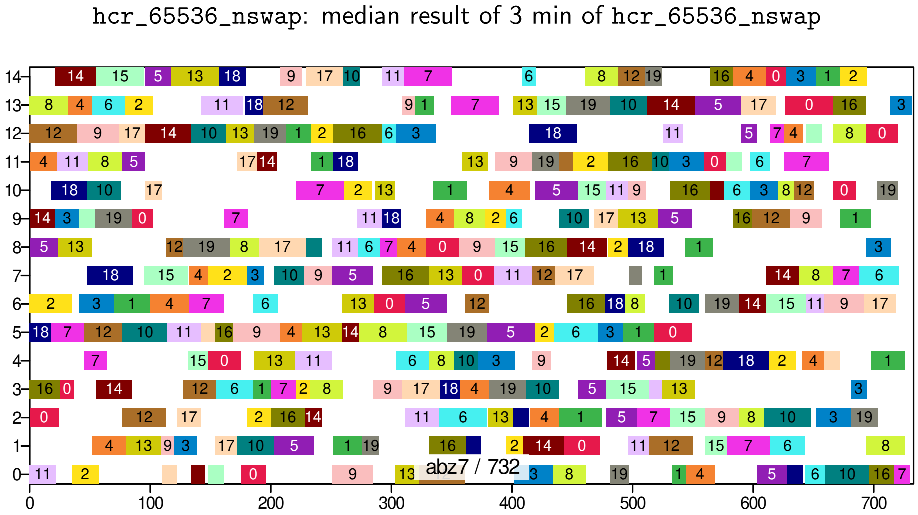

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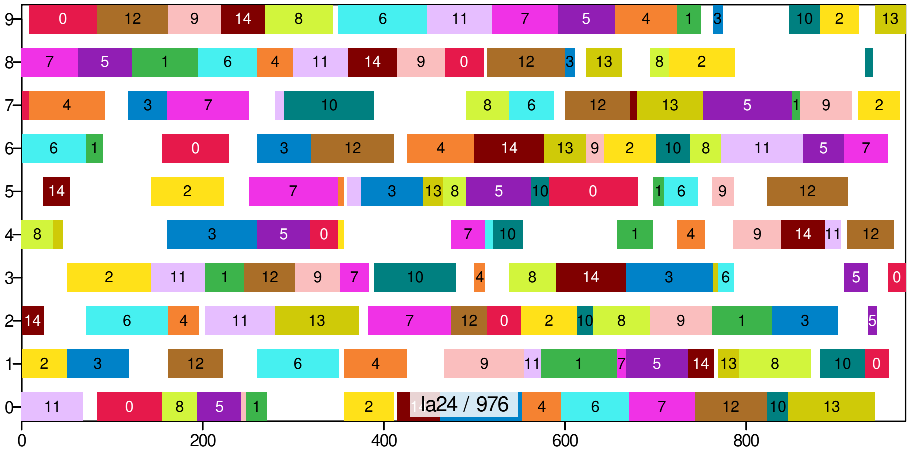

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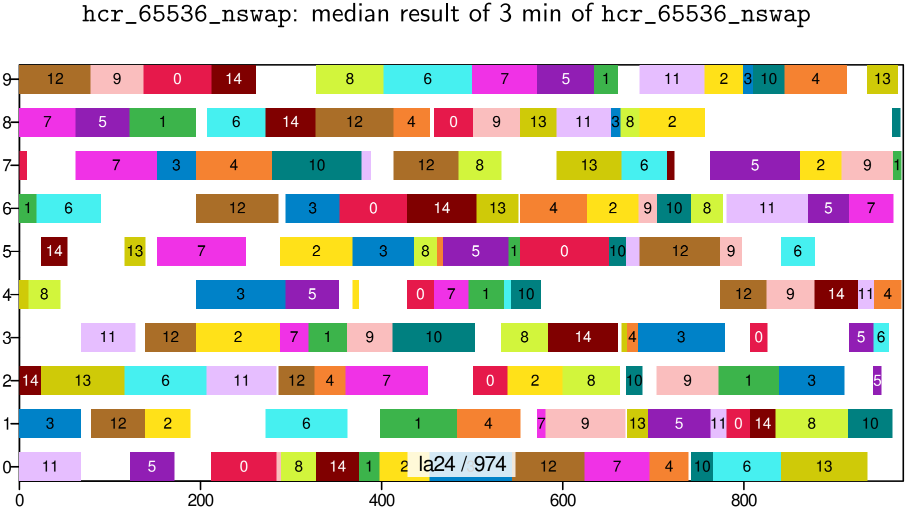

