合肥大學

HEFEI UNIVERSITY

Frequency Fitness Assignment

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Outline

Introduction

Introduction to Optimization

- Optimization means finding superlatives.

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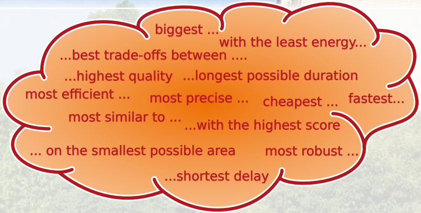

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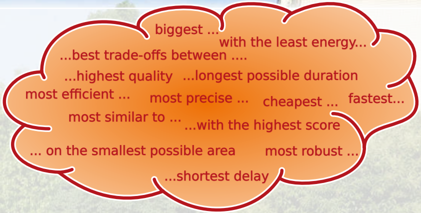

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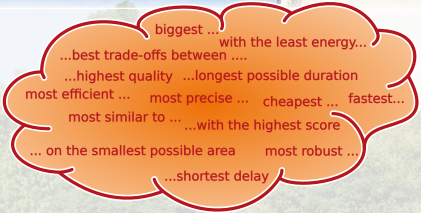

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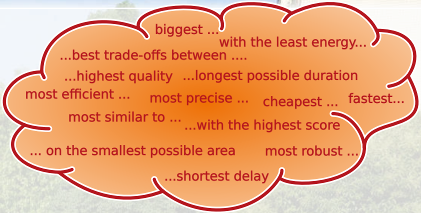

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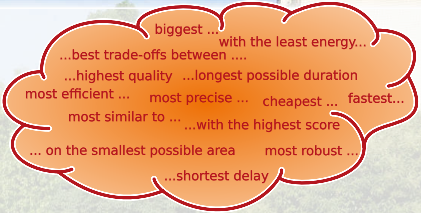

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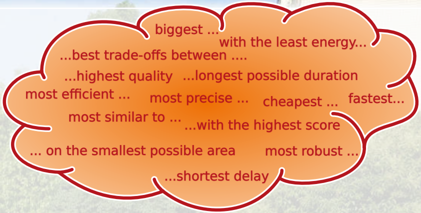

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- Find a strategy to manage the power of the nodes in this sensor network so that full coverage is guaranteed for the longest possible duration.
- And so on.

Views on Optimization

- There are two ways to look at optimization.

Views on Optimization

- The economic view.

Optimization

An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined or required benefit at minimal costs.

Views on Optimization

- The mathematical view.

Optimization

An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined or required benefit at minimal costs.

Solving an optimization problem requires finding an input element $x^\star$ within a set $\mathbb{X}$ of allowed elements for which a mathematical function $f.\mathbb{X} \mapsto \mathbb{R}$ takes on the smallest possible value.

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- The optimal solution $x^{\star} \in \mathbb{X}$ is the shortest possible tour.

Example: Maximum Satisfiability Problem

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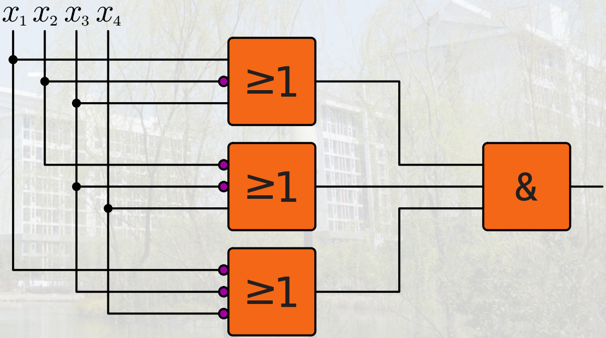

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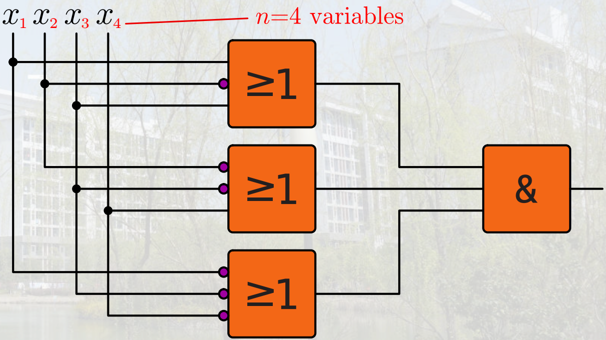

