合肥学院
HEFEI UNIVERSITY

Optimization Algorithms

6. Evolutionary Algorithms

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Outline

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- If they reach a local optimum $x^{\times}$, they get trapped.
- We then can restart them, but this means
    1. to start again back at "zero" and losing all accumulated information and
    2. they may still land again in a local optimum.
- We can use unary operators which sample non-uniformly from larger neighborhoods, like `nswap`, but the search move needed to escape from a good but non-optimal point might be too unlikely.
- Idea: We could investigate multiple points in the search space at once and use the additional information in a clever way?

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- This has a couple of advantages:
  - We are less likely to get trapped in a single local optimum (because we work on multiple points).
  - We might more likely find a better (local) optimum.
  - If we have different good points from the search space in our population, we can try to use this additional information...

Algorithm Concept: Population

$(\mu + \lambda)$ EA

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    1. Generate a population of $\mu + \lambda$ random points in the search space (and map them to solutions and evaluate them).

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  3. Generate λ new "offspring" points by applying a unary search operator (which creates a randomly modified copy from a selected point).

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  3. Generate λ new "offspring" points by applying a unary search operator (which creates a randomly modified copy from a selected point).
  4. Evaluate the λ offsprings, add them to the population, and go back to step 2..

Ingredient: Solution Record

```java
package aitoa.structure;

public class Record<X> {

  /** The comparator to be used for sorting according
      quality */
  public static final Comparator<Record<?>> BY_QUALITY =
      (a, b) -> Double.compare(a.quality, b.quality);

  /** the point in the search space */
  public final X x;
  /** the quality */
  public double quality;

// unnecessary stuff omitted here...
}
```

Evolutionary Algorithm Implementation

```
package aitoa.algorithms;

public class EA<X, Y> {
// abridged code: unnecessary stuff omitted here and in function solve...
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      Record<X> sel  = P[p1];
      this.unary.apply(sel.x, dest.x, random); // generate offspring
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    for (int index = P.length; (--index) >= this.mu;) { // overwrite lambda worst
      if (process.shouldTerminate()) return;
      Record<X> dest = P[index];
      p1 = (p1 + 1) % this.mu; // step the parent 1 index
      Record<X> sel  = P[p1];
      this.unary.apply(sel.x, dest.x, random); // generate offspring
      dest.quality = process.evaluate(dest.x); // evaluate offspring
    } // the end of the offspring generation
//
  } // end solve
} // end class
```