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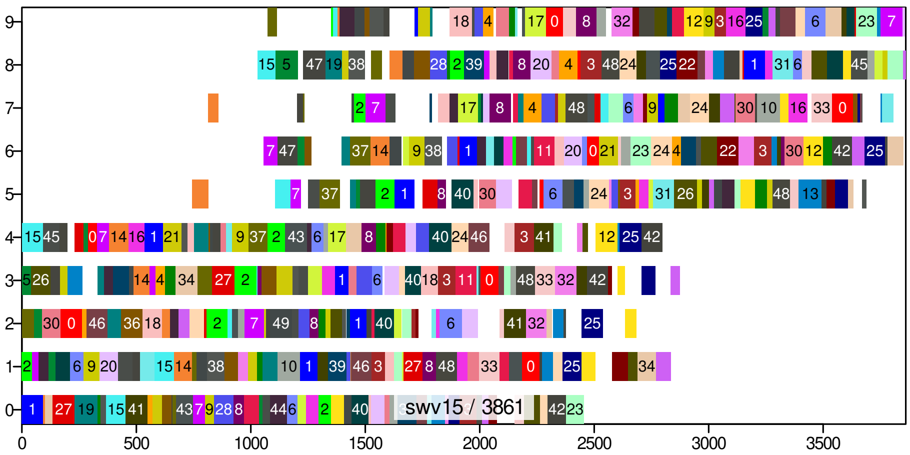

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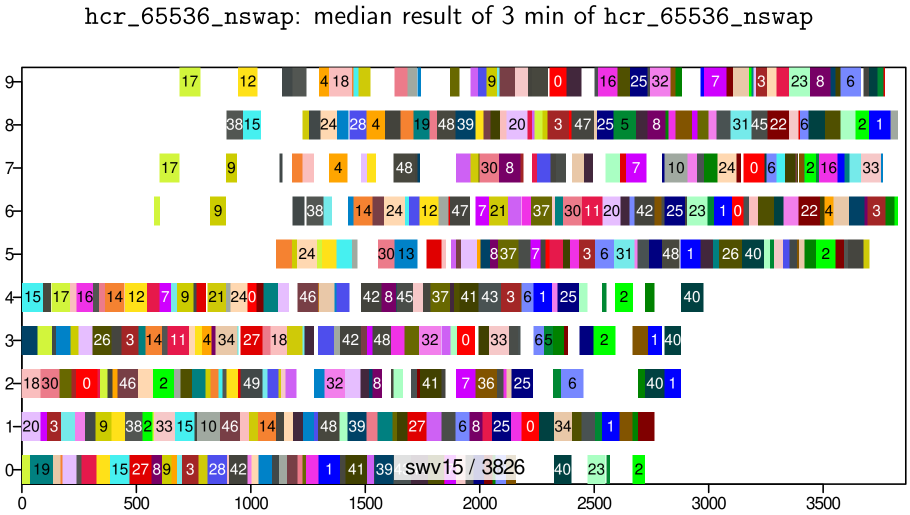

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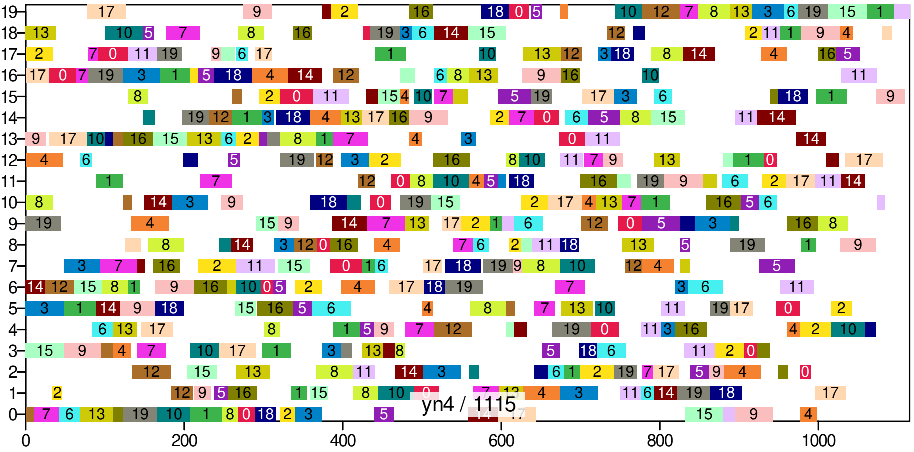