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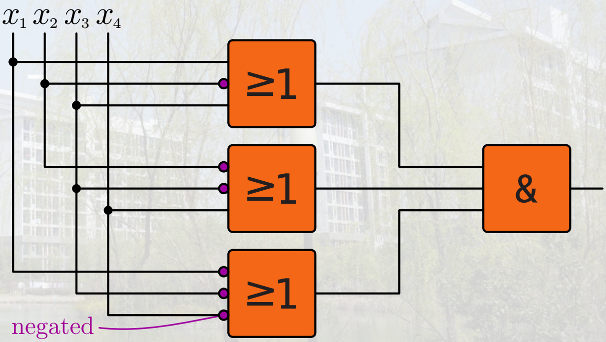

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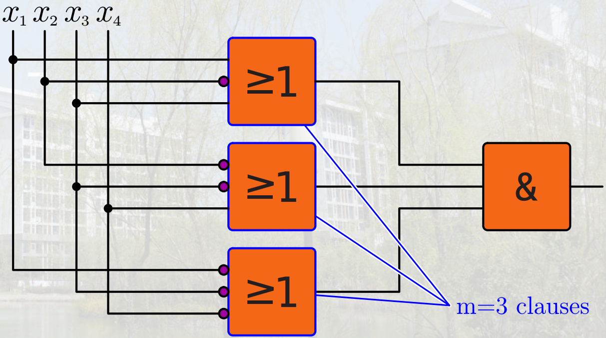

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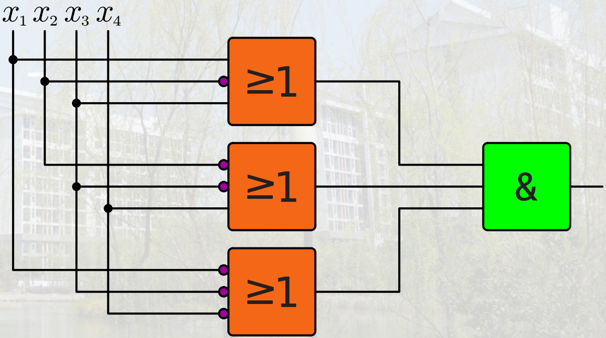

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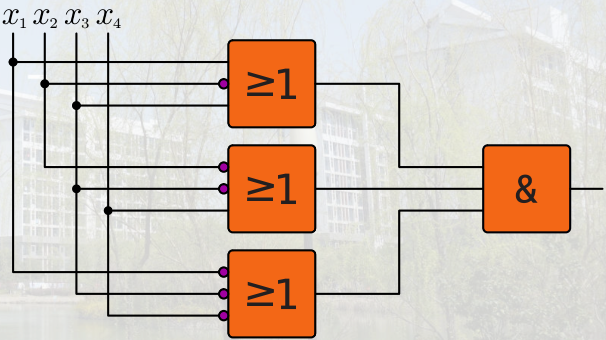

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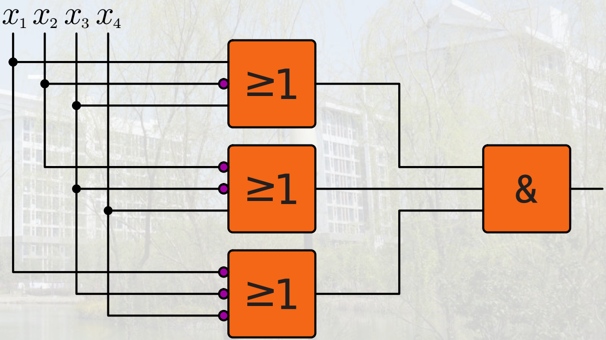