Evolutionary Algorithm Implementation

```java
package aitoa.algorithms;

public class EA<X, Y> extends Metaheuristic2<X, Y> {
// abridged code: unnecessary stuff omitted here and in function solve...
  public void solve(IBlackBoxProcess<X, Y> process) {
    Random      random      = process.getRandom();
    ISpace<X>   searchSpace = process.getSearchSpace();
    Record<X>[] P           = new Record[this.mu + this.lambda];

    for (int i = P.length; (--i) >= 0;) { // first generation: fill P with random points
      X x  = searchSpace.create();   // allocate point
      this.nullary.apply(x, random); // fill with random data
      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

    for (;;) { // main loop: one iteration = one generation
      Arrays.sort(P, Record.BY_QUALITY); // sort the population: mu best at front
      RandomUtils.shuffle(random, P, 0, this.mu); // shuffle parents for fairness
      int p1 = -1; // index to iterate over first parent
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Experiment and Analysis

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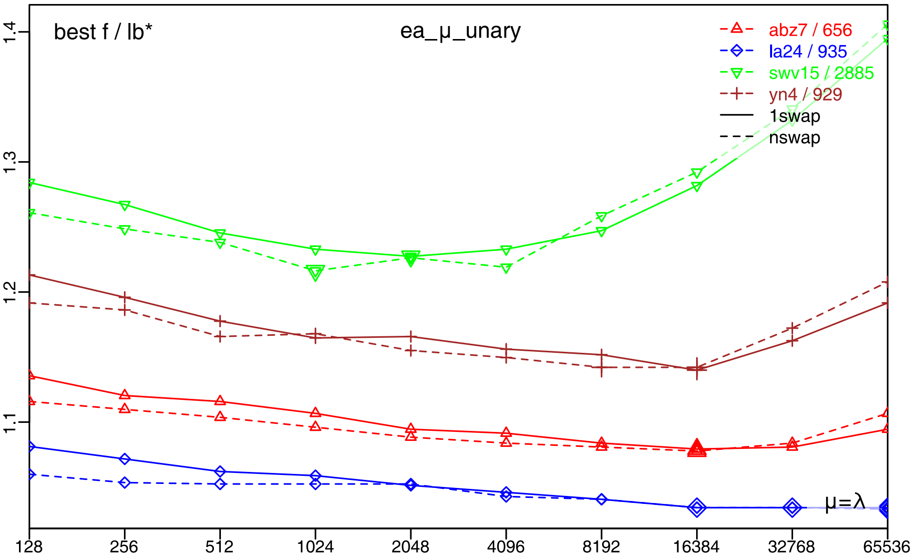

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- Generally, the EA seems to be quite robust and performs well for many parameter settings (except on `swv15`).

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		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	hcr_65536_nswap	712	731	732	**6**	96s	21'189'358
	ea_16384_nswap	**691**	**707**	**707**	8	151s	25'293'859
la24	hcr_65536_nswap	**942**	973	974	**8**	71s	31'466'420
	ea_16384_nswap	945	**968**	**967**	12	39s	10'161'119
swv15	hcr_65536_nswap	3740	3818	3826	**35**	89s	10'783'296
	ea_16384_nswap	**3577**	**3723**	**3728**	50	178s	18'897'833
yn4	hcr_65536_nswap	1068	1109	1110	**12**	78s	18'756'636
	ea_16384_nswap	**1022**	**1063**	**1061**	16	168s	26'699'633

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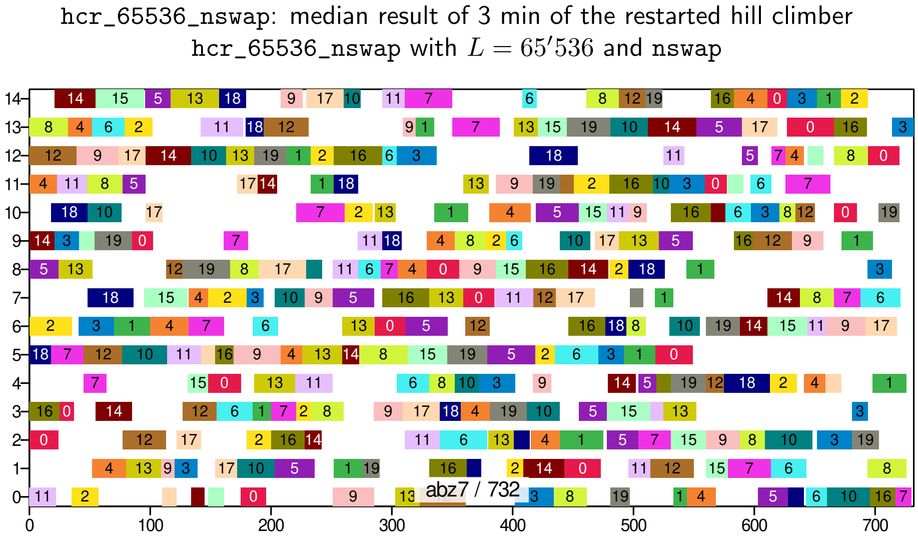

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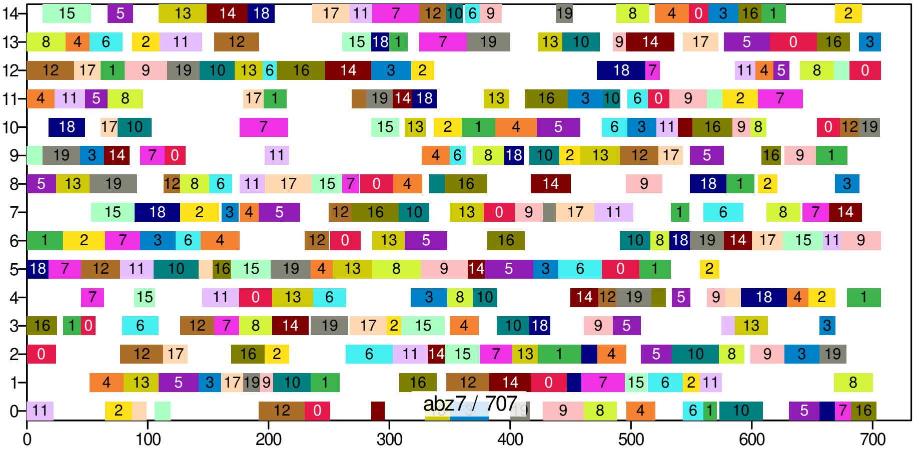

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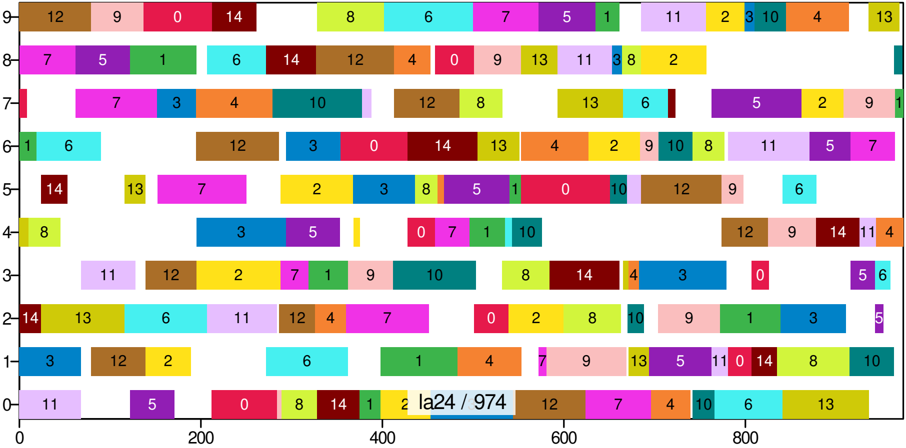

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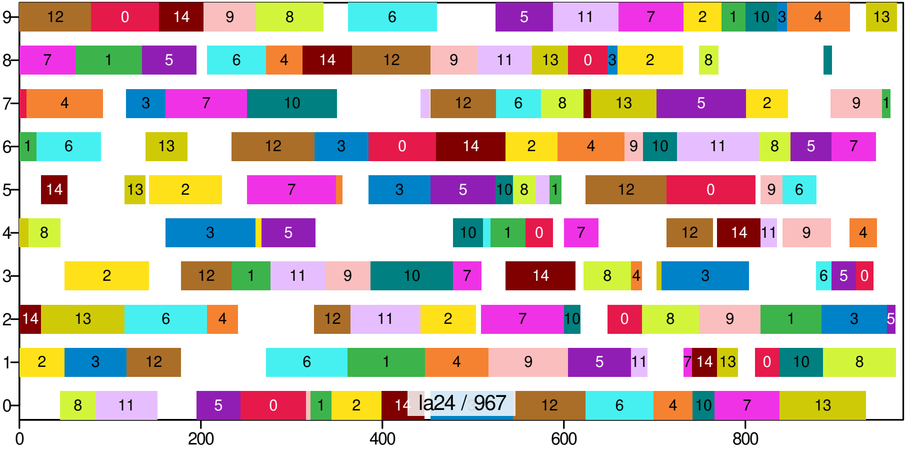

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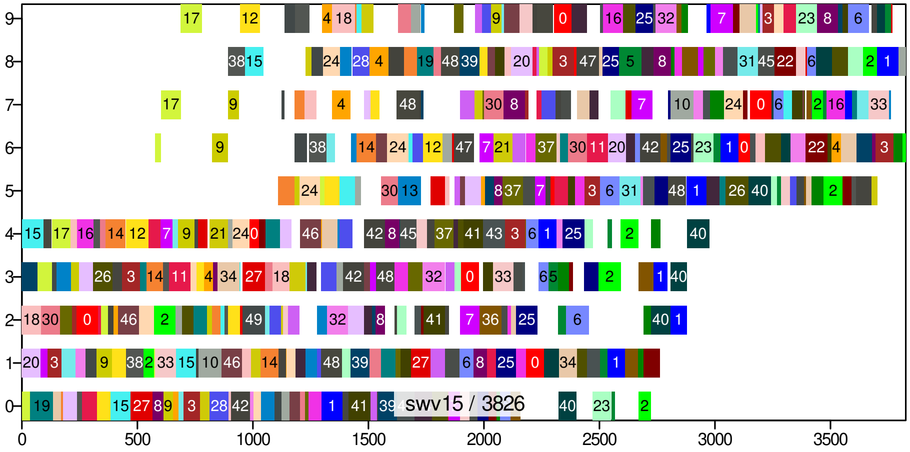

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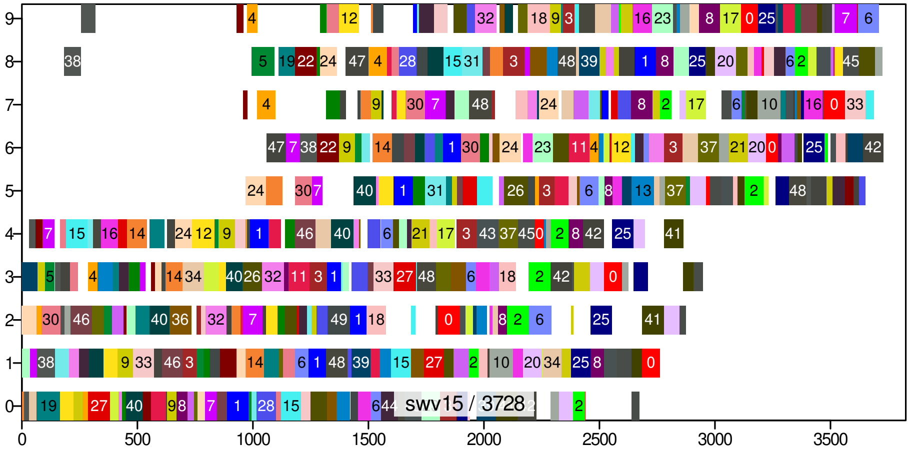

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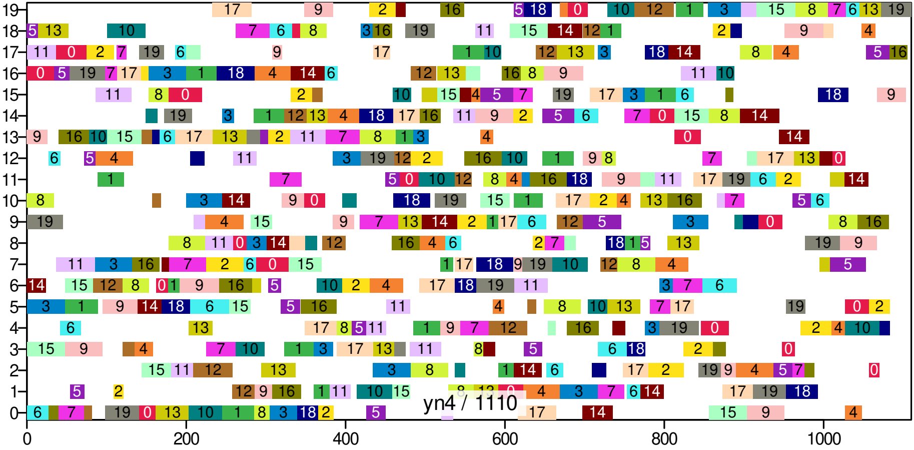

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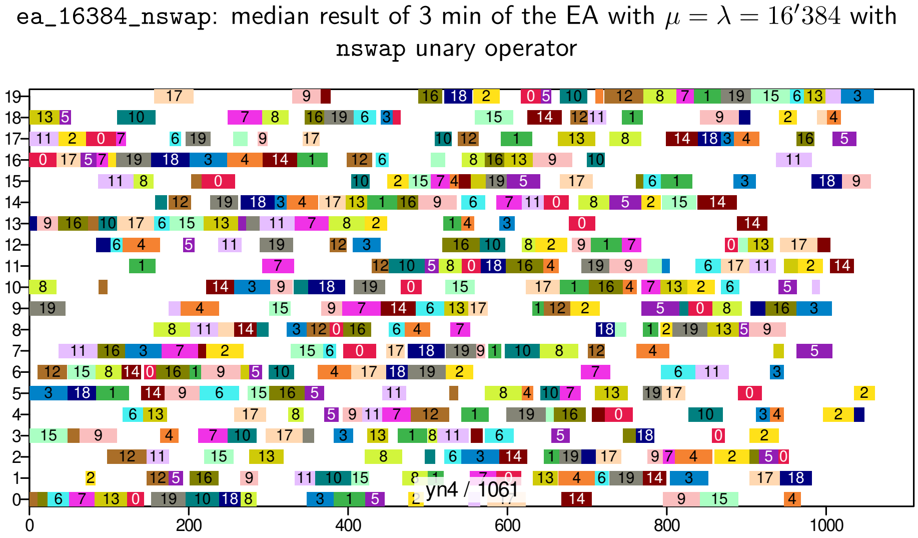

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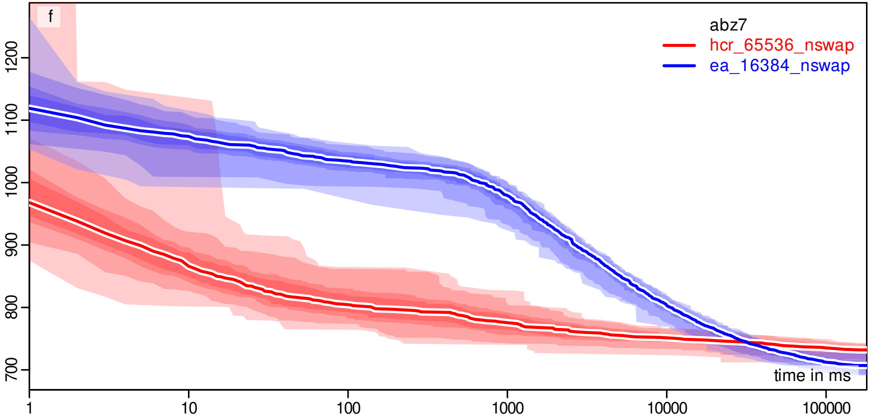

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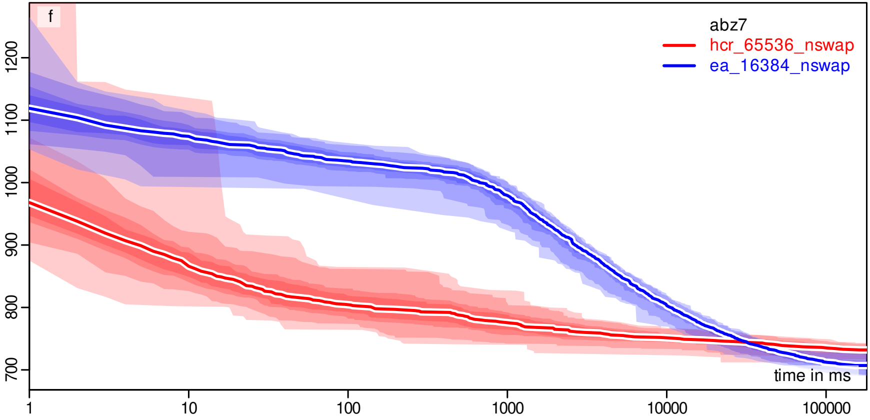

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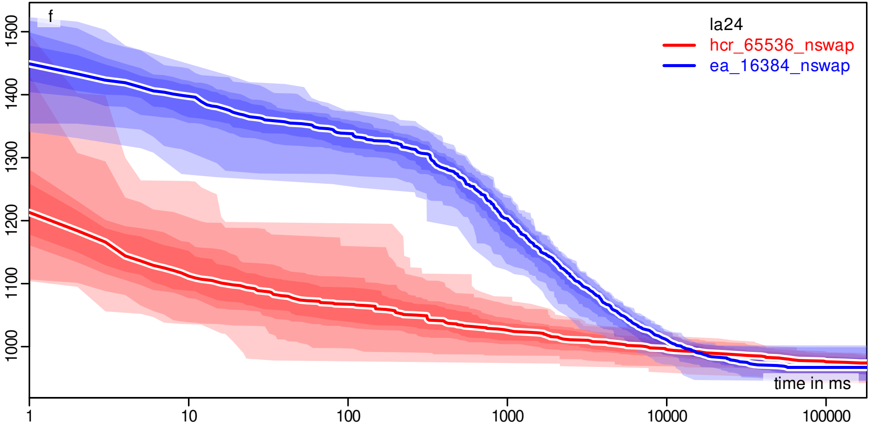

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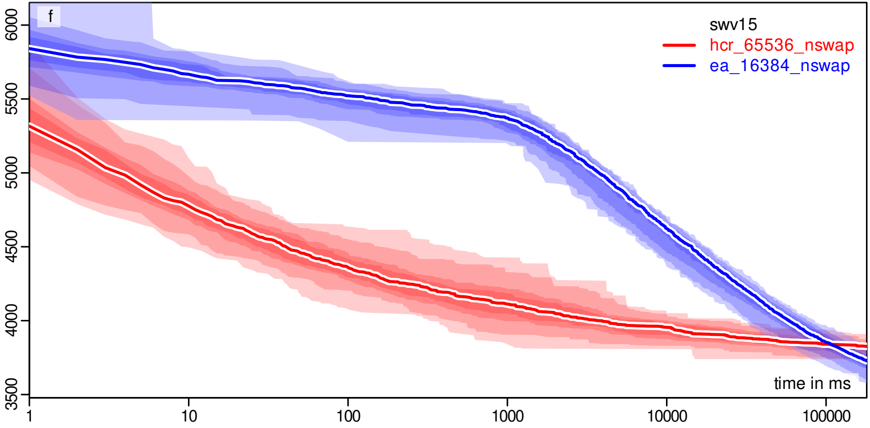

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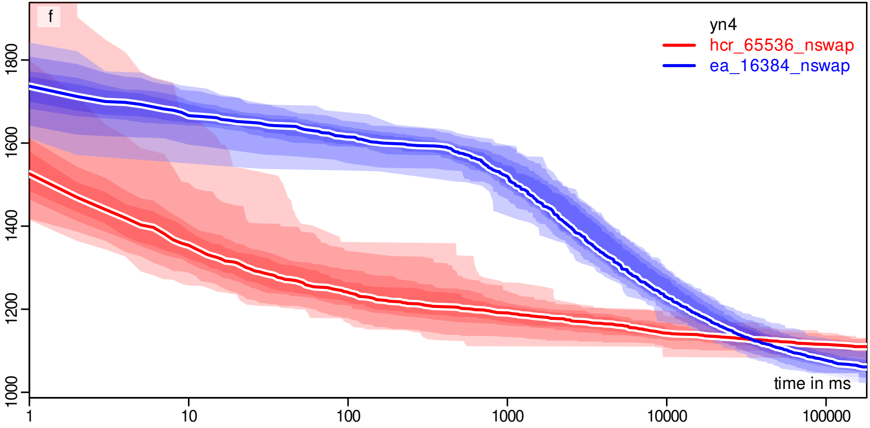

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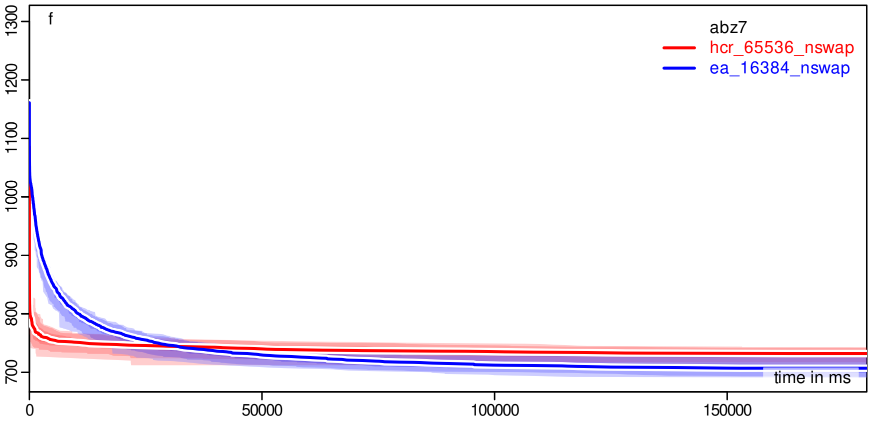

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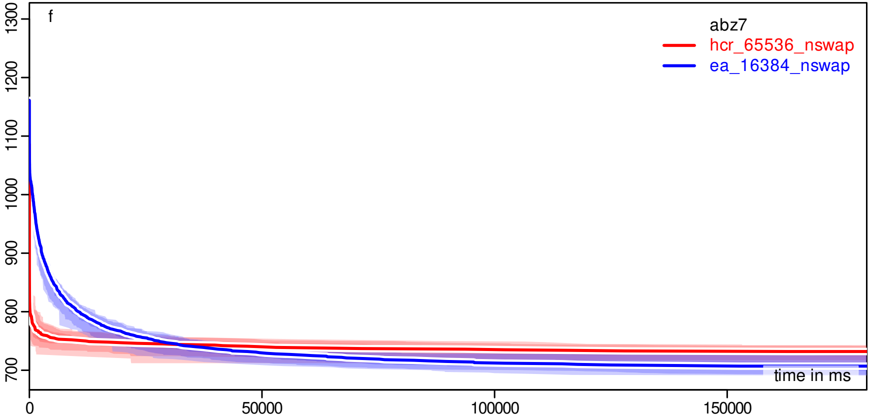

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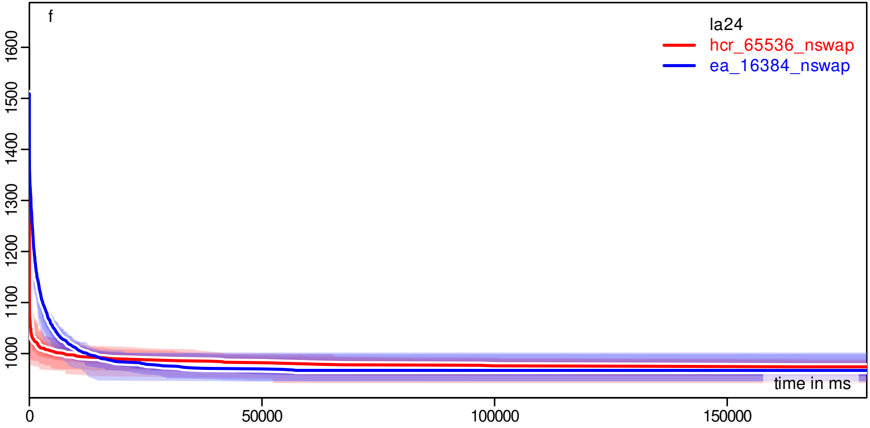

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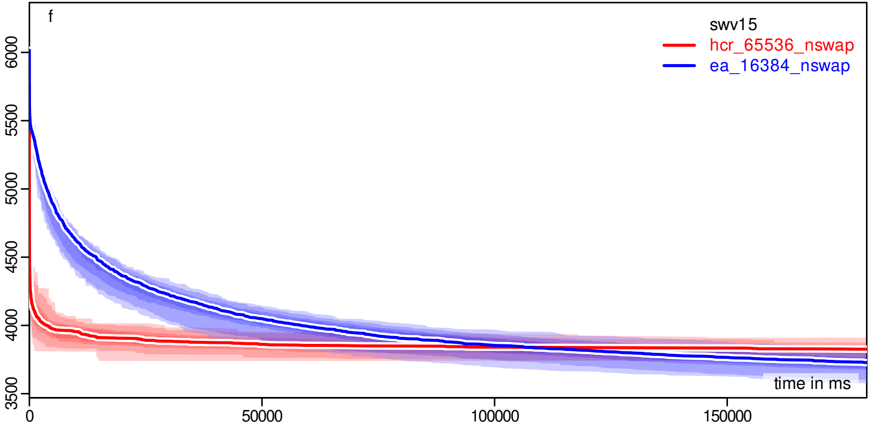

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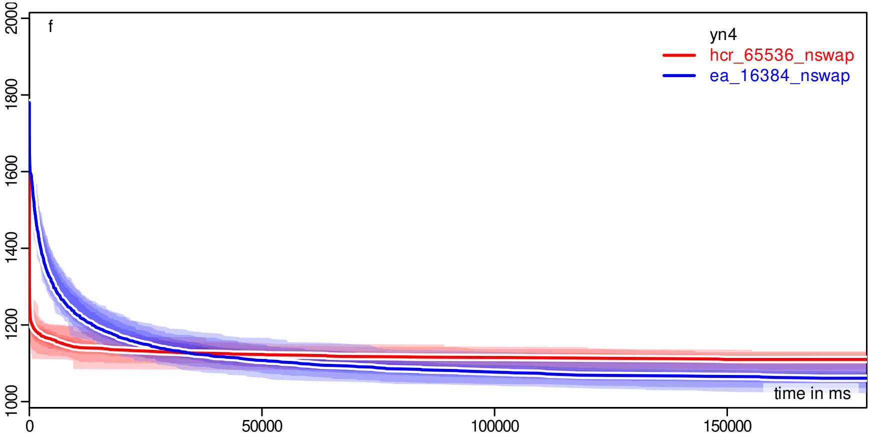

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- This is dilemma of Exploration versus Exploitation.[2 8–10]

Algorithm Concept: Binary Operator

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- This is the idea of the crossover or recombination operator in Evolutionary Algorithms.[2 3 6]

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  2. From the complete population, select the μ best points as "parents" for the next "generation," discard the remaining λ points.
  3. Generate λ new "offspring" points by either
     3.1 applying a binary recombination operator which combines two existing parents to one new offspring with probability cr or
     3.2 applying a unary search operator which creates a randomly modified copy from a parent as offspring.

$(\mu + \lambda)$ **EA with Recombination**

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     - 3.1 applying a binary recombination operator which combines two existing parents to one new offspring with probability cr or
     - 3.2 applying a unary search operator which creates a randomly modified copy from a parent as offspring.
  4. Evaluate the λ offsprings, add them to the population, and go back to step 2.

Implementation