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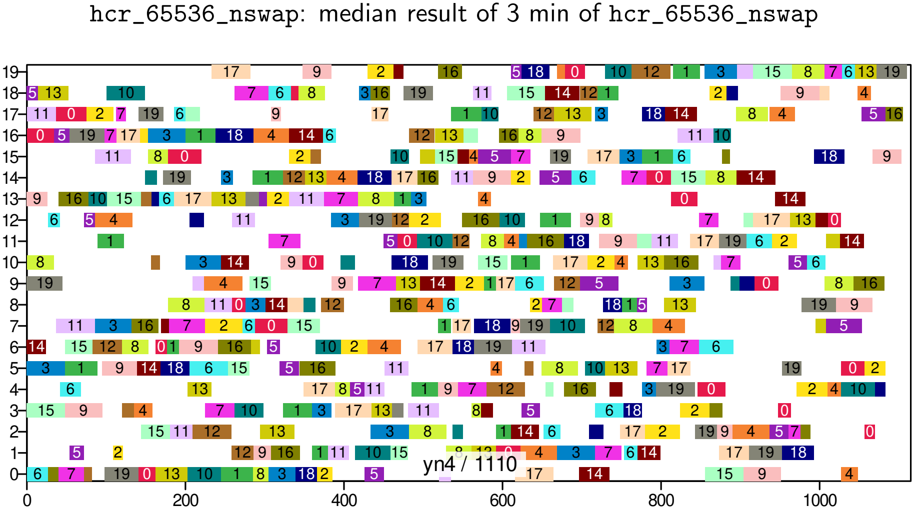

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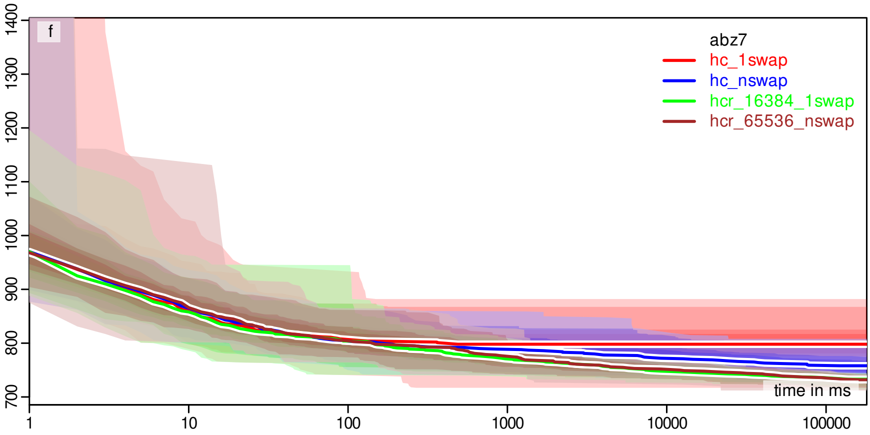

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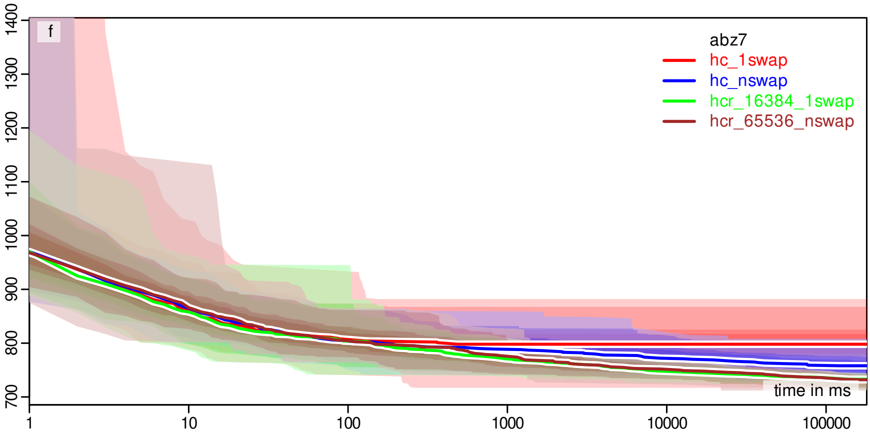

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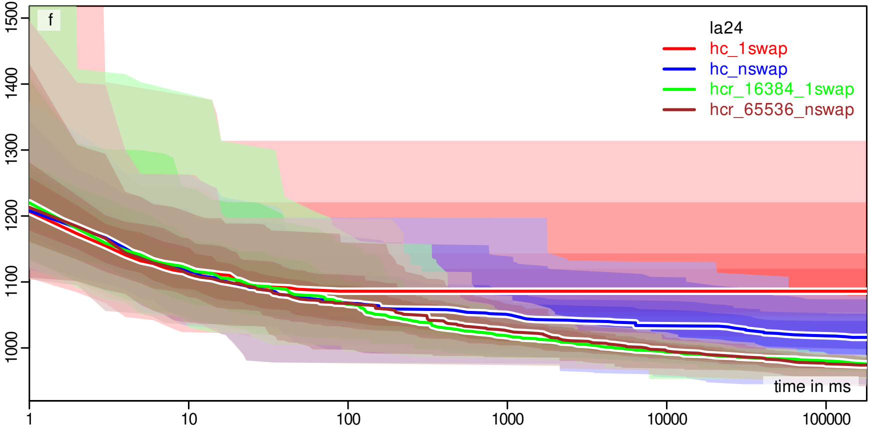