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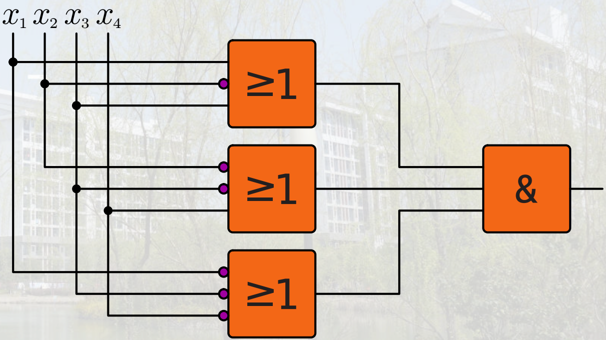
- The objective function $f : \mathbb{X} \mapsto \mathbb{R}$ is the number of unsatisfied OR-clauses.
- The optimum $x^{\star} \in \mathbb{X}$ has $f(x^{\star}) = 0$, i.e., all clauses satisfied, i.e., $F(x^{\star}) = $ True.



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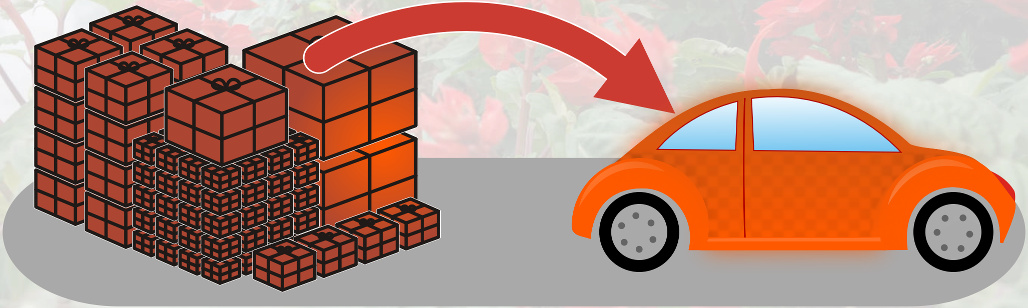

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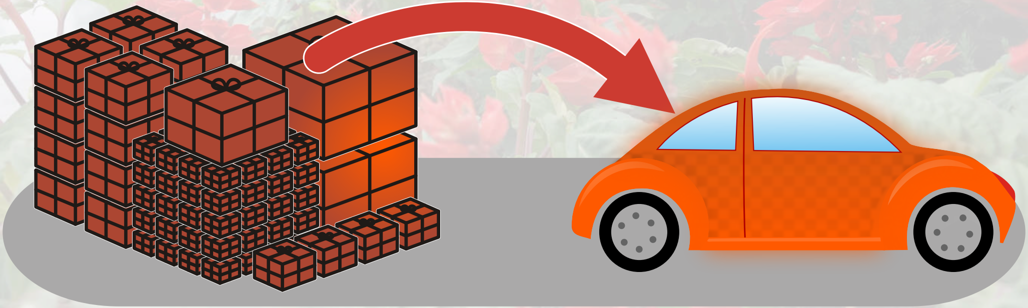

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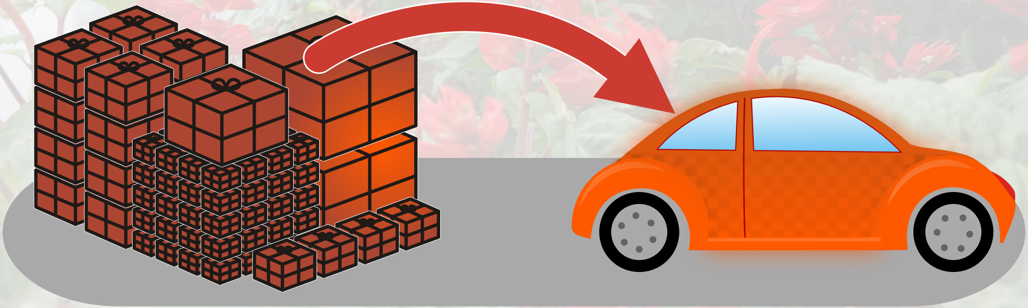