```java
package aitoa.algorithms;

public class EA<X, Y> extends Metaheuristic2<X, Y> {
// abridged code: unnecessary stuff omitted here and in function solve...
  public void solve(IBlackBoxProcess<X, Y> process) {
    Random      random      = process.getRandom();
    ISpace<X>   searchSpace = process.getSearchSpace();
    Record<X>[] P           = new Record[this.mu + this.lambda];

    for (int i = P.length; (--i) >= 0;) { // first generation: fill P with random points
      X x  = searchSpace.create();   // allocate point
      this.nullary.apply(x, random); // fill with random data
      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

    for (;;) { // main loop: one iteration = one generation
      Arrays.sort(P, Record.BY_QUALITY); // sort the population: mu best at front
      RandomUtils.shuffle(random, P, 0, this.mu); // shuffle parents for fairness
      int p1 = -1; // index to iterate over first parent
      for (int index = P.length; (--index) >= this.mu;) { // overwrite lambda worst
        if (process.shouldTerminate()) return;
        Record<X> dest = P[index];
        p1 = (p1 + 1) % this.mu; // step the parent 1 index
        Record<X> sel  = P[p1];
        this.unary.apply(sel.x, dest.x, random); // generate offspring
        dest.quality = process.evaluate(dest.x); // evaluate offspring
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    } // the end of the main loop
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} // end class
```

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        Record<X> sel  = P[p1];
//
//
//
//
//
//
        this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
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      int p1 = -1; // index to iterate over first parent
      for (int index = P.length; (--index) >= this.mu;) { // overwrite lambda worst
        if (process.shouldTerminate()) return;
        Record<X> dest = P[index];
        p1 = (p1 + 1) % this.mu; // step the parent 1 index
        Record<X> sel  = P[p1];
        if (random.nextDouble() <= this.cr) { // crossover!
//
//
//
//
//
        } else this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
        dest.quality = process.evaluate(dest.x); // evaluate offspring
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      RandomUtils.shuffle(random, P, 0, this.mu); // shuffle parents for fairness
      int p1 = -1; // index to iterate over first parent
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        if (process.shouldTerminate()) return;
        Record<X> dest = P[index];
        p1 = (p1 + 1) % this.mu; // step the parent 1 index
        Record<X> sel  = P[p1];
        if (random.nextDouble() <= this.cr) { // crossover!
          int p2;
          do { // find a second, different record
            p2 = random.nextInt(this.mu);
          } while (p2 == p1); // repeat until p1 != p2
//
        } else this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
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      int p1 = -1; // index to iterate over first parent
      for (int index = P.length; (--index) >= this.mu;) { // overwrite lambda worst
        if (process.shouldTerminate()) return;
        Record<X> dest = P[index];
        p1 = (p1 + 1) % this.mu; // step the parent 1 index
        Record<X> sel  = P[p1];
        if (random.nextDouble() <= this.cr) { // crossover!
          int p2;
          do { // find a second, different record
            p2 = random.nextInt(this.mu);
          } while (p2 == p1); // repeat until p1 != p2
          this.binary.apply(sel.x, P[p2].x, dest.x, random); // perform recombination
        } else this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
        dest.quality = process.evaluate(dest.x); // evaluate offspring
      } // the end of the offspring generation
    } // the end of the main loop
  } // end solve
} // end class
```

Recombination for our Representation: One Possible Idea

1. Data structure x' be the destination to hold the new point in the search space that we want to sample.

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3. Repeat

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   3.1 Randomly choose one of the input points $x1$ or $x2$ with equal probability as source x.

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   3.2 Select the first (at the lowest index) operation in x that is not marked yet and store it in variable J.

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   3.3 Set $x'_i = J$.

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   3.5 If $i = n * m$, then all operations have been assigned. We exit and returning x'.

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   3.2 Select the first (at the lowest index) operation in x that is not marked yet and store it in variable J.
   3.3 Set $x'_i = J$.
   3.4 Increase i by one ($i = i + 1$).
   3.5 If $i = n * m$, then all operations have been assigned. We exit and returning x'.
   3.6 Mark the first unmarked occurrence of J as "already assigned" in $x1$.

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   - 3.3 Set $x'_i = J$.
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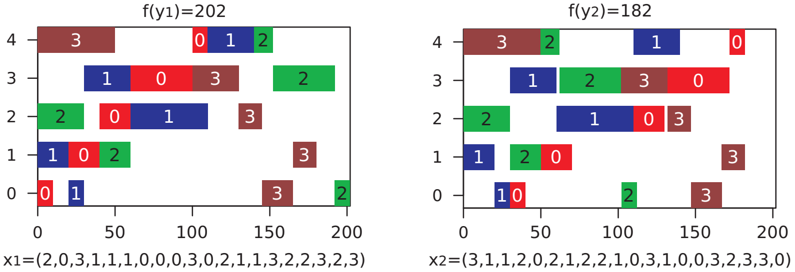
   - 3.5 If $i = n * m$, then all operations have been assigned. We exit and returning x'.
   - 3.6 Mark the first unmarked occurrence of J as "already assigned" in $x1$.
   - 3.7 Mark the first unmarked occurrence of J as "already assigned" in $x2$.

Example for Sequence Recombination

x_1=(2,0,3,1,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3) x_2=(3,1,1,2,0,2,1,2,2,1,0,3,1,0,0,3,2,3,3,0)

Example for Sequence Recombination



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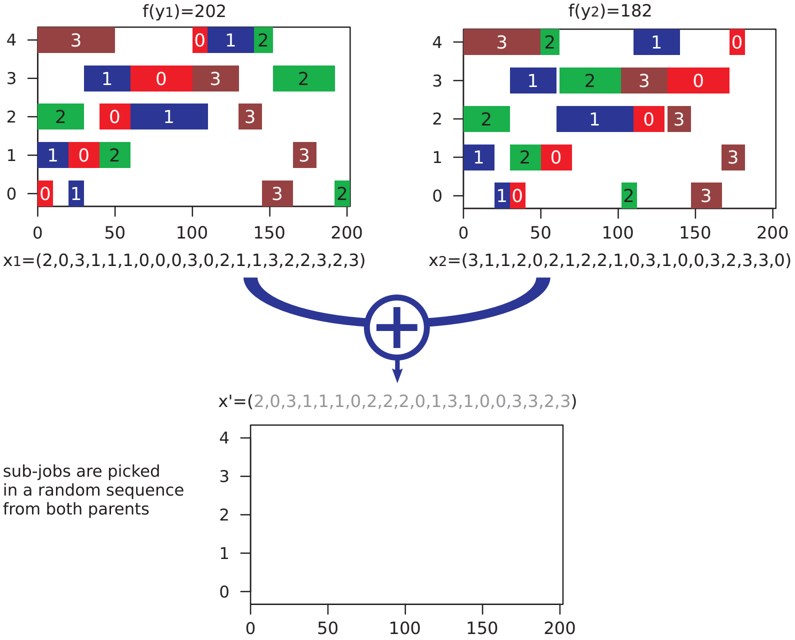

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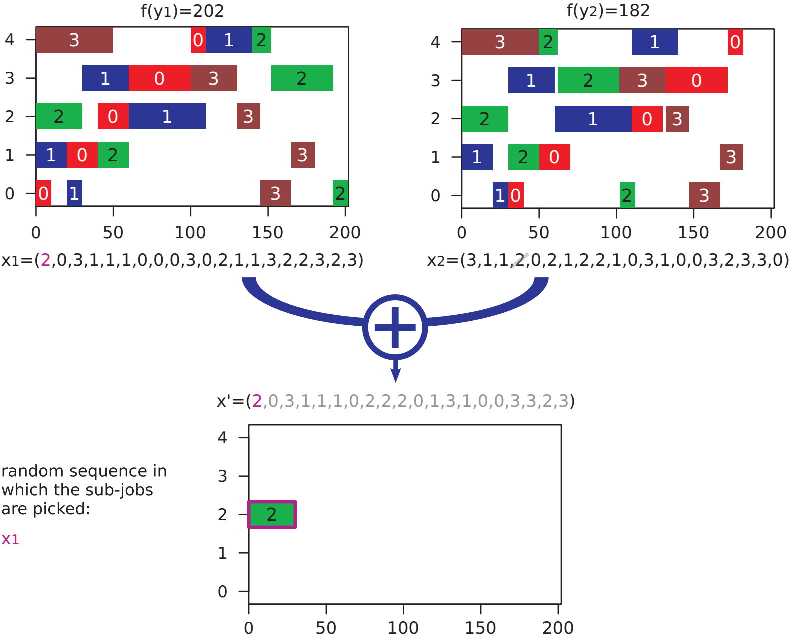

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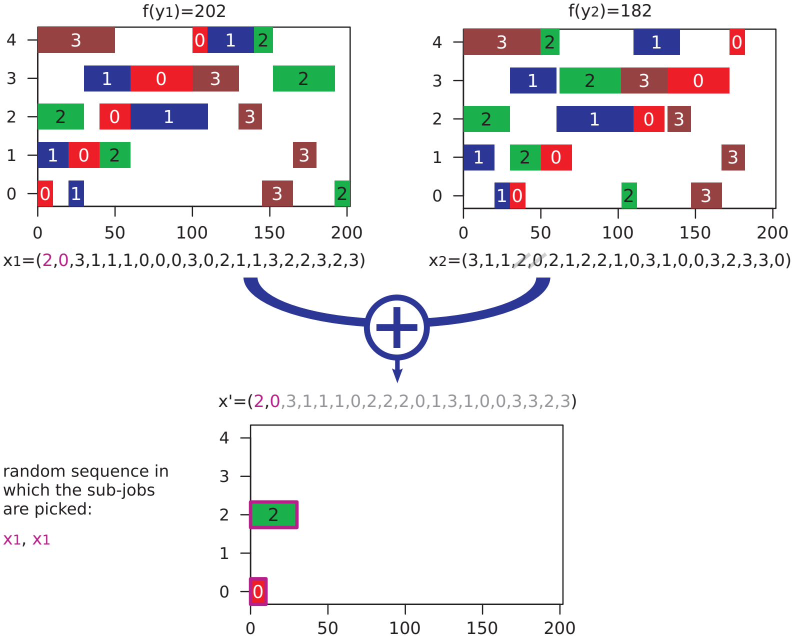

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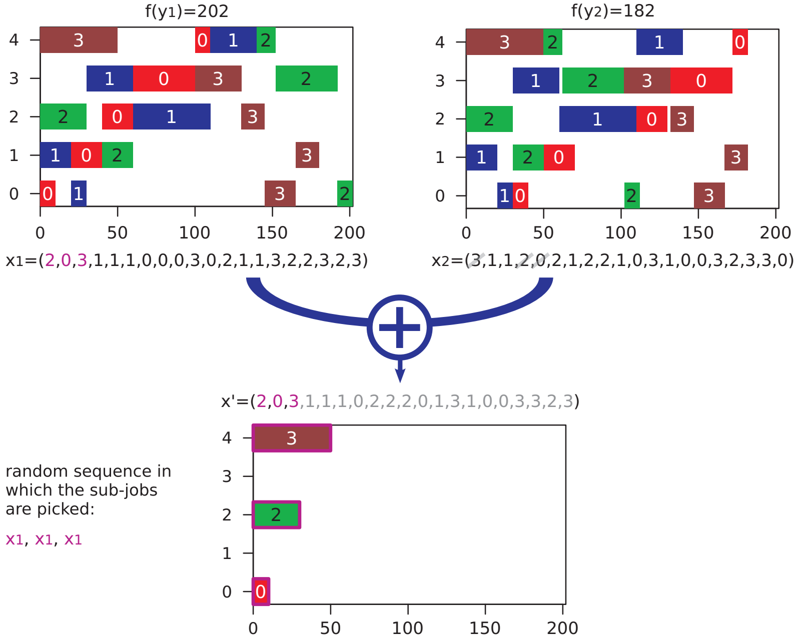

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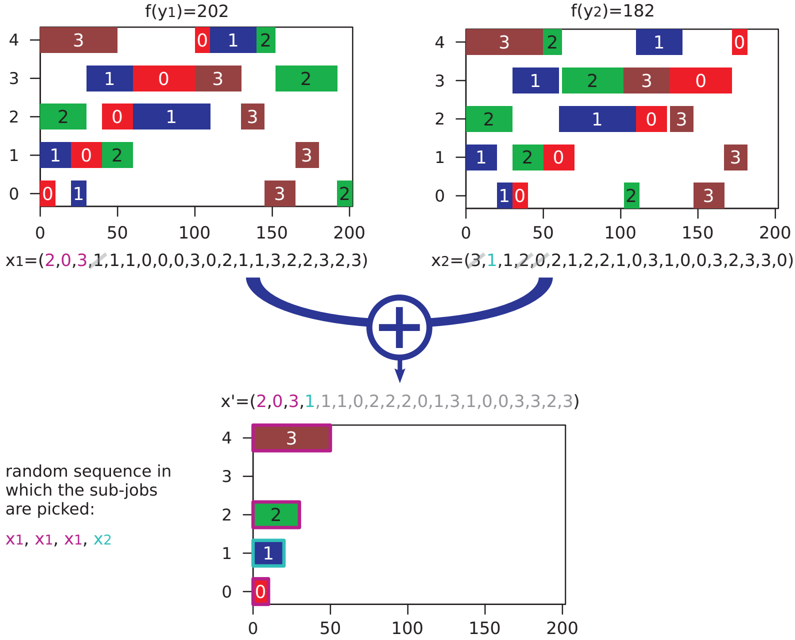

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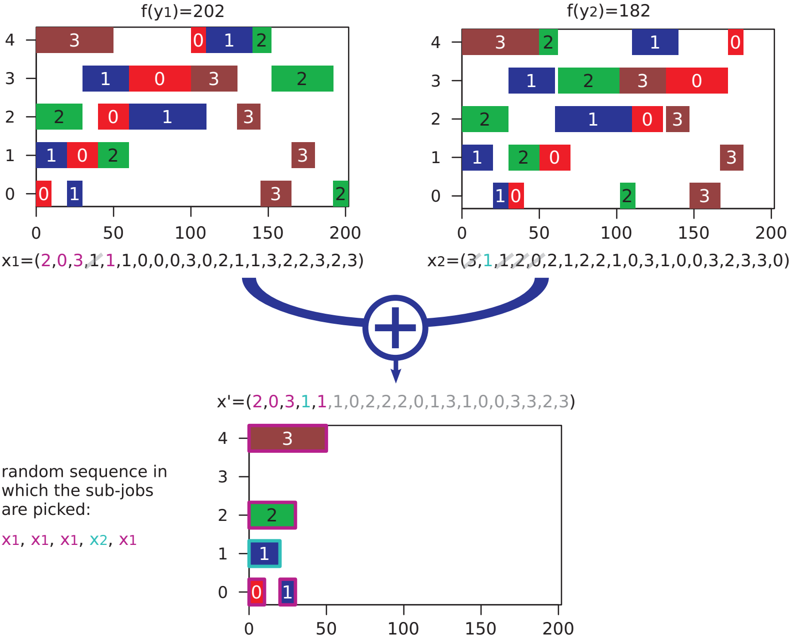

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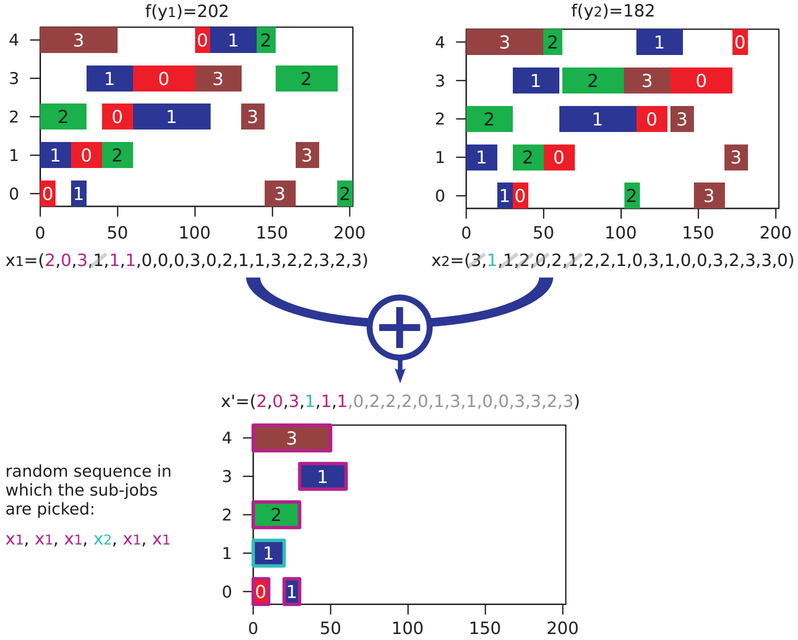