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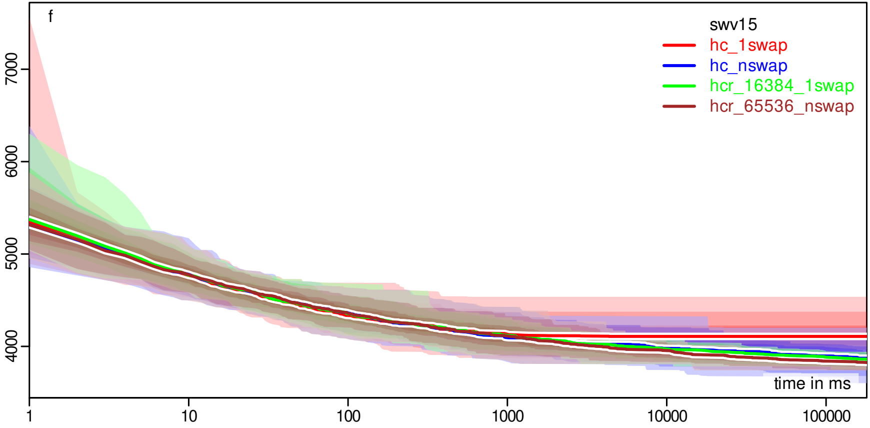

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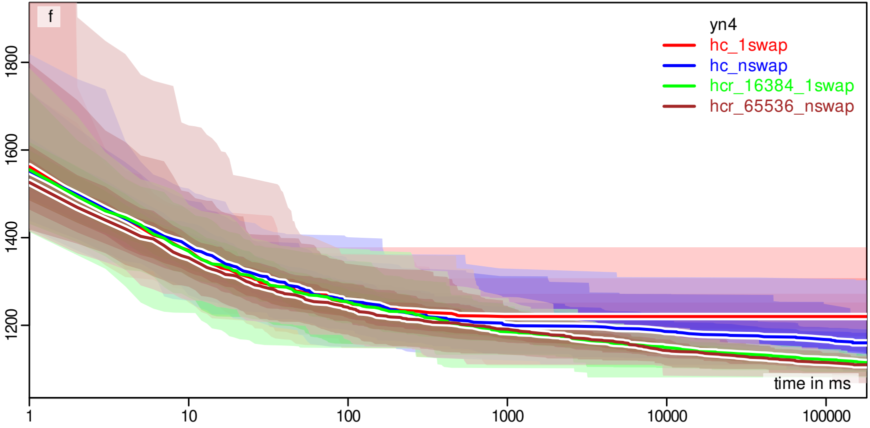

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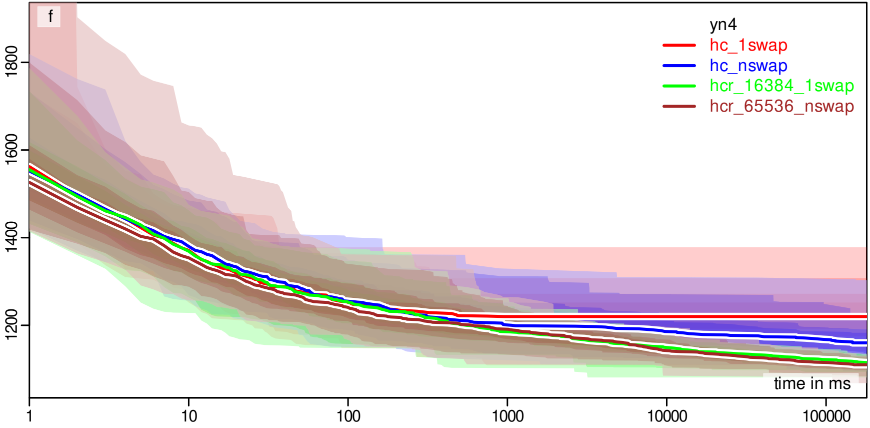

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- Hill climbing is a local search and vulnerable to get trapped in local optima.
- We can try to work around that by implementing good search operators and by restarting the algorithm.

谢谢

Thank you

References I

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