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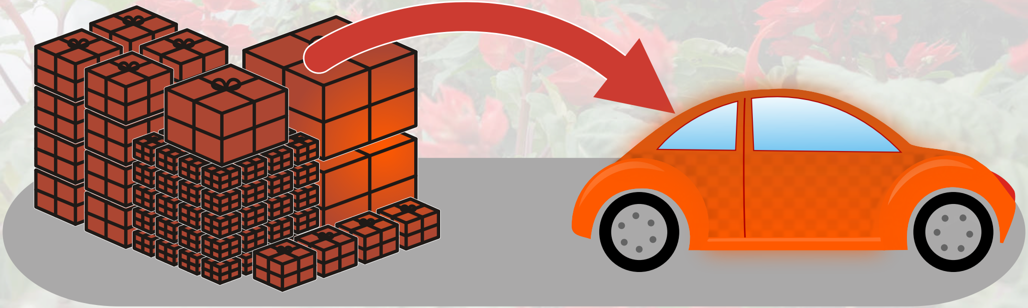

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- In other words, if we want to guarantee to find the best possible solution $x^\star$ for all possible instances of a problem, we often cannot really be much faster than testing all possible candidate solutions $x \in \mathbb{X}$ in the worst case.

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- And then roughly follow this cycle.

Begin with a set $S_0 \subset \mathbb{X}$ of one or multiple randomly sampled solutions

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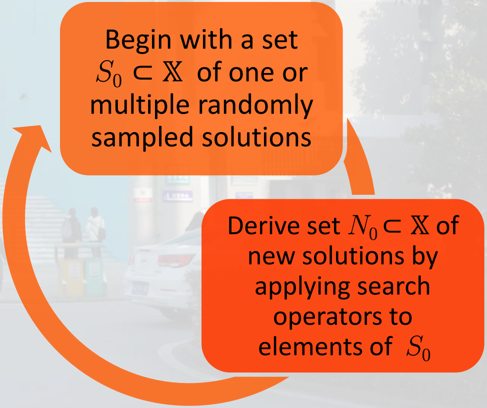

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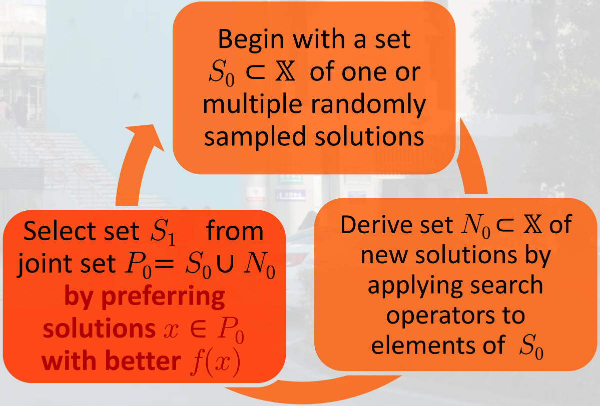

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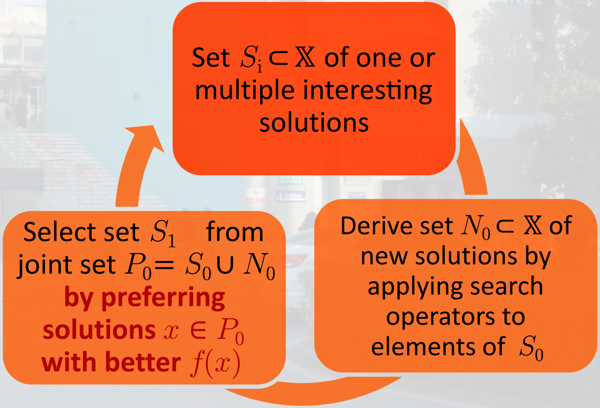