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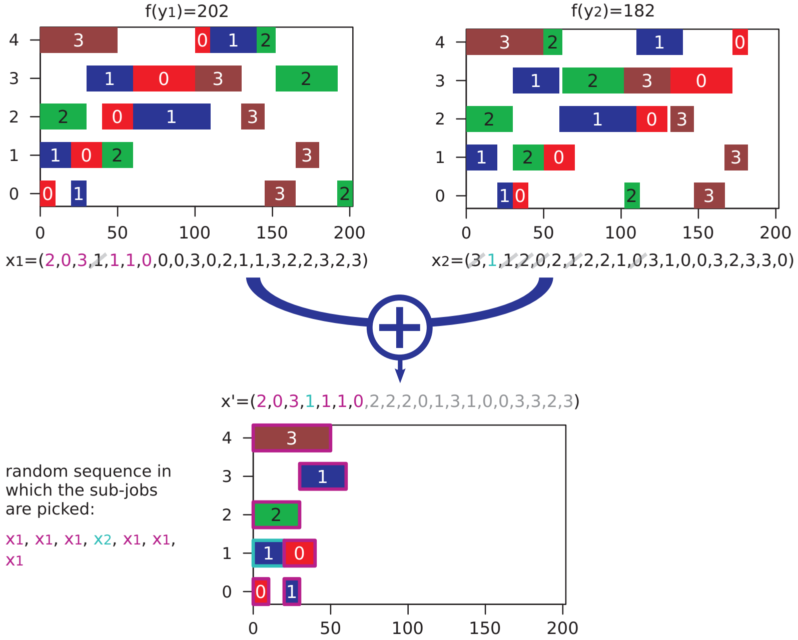

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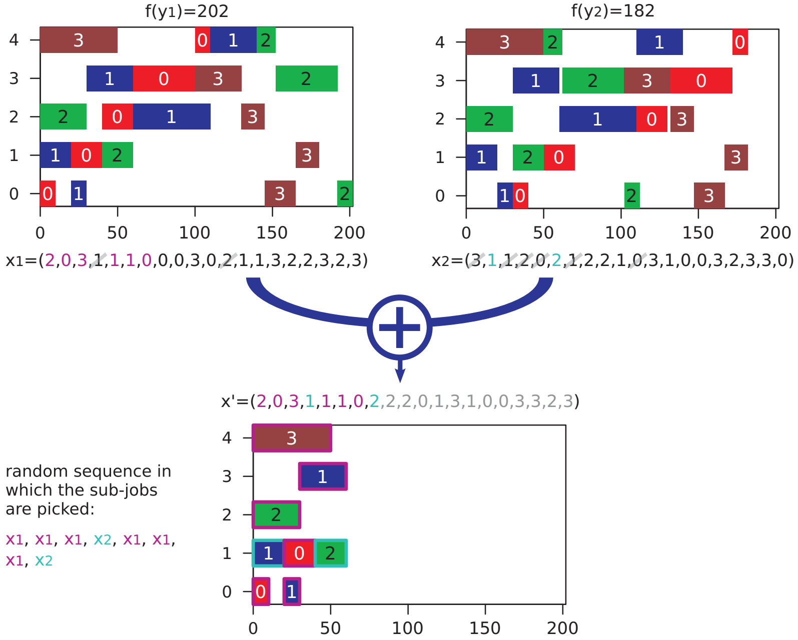

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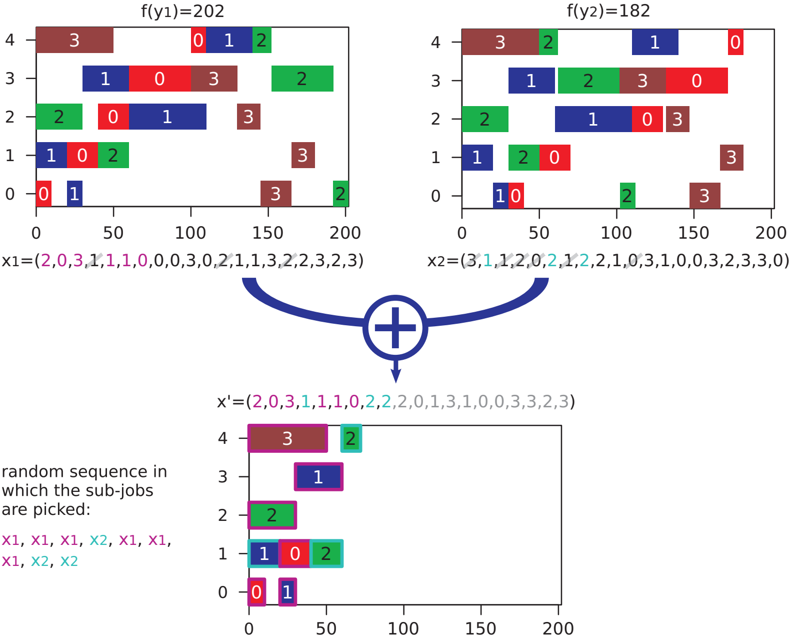

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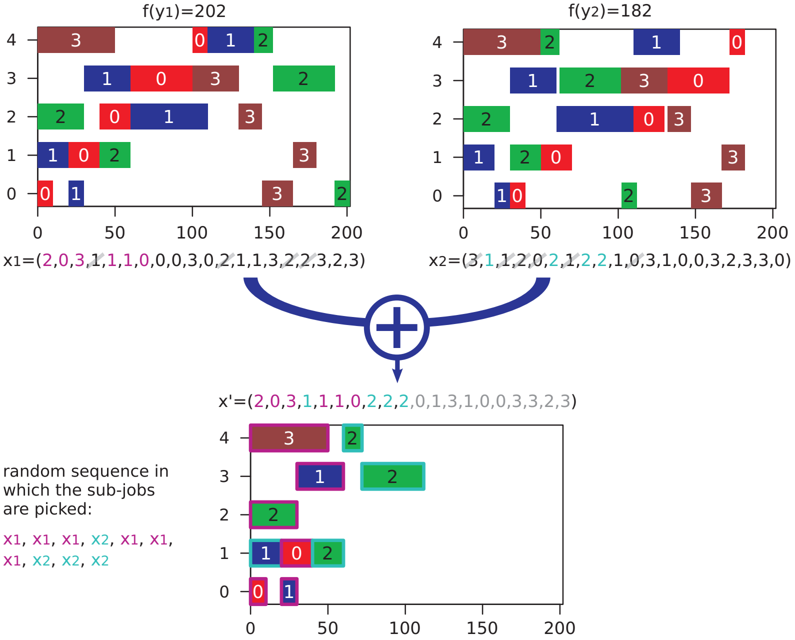

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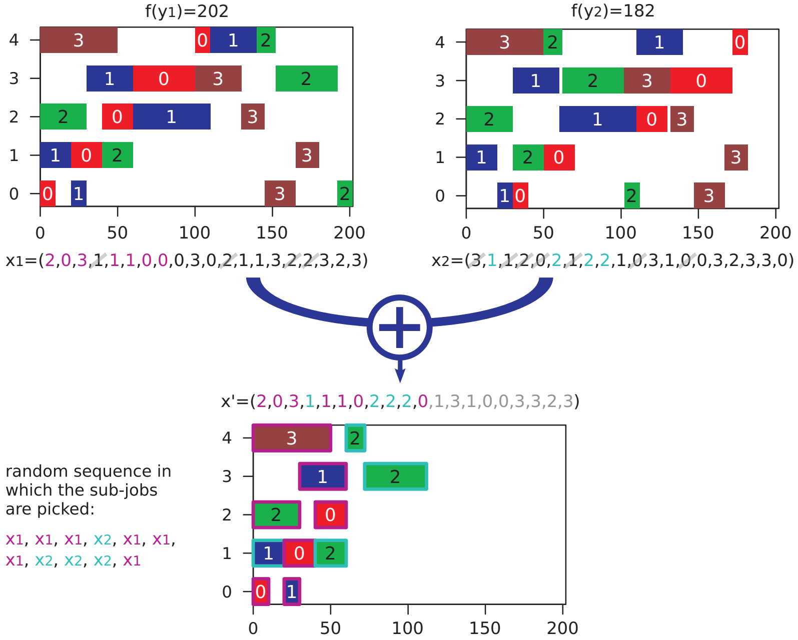

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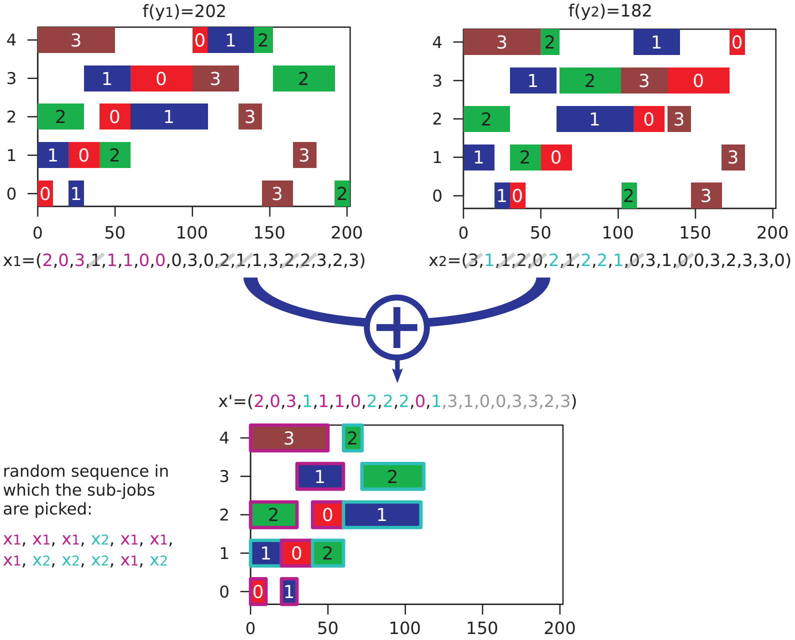

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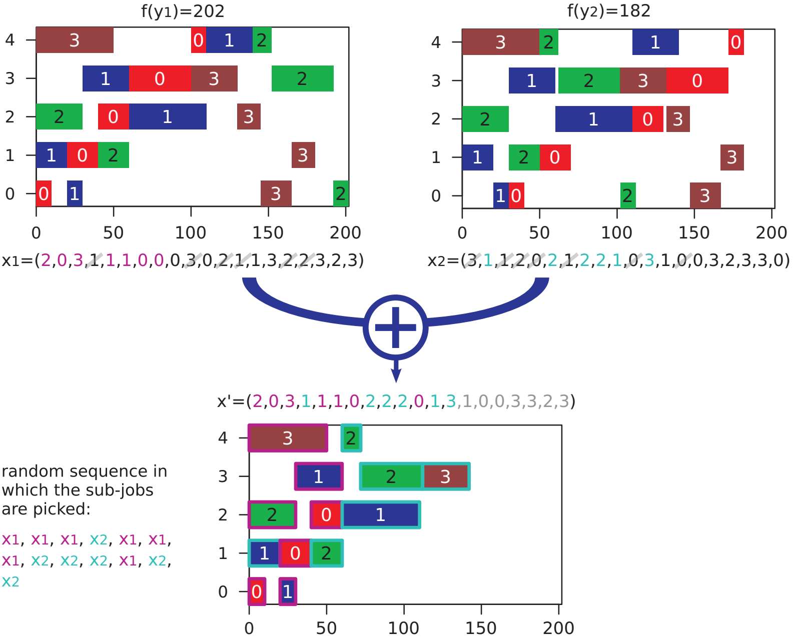

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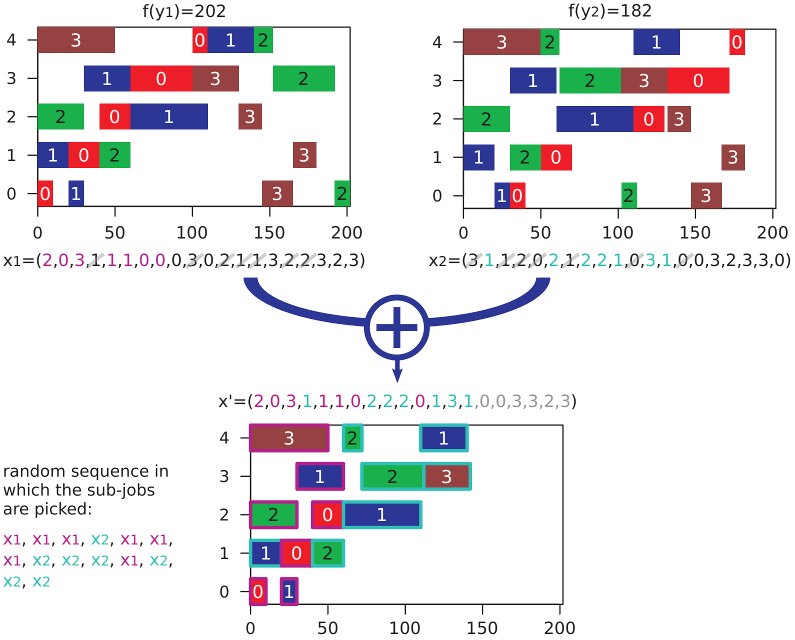

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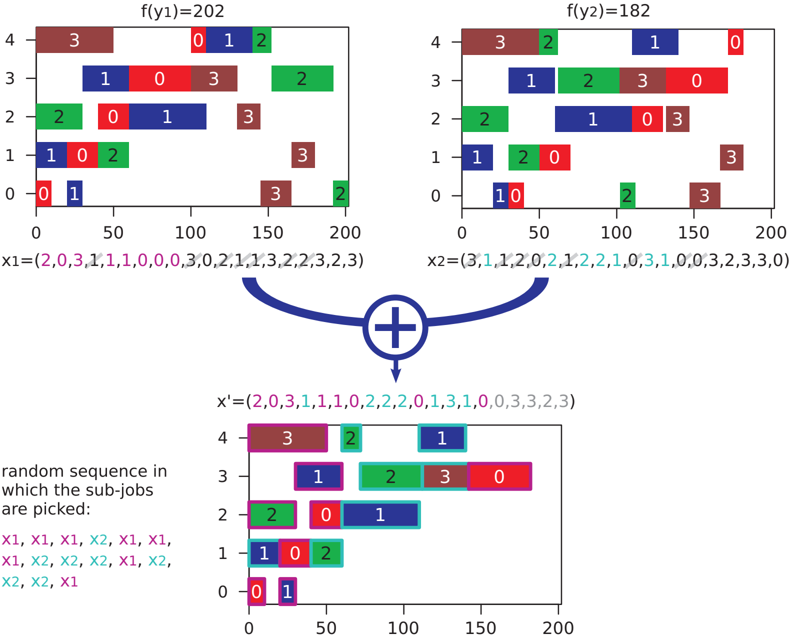

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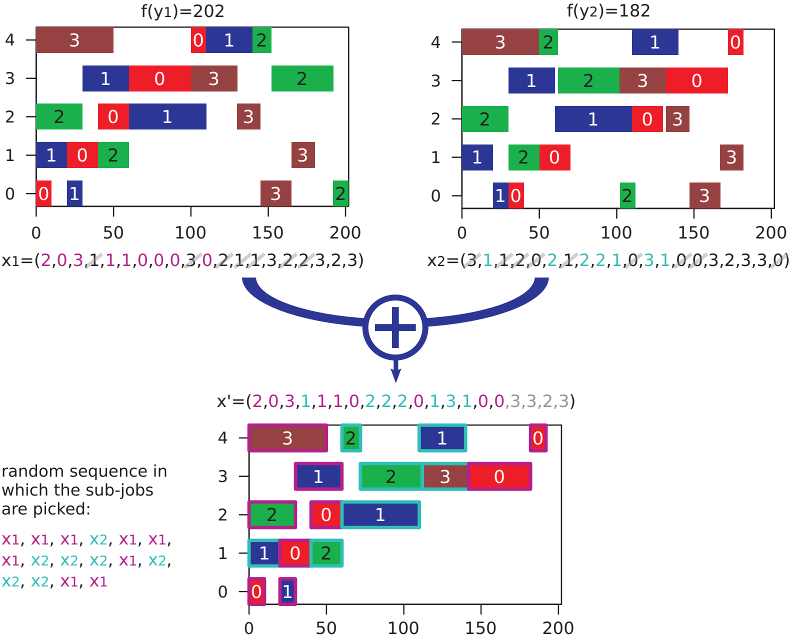