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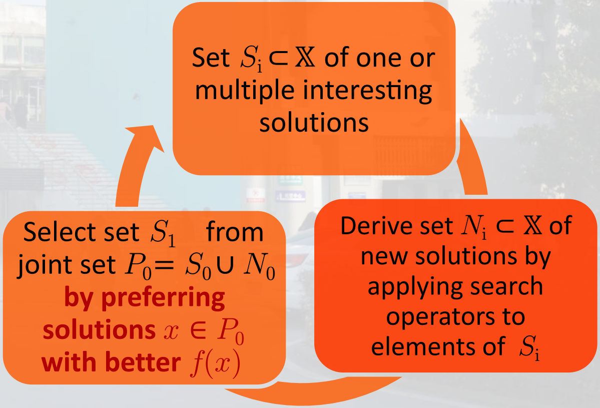

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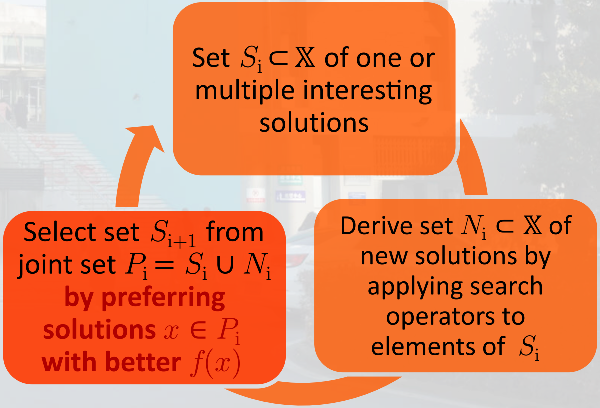

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  return x_c, y_c

Simulated Annealing

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procedure SA($f : \mathbb{X} \mapsto \mathbb{R}$, T_0, ε)
  randomly sample x_c from $\mathbb{X}$; $y_c \leftarrow f(x_c)$;

  while $\neg$ terminate **do**
    $x_n \leftarrow \texttt{move}(x_c)$; $y_n \leftarrow f(x_n)$;

    if $y_n \leq y_c$ **then**
      $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;

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    if $\Re_0^1 < e^{y_c - y_n}$ **then**

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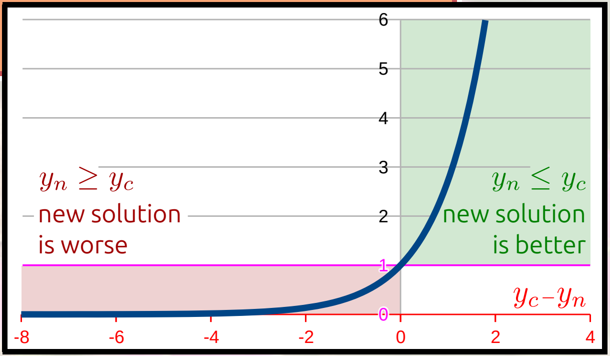

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    if $\Re_0^1 < e^{\frac{y_c - y_n}{T}}$ **then** ▷ always true if $y_n \leq y_c$
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  $\tau \leftarrow 0$; ▷ τ is iteration counter

  while ¬ terminate **do**

    $x_n \leftarrow \texttt{move}(x_c)$; $y_n \leftarrow f(x_n)$;

    $\tau \leftarrow \tau + 1$;

    $T \leftarrow T_0$;

    if $\Re_0^1 < e^{\frac{y_c - y_n}{T}}$ **then** ▷ always true if $y_n \leq y_c$

      $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;

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  $\tau \leftarrow 0$; ▷ τ is iteration counter
  while $\neg$ terminate **do**
    $x_n \leftarrow \texttt{move}(x_c)$; $y_n \leftarrow f(x_n)$;
    $\tau \leftarrow \tau + 1$;
    $T \leftarrow T_0(1-\varepsilon)^{\tau-1}$; ▷ T decreases over time
    if $\Re_0^1 < e^{\frac{y_c - y_n}{T}}$ **then** ▷ always true if $y_n \leq y_c$
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- It also remembers best-so-far solution x_B and its objective value y_B, because it could get lost.