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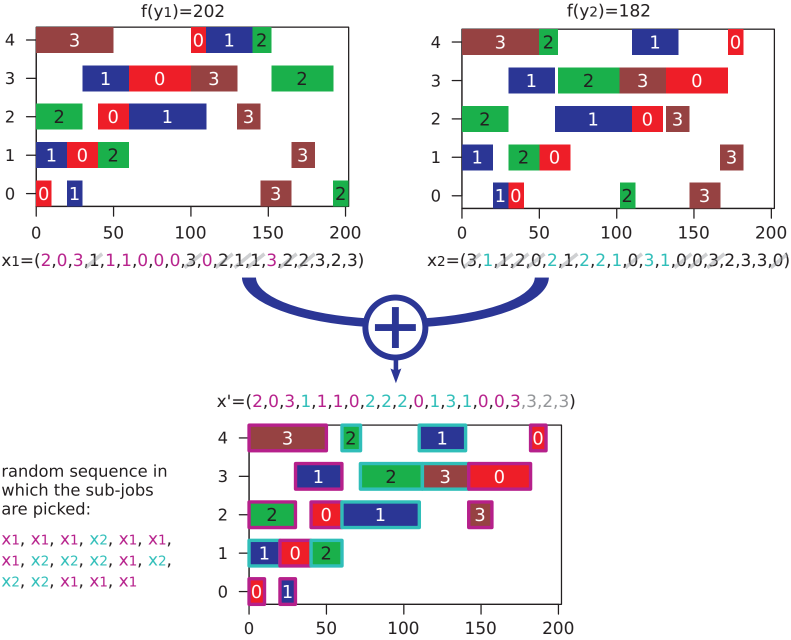

random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1, x2, x2, x2, x1, x2, x2, x2, x1, x1, x1

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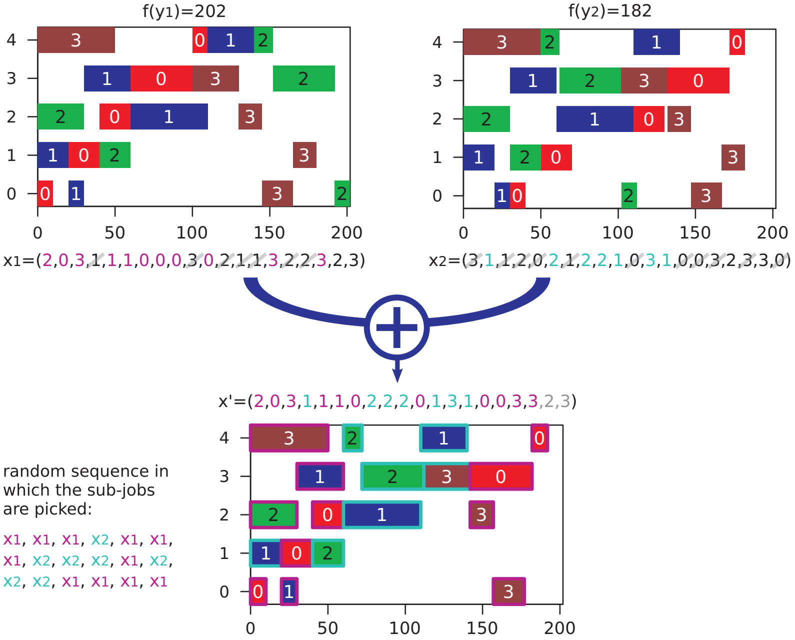

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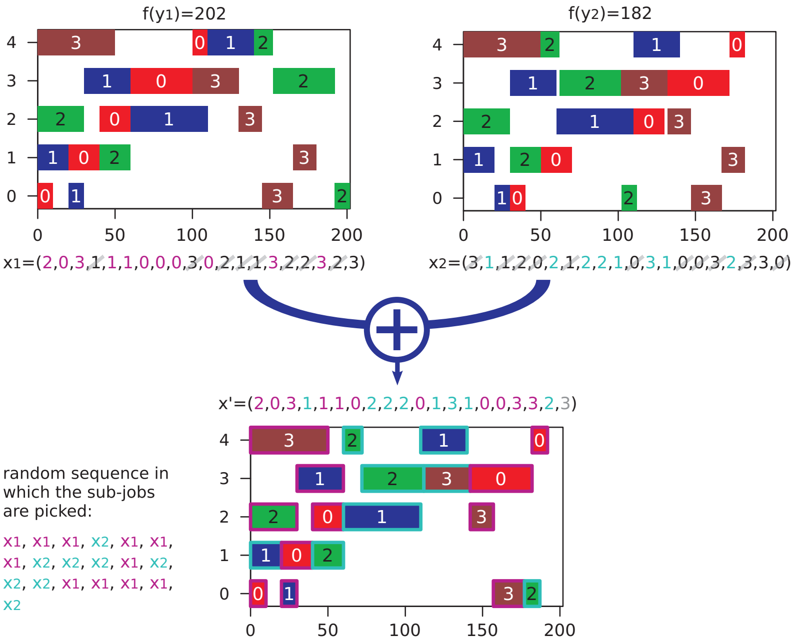

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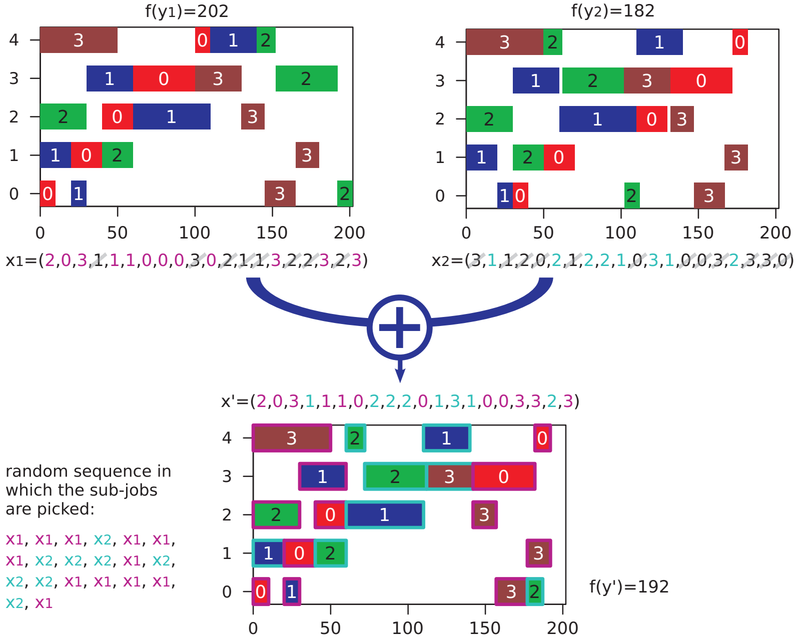

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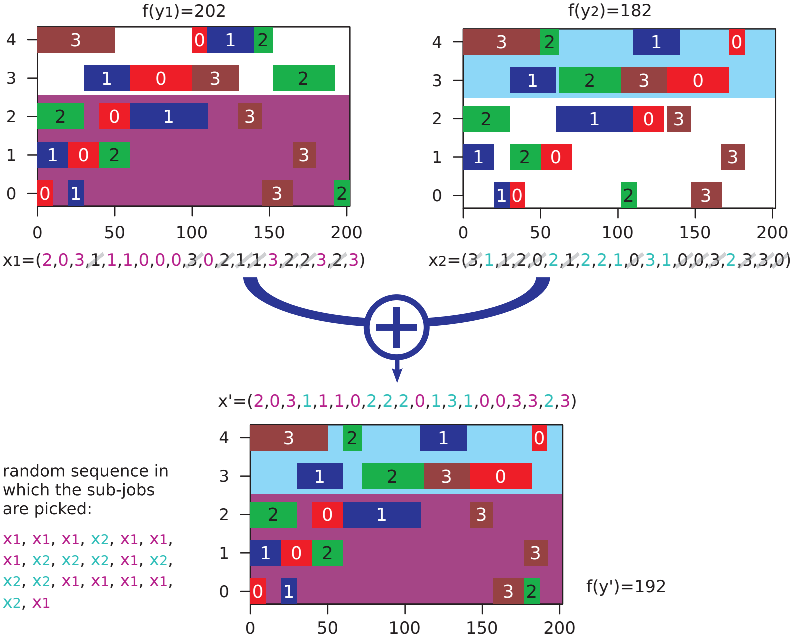

Implementing Sequence Recombination

```
package aitoa.examples.jssp;

public class JSSPBinaryOperatorSequence {
//
//
//

//
//
//
//

//
//
//
//
//

//
//
//
//
//
//
//

//
//
//
//
//
//
//
//
//
}
```

Implementing Sequence Recombination

```
package aitoa.examples.jssp;

public class JSSPBinaryOperatorSequence implements IBinarySearchOperator<int[]> {
//
//
//

//
//
//
//

//
//
//
//
//

//
//
//
//
//
//
//

//
//
//
//
//
//
//
//
//
}
```

Implementing Sequence Recombination

```
package aitoa.examples.jssp;

public class JSSPBinaryOperatorSequence implements IBinarySearchOperator<int[]> {
  public void apply(int[] x0, int[] x1, int[] dest, Random random) {
//
//

//
//
//
//

//
//
//
//
//

//
//
//
//
//
//
//

//
//
//
//
//
//
//
//
  } // end of function
}
```