procedure SA($f : \mathbb{X} \mapsto \mathbb{R}$, T_0, ε)
  randomly sample x_c from $\mathbb{X}$; $y_c \leftarrow f(x_c)$;
  $x_B \leftarrow x_c$; $y_B \leftarrow y_c$; ▷ preserve best!
  $\tau \leftarrow 0$; ▷ τ is iteration counter
  while $\neg$ terminate **do**
    $x_n \leftarrow \texttt{move}(x_c)$; $y_n \leftarrow f(x_n)$;
    $\tau \leftarrow \tau + 1$;
    $T \leftarrow T_0(1-\varepsilon)^{\tau-1}$; ▷ T decreases over time
    if $\Re_0^1 < e^{\frac{y_c - y_n}{T}}$ **then** ▷ always true if $y_n \leq y_c$
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```
procedure SA(f : 𝕏 ↦ ℝ, T₀, ε)
    randomly sample x_c from 𝕏;  y_c ← f(x_c);
    x_B ← x_c;  y_B ← y_c;                          ▷ preserve best!
    τ ← 0;                                          ▷ τ is iteration counter
    while ¬ terminate do
        x_n ← move(x_c);  y_n ← f(x_n);
        τ ← τ + 1;
        T ← T₀(1 − ε)^(τ−1);                        ▷ T decreases over time
        if ℜ₀¹ < e^((y_c − y_n)/T) then             ▷ always true if y_n ≤ y_c
            x_c ← x_n;  y_c ← y_n;
            if y_c < y_B then x_B ← x_c;  y_B ← y_c;
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procedure SA($f : \mathbb{X} \mapsto \mathbb{R}$, T_0, ε)
  randomly sample x_c from $\mathbb{X}$; $y_c \leftarrow f(x_c)$;
  $x_\mathrm{B} \leftarrow x_c$; $y_\mathrm{B} \leftarrow y_c$; ▷ preserve best!
  $\tau \leftarrow 0$; ▷ τ is iteration counter
  while $\neg$ terminate **do**
    $x_n \leftarrow \mathtt{move}(x_c)$; $y_n \leftarrow f(x_n)$;
    $\tau \leftarrow \tau + 1$;
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    if $\Re_0^1 < e^{\frac{y_c - y_n}{T}}$ **then** ▷ always true if $y_n \leq y_c$
      $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
      if $y_c < y_\mathrm{B}$ **then** $x_\mathrm{B} \leftarrow x_c$; $y_\mathrm{B} \leftarrow y_c$;
  return x_B, y_B

Standard Genetic Algorithm with Roulette Wheel Selection

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for $i \in 0 \dots \infty$ **do** ▷ iterate "generations"
  for $j \in 1 \dots ps$ **do** ▷ new pop. via mutation

    $N_i[j].x \leftarrow \texttt{move}(S_i[\lfloor \mathfrak{R}_1^{ps} \rfloor].x)$;

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        for $j \in 1 \dots ps$ **do** ▷ new pop. via mutation and crossover
            if $\mathfrak{R}_0^1 < cr$ **then** $N_i[j].x \leftarrow \texttt{binary}(S_i[\lfloor \mathfrak{R}_1^{ps} \rfloor].x, S_i[\lfloor \mathfrak{R}_1^{ps} \rfloor].x)$;
            else $N_i[j].x \leftarrow \texttt{move}(S_i[\lfloor \mathfrak{R}_1^{ps} \rfloor].x)$;

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$S_{i+1} \leftarrow$ *Roulette Wheel:* select ps records from $P_i = S_i \cup N_i$ such that, for each of the ps slots, the probability of $P_i[j]$ to be chosen is **proportional to $P_i[j].y$.**

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procedure SGA($f : \mathbb{X} \mapsto \mathbb{R}^+$, ps, cr) ▷ **for maximization!**

  $x_B \leftarrow \emptyset$; $y_B \leftarrow -\infty$; ▷ best-so-far solution

  for $j \in 1 \dots ps$ **do** ▷ random initial population

    randomly sample $S_0[j].x$ from $\mathbb{X}$; $S_0[j].y \leftarrow f(S_0[j].x)$;