Implementing Sequence Recombination

```
package aitoa.examples.jssp;

public class JSSPBinaryOperatorSequence implements IBinarySearchOperator<int[]> {
  public void apply(int[] x0, int[] x1, int[] dest, Random random) {
    boolean[] doneX0 = new boolean[x0.length];
//

//
//
//
//

//
//
//
//
//

//
//
//
//
//
//
//

//
//
//
//
//
//
//
//
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```

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    boolean[] doneX0 = new boolean[x0.length];
    boolean[] doneX1 = new boolean[x0.length];

//
//
//
//

//
//
//
//
//

//
//
//
//
//
//
//

//
//
//
//
//
//
//
//
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public class JSSPBinaryOperatorSequence implements IBinarySearchOperator<int[]> {
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    boolean[] doneX0 = new boolean[x0.length]; // can be stored as reuseable
    boolean[] doneX1 = new boolean[x0.length]; // member variable instead

//
//
//
//

//
//
//
//
//

//
//
//
//
//
//
//

//
//
//
//
//
//
//
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Experiment and Analysis

Configuring the Algorithm

- We now have everything together, the EA that can use a binary operator and a simple idea for a binary operator.

Configuring the Algorithm

- We now have everything together, the EA that can use a binary operator and a simple idea for a binary operator.
- But now we have five parameters!

Configuring the Algorithm

- We now have everything together, the EA that can use a binary operator and a simple idea for a binary operator.
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Configuring the Algorithm

- We now have everything together, the EA that can use a binary operator and a simple idea for a binary operator.
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Configuring the Algorithm

- We now have everything together, the EA that can use a binary operator and a simple idea for a binary operator.
- But now we have five parameters μ, λ, the unary operator, the binary operator, and the crossover rate cr.

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- Let's stick with $\mu = \lambda$, `nswap`, and our `sequence` recombination operator.

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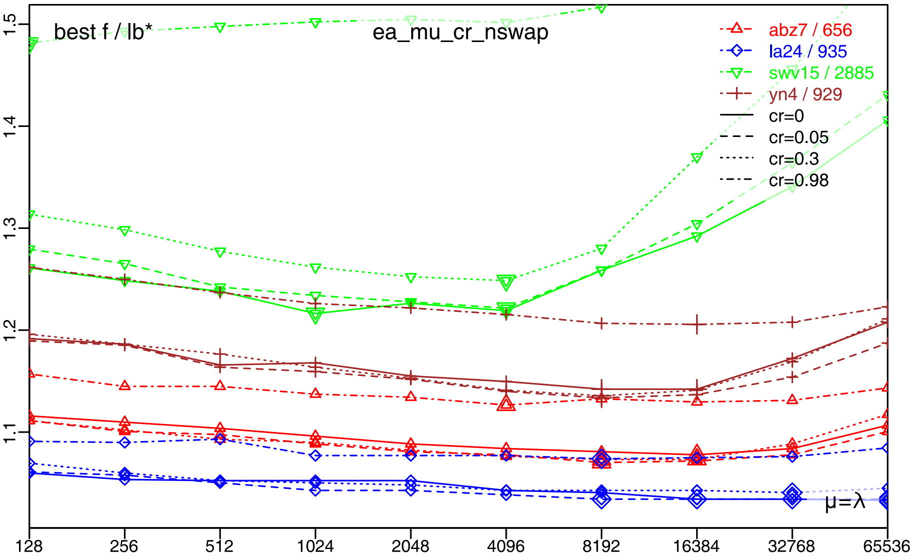
- This leaves us to choose the value of λ and cr.

Configuring the Algorithm



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- This leaves us to choose the value of λ and cr.
- The improvements that the binary operator offered us in this scenario are quite small.
- Nevertheless, creating 5% of the offspring with it seems a reasonable idea at $\lambda = \mu = 8192$.

So what do we get?

- I execute the program 101 times for each of the instances `abz7`, `la24`, `swv15`, and `yn4`

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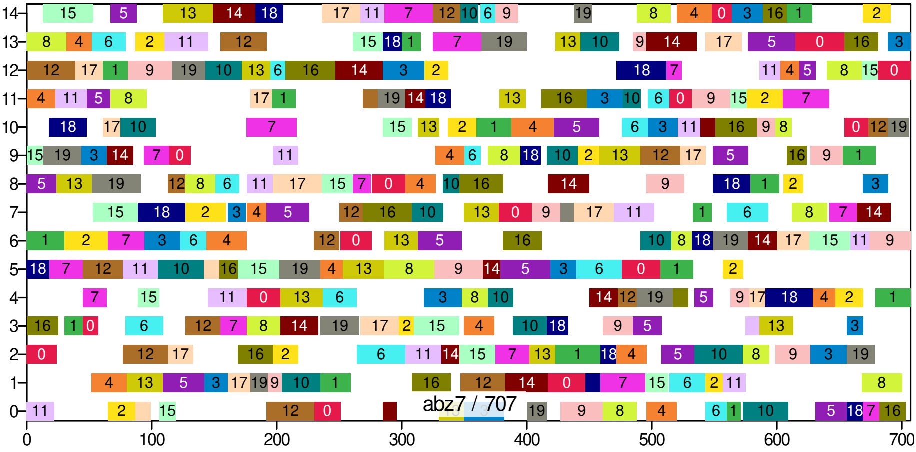
- I execute the program 101 times for each of the instances abz7, la24, swv15, and yn4

		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	hcr_65536_nswap	712	731	732	**6**	96s	21'189'358
	ea_16384_nswap	691	707	707	8	151s	25'293'859
	ea_8192_5%_nswap	**684**	**703**	**702**	8	54s	10'688'314
la24	hcr_65536_nswap	**942**	973	974	**8**	71s	31'466'420
	ea_16384_nswap	945	968	**967**	12	39s	10'161'119
	ea_8192_5%_nswap	943	**967**	**967**	11	18s	4'990'002
swv15	hcr_65536_nswap	3740	3818	3826	**35**	89s	10'783'296
	ea_16384_nswap	3577	3723	3728	50	178s	18'897'833
	ea_8192_5%_nswap	**3498**	**3631**	**3632**	65	178s	17'747'983
yn4	hcr_65536_nswap	1068	1109	1110	**12**	78s	18'756'636
	ea_16384_nswap	**1022**	1063	1061	16	168s	26'699'633
	ea_8192_5%_nswap	1026	**1056**	**1053**	17	114s	13'206'552

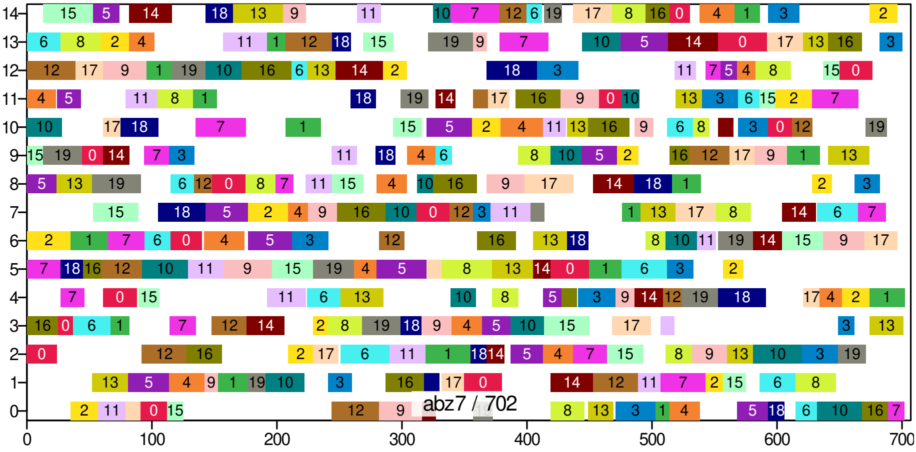
So what do we get?

ea_16384_nswap: median result of 3 min of the EA with $\mu = \lambda = 16'384$ with nswap unary operator



So what do we get?

ea_8192_5%_nswap: median result of 3 min of the EA with $\mu = \lambda = 8'192$ with nswap unary operator and 5% sequence recombination



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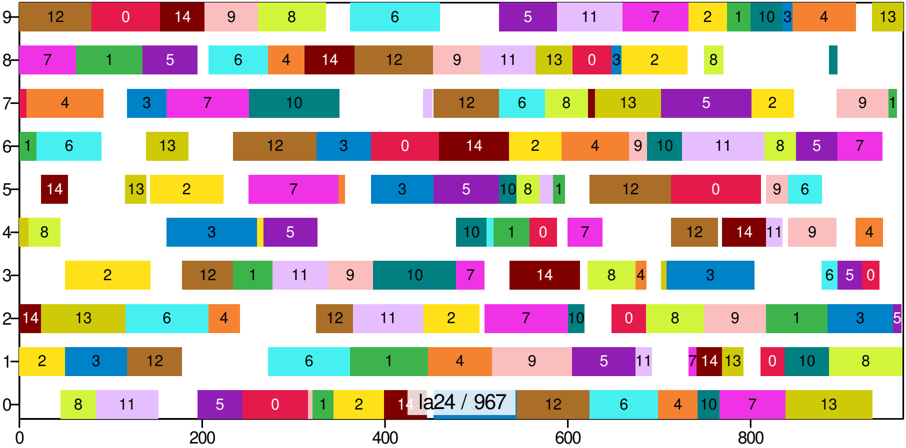
`ea_16384_nswap`: median result of 3 min of the EA with $\mu = \lambda = 16'384$ with `nswap` unary operator



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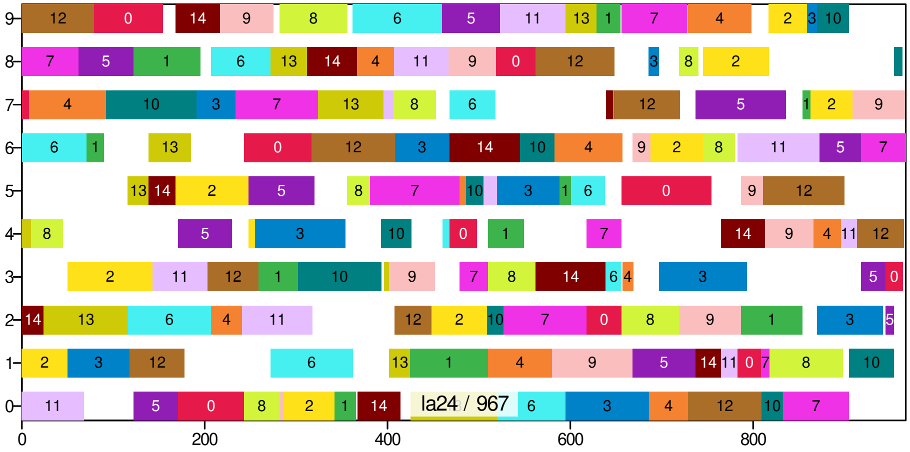

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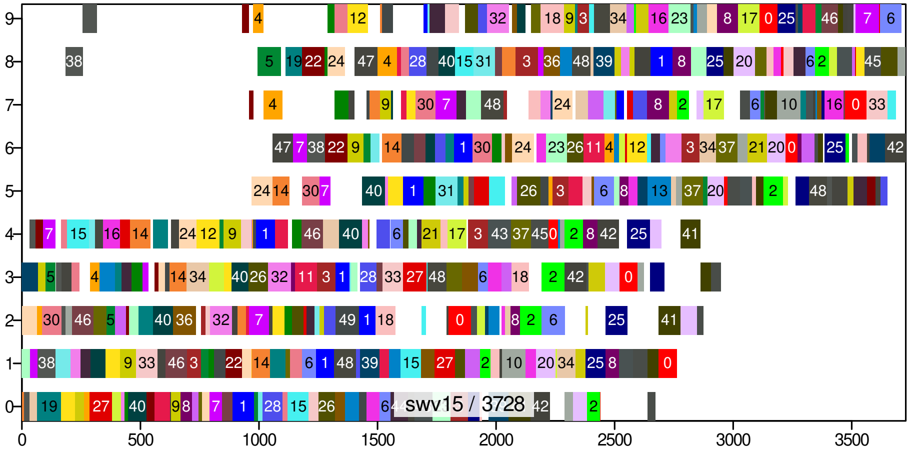

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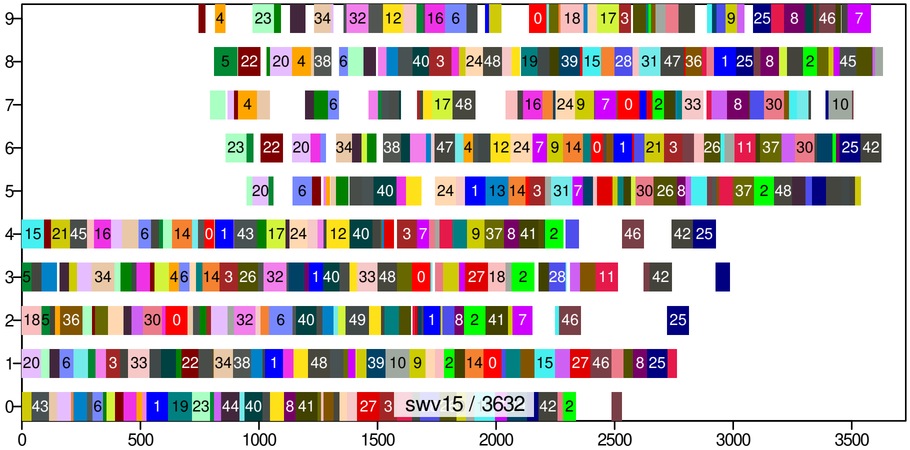

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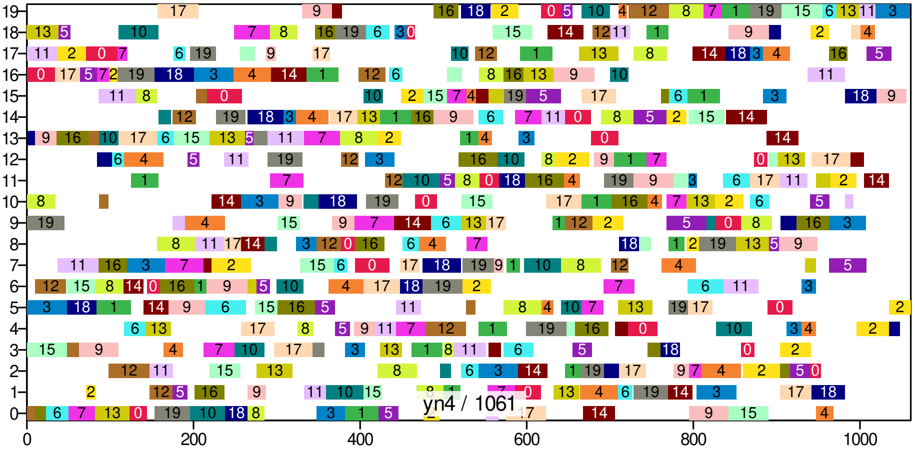

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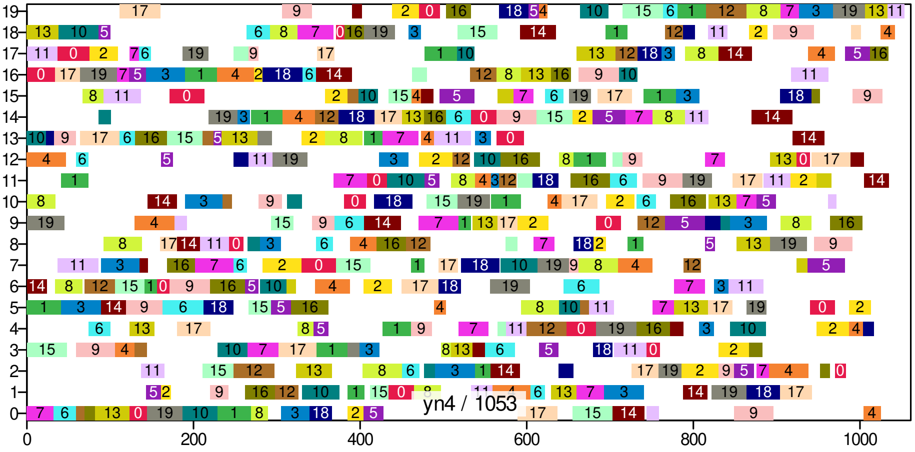

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What progress does the algorithm make over time?

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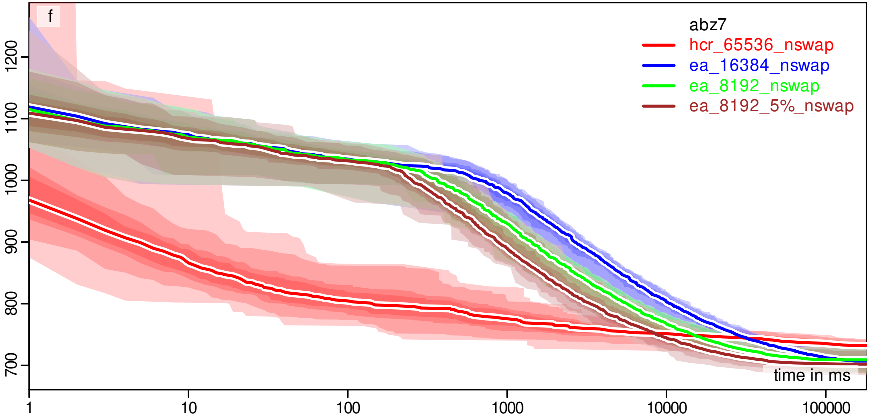

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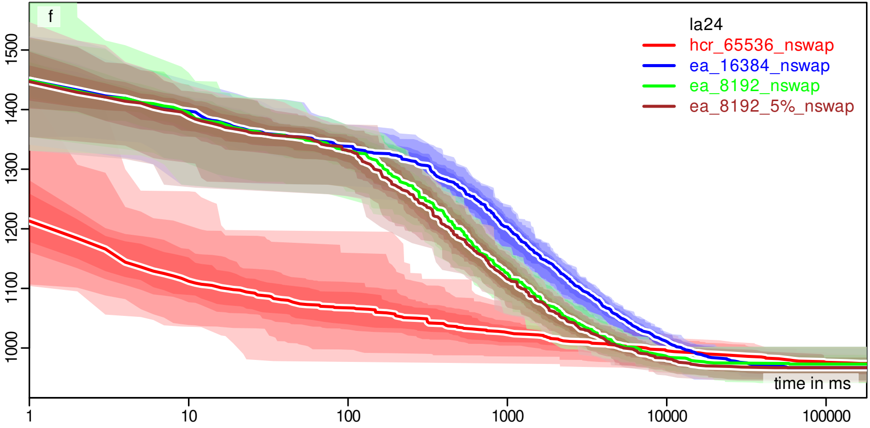

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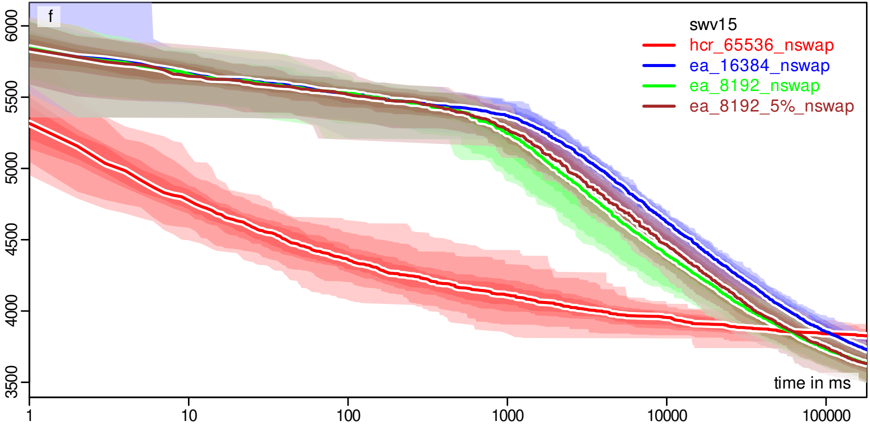

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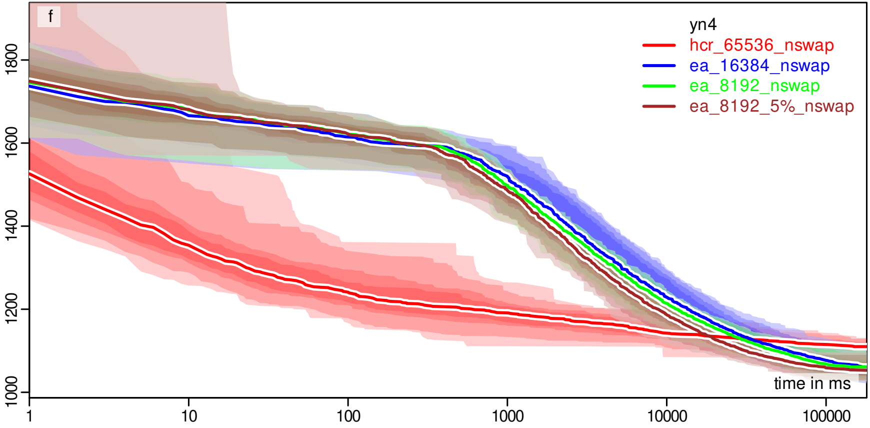

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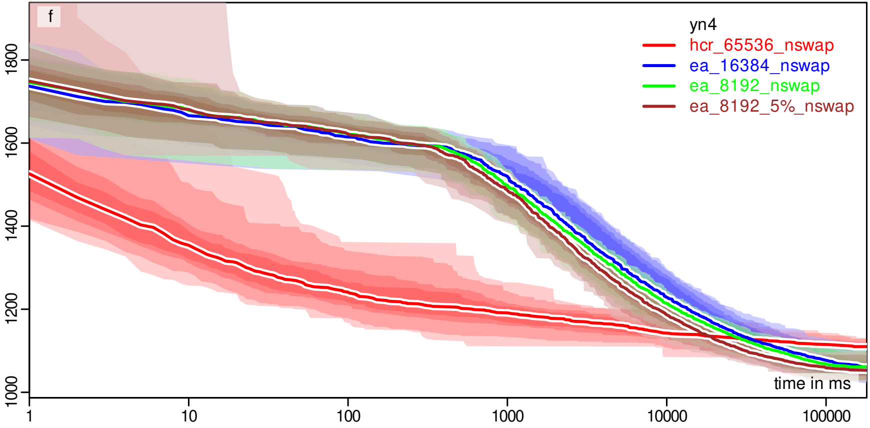

There is no big difference between the EA with and without recombination – but the one with recombination is a little bit better.

Recombination

- In some application areas, the binary operator can very significantly improve the result quality.

Recombination

- In some application areas, the binary operator can very significantly improve the result quality.
- Here, our idea does not work that well, although it is a bit helpful.

Algorithm Concept: Increased Diversity via Clearing

Diversity

- When is the population of the EA useless?

Diversity

- When is the population of the EA useless?
- If all the solutions in it are the same!

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Diversity

- When is the population of the EA useless?
- If all the solutions in it are the same!
- When is a population of the EA useful?
- When the elements of it represent different good solution traits – when they are diverse.
- Many methods have been devised to ensure the diversity of a population, to prevent the population from collapsing to a single point in the search space.[11–13]

Clearing

- We will here consider a very simple approach to preserve population diversity: clearing[11 14].

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Clearing

- We will here consider a very simple approach to preserve population diversity: clearing[11, 14].
- Furthermore, we will apply the simplest version of this approach.
- Every time, when μ out of the $\mu + \lambda$ records are selected, one prior step is applied: we ensure that there is only one record per objective value in the population.
- We call the EA with clearing and recombination eac.

Ingredient: Clearing

```
package aitoa.algorithms;

public class Utils {
//
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//

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//
}
```

Ingredient: Clearing

```
package aitoa.algorithms;

public class Utils {
// useless stuff omitted
  public static int qualityBasedClearing(Record<?>[] array, int max) {
//

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        lastQuality = currentQuality; // update new quality
        if ((++unique) >= max) { // are we finished?
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        lastQuality = currentQuality; // update new quality
        if ((++unique) >= max) { // are we finished?
          return unique; // then quit: unique == max
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        }
        lastQuality = currentQuality; // update new quality
        if ((++unique) >= max) { // are we finished?
          return unique; // then quit: unique == max
        }
      }
    }
    return unique; // return number of unique: 1<=unique<=max
  }
}
```

Implementation: EA with Recombination and Clearing

```java
package aitoa.algorithms;

public class EA<X, Y> extends Metaheuristic2<X, Y> {

  public void solve(IBlackBoxProcess<X, Y> process) {
    Random      random      = process.getRandom();
    ISpace<X>   searchSpace = process.getSearchSpace();
    Record<X>[] P           = new Record[this.mu + this.lambda];

    for (int i = P.length; (--i) >= 0;) { // first generation: fill P with random points
      X x  = searchSpace.create();   // allocate point
      this.nullary.apply(x, random); // fill with random data
      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

    for (;;) { // main loop: one iteration = one generation
//
      Arrays.sort(P, Record.BY_QUALITY); // sort the population: mu best at front
      RandomUtils.shuffle(random, P, 0, this.mu); // shuffle parents for fairness
      int p1 = -1; // index to iterate over first parent
      for (int index = P.length; (--index) >= this.mu;) { // overwrite lambda worst
        if (process.shouldTerminate()) return;
        Record<X> dest = P[index];
        p1 = (p1 + 1) % this.mu; // step the parent 1 index
        Record<X> sel  = P[p1];
        if (random.nextDouble() <= this.cr) { // crossover!
          int p2;
          do { // find a second, different record
            p2 = random.nextInt(this.mu);
          } while (p2 == p1); // repeat until p1 != p2
          this.binary.apply(sel.x, P[p2].x, dest.x, random); // perform recombination
        } else this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
        dest.quality = process.evaluate(dest.x); // evaluate offspring
      } // the end of the offspring generation
    } // the end of the main loop
  } // end solve
} // end class
```

Implementation: EA with Recombination and Clearing

```java
package aitoa.algorithms;

public class EAWithClearing<X, Y> extends Metaheuristic2<X, Y> {

  public void solve(IBlackBoxProcess<X, Y> process) {
    Random      random      = process.getRandom();
    ISpace<X>   searchSpace = process.getSearchSpace();
    Record<X>[] P           = new Record[this.mu + this.lambda];

    for (int i = P.length; (--i) >= 0;) { // first generation: fill P with random points
      X x  = searchSpace.create();   // allocate point
      this.nullary.apply(x, random); // fill with random data
      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

    for (;;) { // main loop: one iteration = one generation
      RandomUtils.shuffle(random, P, 0, P.length); // make fair
      int u = Utils.qualityBasedClearing(P, this.mu);
      RandomUtils.shuffle(random, P, 0, u); // for fairness
      int p1 = -1; // index to iterate over first parent
      for (int index = P.length; (--index) >= u;) { // overwrite non-unique and worst
        if (process.shouldTerminate()) return;
        Record<X> dest = P[index];
        p1 = (p1 + 1) % u; // step the parent 1 index
        Record<X> sel  = P[p1];
        if (random.nextDouble() <= this.cr) { // crossover!
          int p2;
          do { // find a second, different record
            p2 = random.nextInt(u);
          } while (p2 == p1); // repeat until p1 != p2
          this.binary.apply(sel.x, P[p2].x, dest.x, random); // perform recombination
        } else this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
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Implementation: EA with Recombination and Clearing

```
package aitoa.algorithms;

public class EAWithClearing<X, Y> extends Metaheuristic2<X, Y> {

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Implementation: EA with Recombination and Clearing

```
package aitoa.algorithms;

public class EAWithClearing<X, Y> extends Metaheuristic2<X, Y> {

  public void solve(IBlackBoxProcess<X, Y> process) {
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```

Implementation: EA with Recombination and Clearing

```
package aitoa.algorithms;

public class EAWithClearing<X, Y> extends Metaheuristic2<X, Y> {

  public void solve(IBlackBoxProcess<X, Y> process) {
    Random      random      = process.getRandom();
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public class EAWithClearing<X, Y> extends Metaheuristic2<X, Y> {

  public void solve(IBlackBoxProcess<X, Y> process) {
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Implementation: EA with Recombination and Clearing

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      X x  = searchSpace.create();   // allocate point
      this.nullary.apply(x, random); // fill with random data
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      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

//
    RandomUtils.shuffle(random, P, 0, P.length); // make fair
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      this.nullary.apply(x, random); // fill with random data
      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

//
    RandomUtils.shuffle(random, P, 0, P.length); // make fair
    int u = Utils.qualityBasedClearing(P, this.mu);
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      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
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//
    RandomUtils.shuffle(random, P, 0, P.length); // make fair
    int u = Utils.qualityBasedClearing(P, this.mu);
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      Record<X> dest = P[index];
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      Record<X> dest = P[index];
      p1 = (p1 + 1) % u; // step the parent 1 index
      Record<X> sel  = P[p1];
      if (random.nextDouble() <= this.cr) { // crossover!
//
//
//
//
//
      }
//
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} // end class
```

Implementation: EA with Recombination and Clearing

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      Record<X> dest = P[index];
      p1 = (p1 + 1) % u; // step the parent 1 index
      Record<X> sel  = P[p1];
      if (random.nextDouble() <= this.cr) { // crossover!
        int p2 = random.nextInt(u);
//
//
//
//
      }
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    } // the end of the offspring generation
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  } // end solve
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```

Implementation: EA with Recombination and Clearing

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    RandomUtils.shuffle(random, P, 0, u); // for fairness
    int p1 = -1; // index to iterate over first parent
    for (int index = P.length; (--index) >= u;) { // overwrite non-unique and worst
      if (process.shouldTerminate()) return;
      Record<X> dest = P[index];
      p1 = (p1 + 1) % u; // step the parent 1 index
      Record<X> sel  = P[p1];
      if (random.nextDouble() <= this.cr) { // crossover!
        int p2;
        do { // find a second, different record
          p2 = random.nextInt(u);
        } while (p2 == p1); // repeat until p1 != p2
//
      }
//
    } // the end of the offspring generation
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```

Implementation: EA with Recombination and Clearing

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      this.nullary.apply(x, random); // fill with random data
      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

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    RandomUtils.shuffle(random, P, 0, P.length); // make fair
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    for (int index = P.length; (--index) >= u;) { // overwrite non-unique and worst
      if (process.shouldTerminate()) return;
      Record<X> dest = P[index];
      p1 = (p1 + 1) % u; // step the parent 1 index
      Record<X> sel  = P[p1];
      if (random.nextDouble() <= this.cr) { // crossover!
        int p2;
        do { // find a second, different record
          p2 = random.nextInt(u);
        } while (p2 == p1); // repeat until p1 != p2
        this.binary.apply(sel.x, P[p2].x, dest.x, random); // perform recombination
      }
//
    } // the end of the offspring generation
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  } // end solve
} // end class
```

Implementation: EA with Recombination and Clearing

```java
package aitoa.algorithms;

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      if (random.nextDouble() <= this.cr) { // crossover!
        int p2;
        do { // find a second, different record
          p2 = random.nextInt(u);
        } while (p2 == p1); // repeat until p1 != p2
        this.binary.apply(sel.x, P[p2].x, dest.x, random); // perform recombination
      } else this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
//
    } // the end of the offspring generation
//
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```

Implementation: EA with Recombination and Clearing

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      X x  = searchSpace.create();   // allocate point
      this.nullary.apply(x, random); // fill with random data
      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

//
    RandomUtils.shuffle(random, P, 0, P.length); // make fair
    int u = Utils.qualityBasedClearing(P, this.mu);
    RandomUtils.shuffle(random, P, 0, u); // for fairness
    int p1 = -1; // index to iterate over first parent
    for (int index = P.length; (--index) >= u;) { // overwrite non-unique and worst
      if (process.shouldTerminate()) return;
      Record<X> dest = P[index];
      p1 = (p1 + 1) % u; // step the parent 1 index
      Record<X> sel  = P[p1];
      if (random.nextDouble() <= this.cr) { // crossover!
        int p2;
        do { // find a second, different record
          p2 = random.nextInt(u);
        } while (p2 == p1); // repeat until p1 != p2
        this.binary.apply(sel.x, P[p2].x, dest.x, random); // perform recombination
      } else this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
      dest.quality = process.evaluate(dest.x); // evaluate offspring
    } // the end of the offspring generation
//
  } // end solve
} // end class
```

Implementation: EA with Recombination and Clearing

```java
package aitoa.algorithms;

public class EAWithClearing<X, Y> extends Metaheuristic2<X, Y> {

  public void solve(IBlackBoxProcess<X, Y> process) {
    Random      random      = process.getRandom();
    ISpace<X>   searchSpace = process.getSearchSpace();
    Record<X>[] P           = new Record[this.mu + this.lambda];

    for (int i = P.length; (--i) >= 0;) { // first generation: fill P with random points
      X x  = searchSpace.create();   // allocate point
      this.nullary.apply(x, random); // fill with random data
      P[i] = new Record<>(x, process.evaluate(x)); // evaluate
      if (process.shouldTerminate()) return;
    } // end of filling the first population