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  $x_B \leftarrow \emptyset$; $y_B \leftarrow -\infty$; ▷ best-so-far solution
  for $j \in 1 \dots ps$ **do** ▷ random initial population
    randomly sample $S_0[j].x$ from $\mathbb{X}$; $S_0[j].y \leftarrow f(S_0[j].x)$;
    if $S_0[j].y > y_B$ **then** $x_B \leftarrow S_0[j].x$; $y_B \leftarrow S_0[j].y$;
  for $i \in 0 \dots \infty$ **do** ▷ iterate "generations"
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procedure SGA($f : \mathbb{X} \mapsto \mathbb{R}^{+}$, ps, cr) ▷ **for maximization!**

$x_B \leftarrow \emptyset$; $y_B \leftarrow -\infty$; ▷ best-so-far solution

for $j \in 1 \dots ps$ **do** ▷ random initial population

randomly sample $S_0[j].x$ from $\mathbb{X}$; $S_0[j].y \leftarrow f(S_0[j].x)$;

if $S_0[j].y > y_B$ **then** $x_B \leftarrow S_0[j].x$; $y_B \leftarrow S_0[j].y$;

for $i \in 0 \dots \infty$ **do** ▷ iterate "generations"

for $j \in 1 \dots ps$ **do** ▷ new pop. via mutation and crossover

if $\mathfrak{R}_0^1 < cr$ **then** $N_i[j].x \leftarrow \texttt{binary}(S_i[\lfloor\mathfrak{R}_1^{ps}\rfloor].x, S_i[\lfloor\mathfrak{R}_1^{ps}\rfloor].x)$;

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$S_{i+1} \leftarrow$ *Roulette Wheel:* select ps records from $P_i = S_i \cup N_i$ such that, for each of the ps slots, the probability of $P_i[j]$ to be chosen is **proportional to $P_i[j].y$.**

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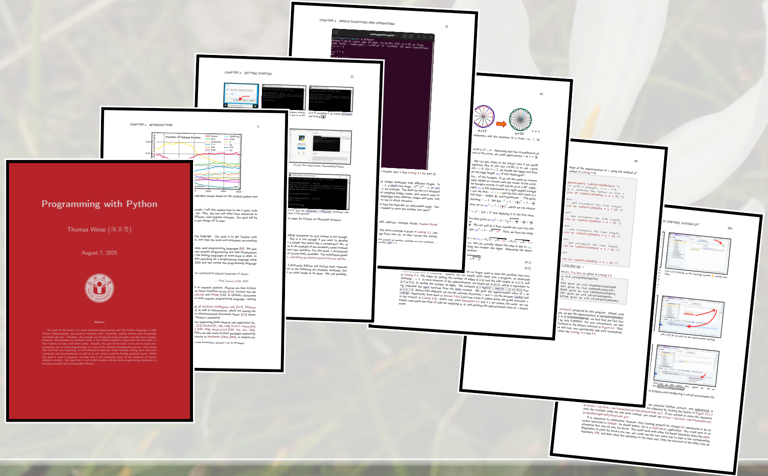
```
procedure SGA(f : 𝕏 ↦ ℝ⁺, ps, cr)                              ▷ for maximization!
    x_B ← ∅;  y_B ← −∞;                                        ▷ best-so-far solution
    for j ∈ 1...ps do                                          ▷ random initial population
        randomly sample S_0[j].x from 𝕏;  S_0[j].y ← f(S_0[j].x);
        if S_0[j].y > y_B then x_B ← S_0[j].x;  y_B ← S_0[j].y;
    for i ∈ 0...∞ do                                           ▷ iterate "generations"
        for j ∈ 1...ps do                    ▷ new pop. via mutation and crossover
            if ℜ_0^1 < cr then N_i[j].x ← binary(S_i[⌊ℜ_1^ps⌋].x, S_i[⌊ℜ_1^ps⌋].x);
            else N_i[j].x ← move(S_i[⌊ℜ_1^ps⌋].x);
            N_i[j].y ← f(N_i[j].x);
            if N_i[j].y > y_B then x_B ← N_i[j].x;  y_B ← N_i[j].y;
        S_{i+1} ← Roulette Wheel: select ps records from P_i = S_i ∪ N_i
                  such that, for each of the ps slots, the probability
                  of P_i[j] to be chosen is proportional to P_i[j].y.
    return x_B, y_B
```

Advertisement

Programming with Python

We have a freely available course book on *Programming with Python* at https://thomasweise.github.io/programmingWithPython, with focus on practical software development using the Python ecosystem of tools[29].