    for (;;) { // main loop: one iteration = one generation
      RandomUtils.shuffle(random, P, 0, P.length); // make fair
      int u = Utils.qualityBasedClearing(P, this.mu);
      RandomUtils.shuffle(random, P, 0, u); // for fairness
      int p1 = -1; // index to iterate over first parent
      for (int index = P.length; (--index) >= u;) { // overwrite non-unique and worst
        if (process.shouldTerminate()) return;
        Record<X> dest = P[index];
        p1 = (p1 + 1) % u; // step the parent 1 index
        Record<X> sel  = P[p1];
        if (random.nextDouble() <= this.cr) { // crossover!
          int p2;
          do { // find a second, different record
            p2 = random.nextInt(u);
          } while (p2 == p1); // repeat until p1 != p2
          this.binary.apply(sel.x, P[p2].x, dest.x, random); // perform recombination
        } else this.unary.apply(sel.x, dest.x, random); // generate offspring via unary
        dest.quality = process.evaluate(dest.x); // evaluate offspring
      } // the end of the offspring generation
    } // the end of the main loop
  } // end solve
} // end class
```

Experiment and Analysis

So what do we get?

- I execute the program 101 times for each of the instances `abz7`, `la24`, `swv15`, and `yn4`

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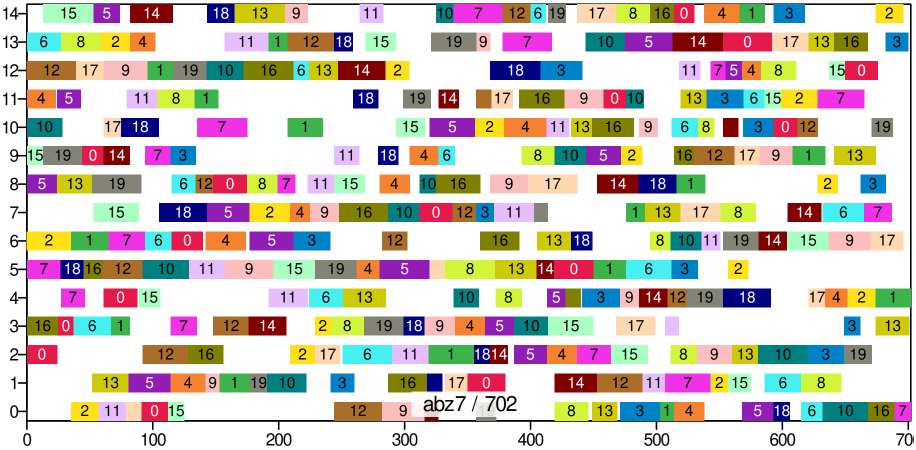
		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
`abz7`	`ea_8192_5%_nswap`	684	703	702	**8**	54s	10'688'314
	`eac_4_5%_nswap`	**672**	**690**	**690**	9	68s	12'474'571
`la24`	`ea_8192_5%_nswap`	943	967	967	**11**	18s	4'990'002
	`eac_4_5%_nswap`	**935**	**963**	**961**	16	30s	9'175'579
`swv15`	`ea_8192_5%_nswap`	3498	3631	3632	**65**	178s	17'747'983
	`eac_4_5%_nswap`	**3102**	**3220**	**3224**	65	168s	18'245'534
`yn4`	`ea_8192_5%_nswap`	1026	1056	1053	**17**	114s	13'206'552
	`eac_4_5%_nswap`	**1000**	**1038**	**1037**	18	118s	15'382'072

So what do we get?

`ea_8192_5%_nswap`: median result of 3 min of the EA with $\mu = \lambda = 8'192$ with `nswap` unary operator and 5% sequence recombination



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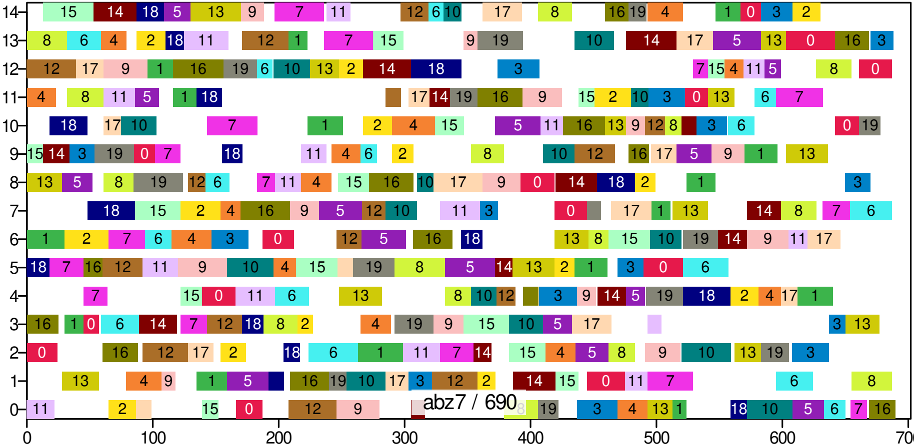
`eac_4_5%_nswap`: median result of 3 min of the EA with clearing and $\mu = \lambda = 4$ with `nswap` unary operator and 5% sequence recombination



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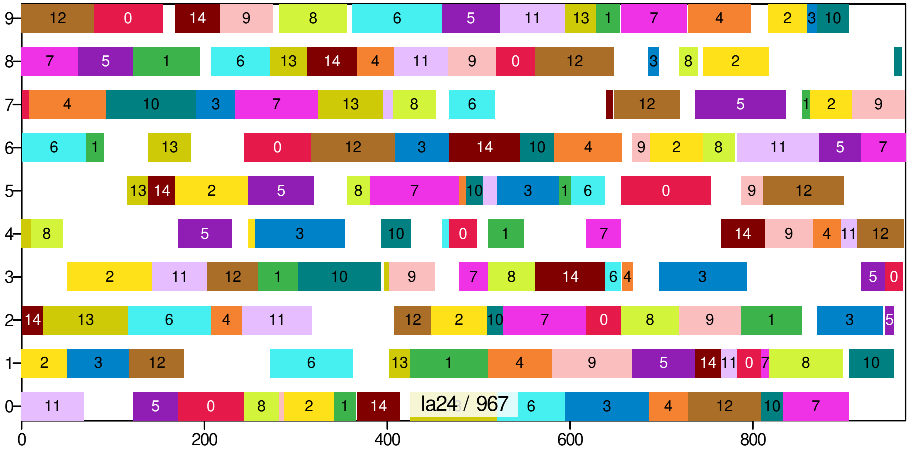

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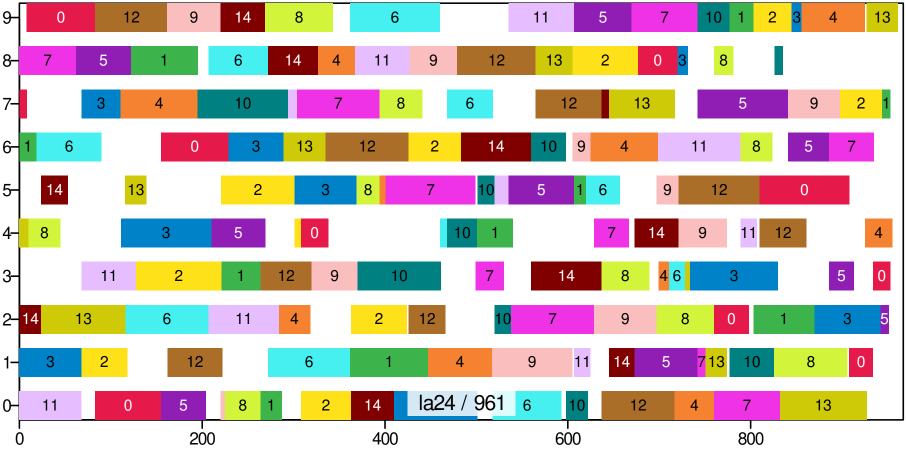

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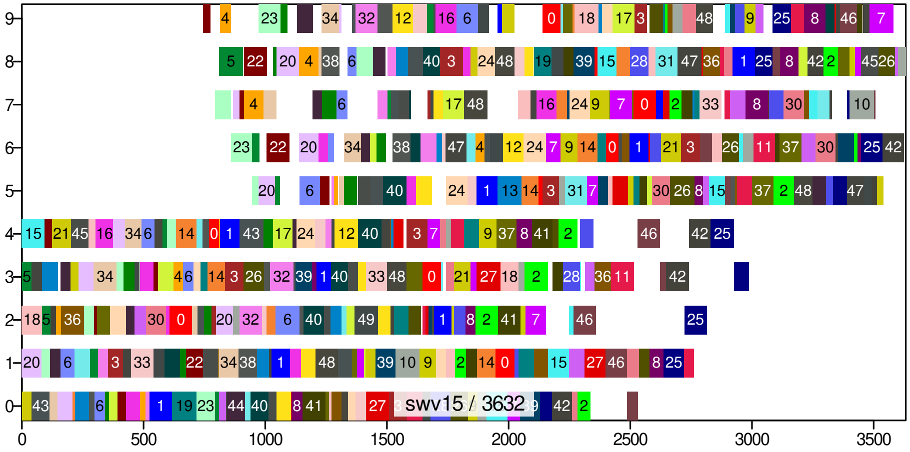

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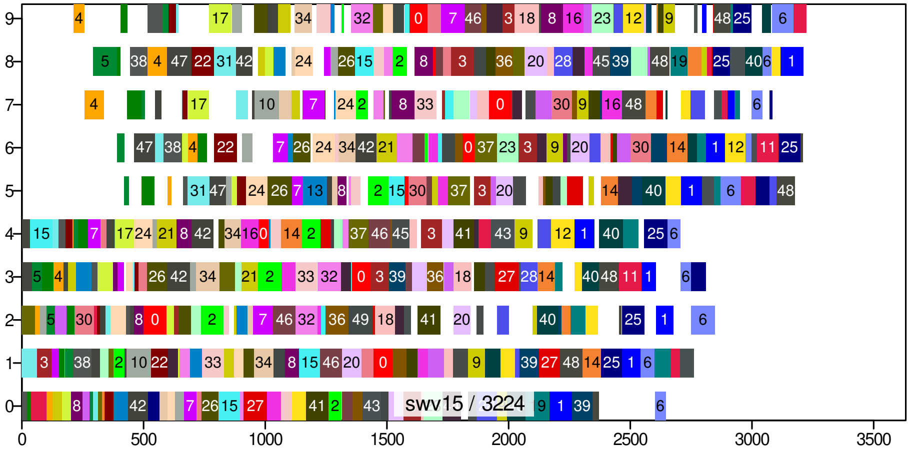

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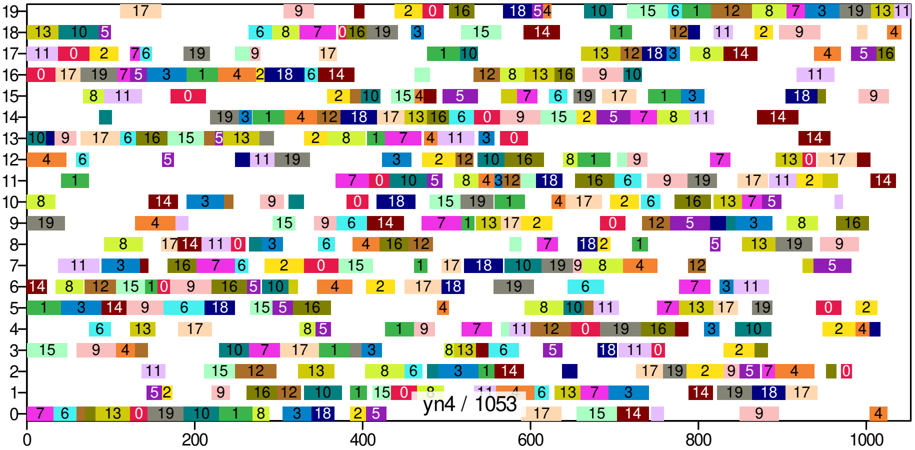

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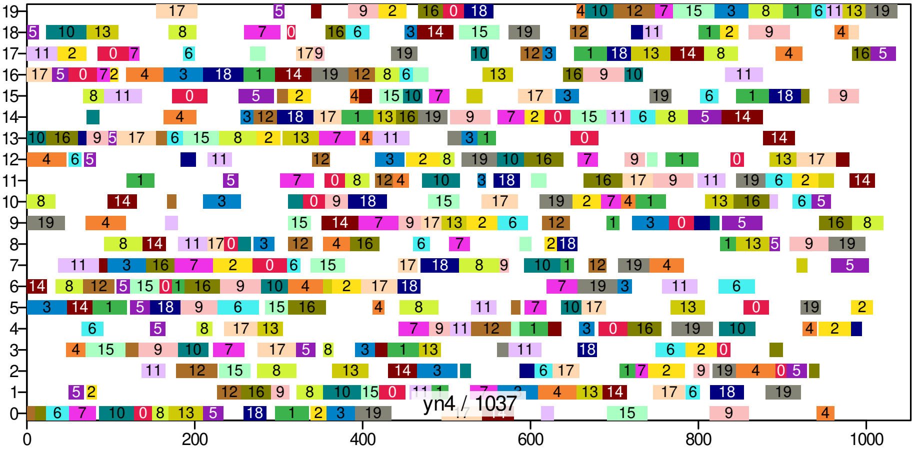

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What progress does the algorithm make over time?

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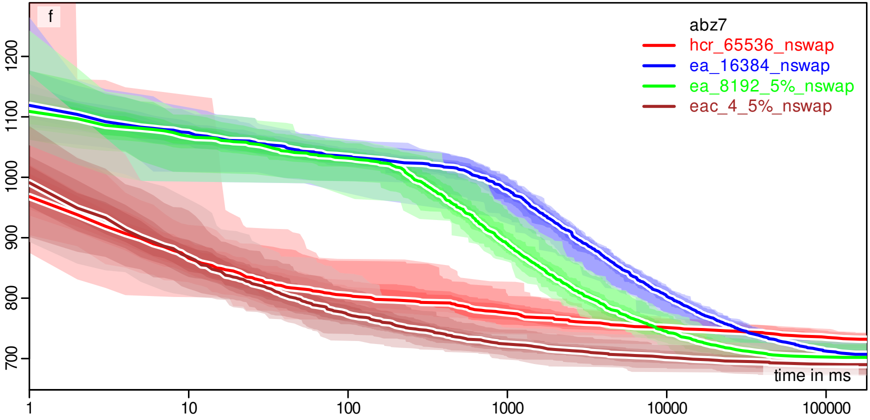

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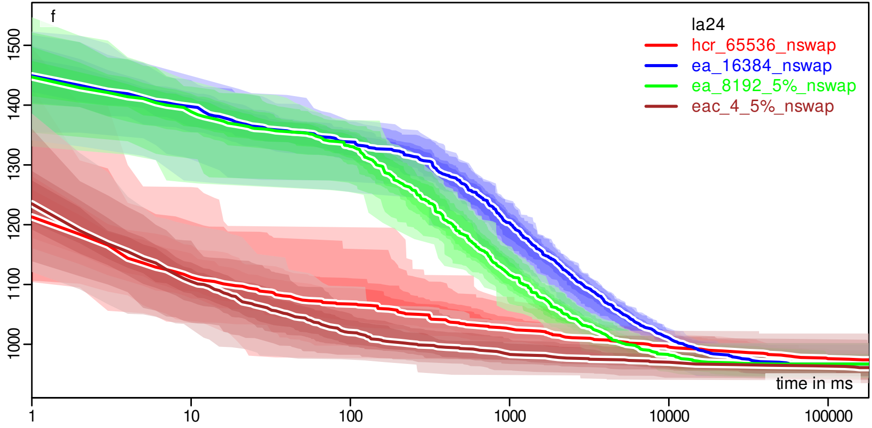

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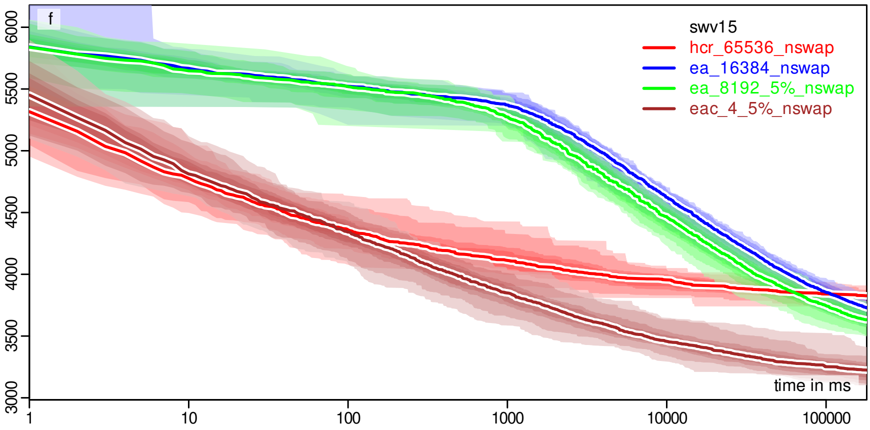

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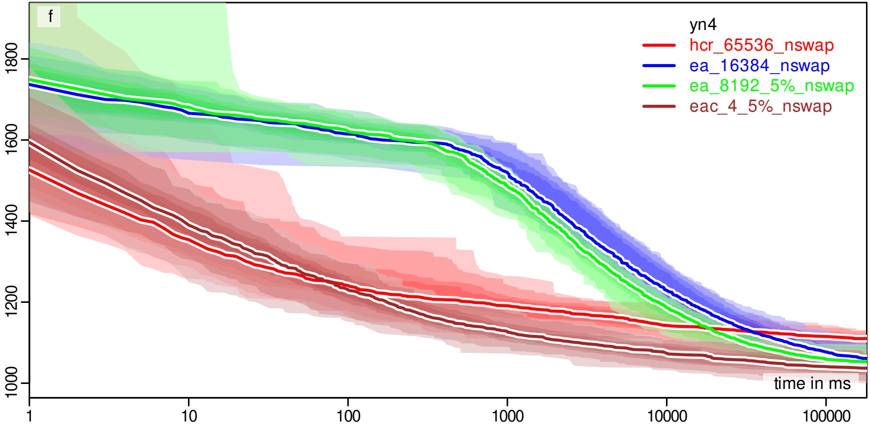

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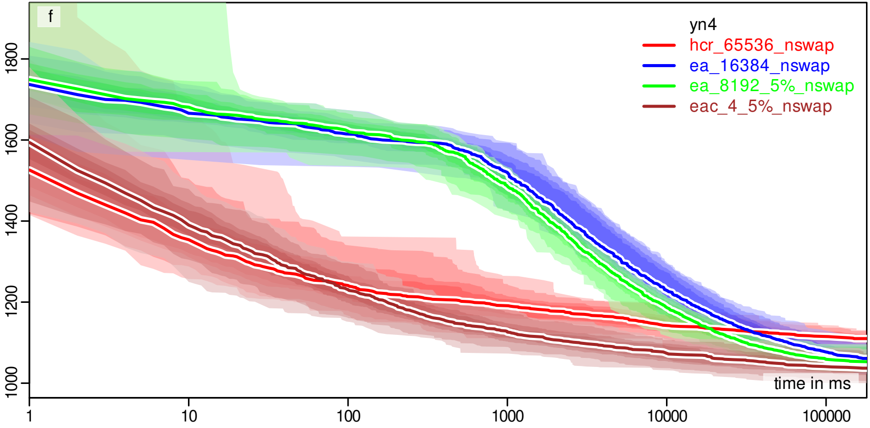

The EA with clearing performs much better than the EA without, at a much smaller population size.

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- . . . but even small improvements might be worthwhile.
- Preserving the diversity in a population can improve the EA performance significantly.

谢谢

Thank you

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