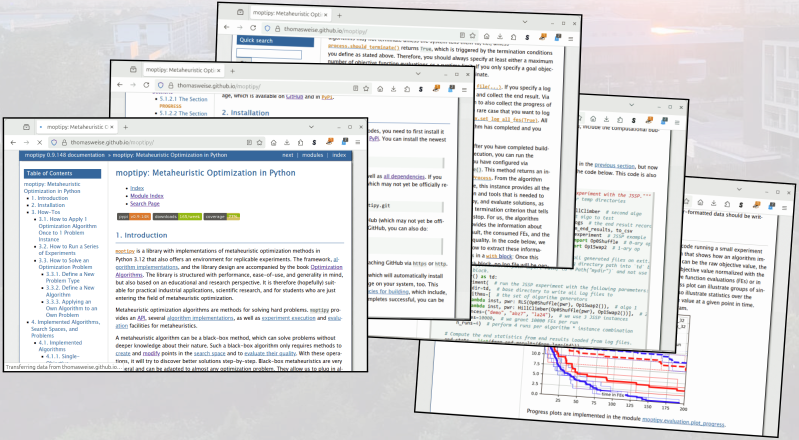


Databases

We have a freely available course book on *Databases* at `https://thomasweise.github.io/databases`, with actual practical examples using a real database management system (DBMS)[27].

Metaheuristic Optimization in Python: moptipy

We offer moptipy[31] a mature open source Python package for metaheuristic optimization, which implements several algorithms, can run self-documenting experiments in parallel and in a distributed fashion, and offers statistical evaluation tools.



谢谢您门！

Thank you!

Vielen Dank!

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Glossary I

EA An *evolutionary algorithm* is a metaheuristic optimization method that maintains a population of candidate solutions, which undergo selection (where better solutions are chosen with higher probability) and reproduction (where mutation and recombination create a new candidate solution from one or two existing ones, repectively)[2,28].

$(\mu+\lambda)$ EA The $(\mu+\lambda)$ EA is an evolutionary algorithm (EA) where, in each generation, λ offspring solutions are generated from the current population of μ parent solutions. The offspring and parent populations are merged, yielding $\mu+\lambda$ solutions, from which then the best μ solutions are ratained to form the parent population of the next generation. If the search space is the bit strings of length n, then this algorithm usually applies a mutation operator flipping each bit independently with probability $1/n$.

$(1+1)$ EA The $(1+1)$ EA is a local search algorithm that retains the best solution x_c discovered so far during the search[4,9]. In each step, it applies a unary search operator to this best-so-far solution x_c and derives a new solution x_n. If the new solution x_n is *better or equally good* when compared with x_c, i.e., not worse, then it replaces it, i.e., is stored as the new x_c. If the search space are bit strings of length n, then the $(1+1)$ EA uses a unary search operator that flips each bit independently with probability m/n, where usually $m=1$. This operator is the main difference to randomized local search (RLS). The $(1+1)$ EA is a special case of the $(\mu+\lambda)$ evolutionary algorithm ($(\mu+\lambda)$ EA) where $\mu=\lambda=1$.

DB A *database* is an organized collection of structured information or data, typically stored electronically in a computer system. Databases are discussed in our book *Databases*[27].

DBMS A *database management system* is the software layer located between the user or application and the database (DB). The DBMS allows the user/application to create, read, write, update, delete, and otherwise manipulate the data in the DB[33].

JSSP The *Job Shop Scheduling Problem*[3,18] is one of the most prominent and well-studied scheduling tasks. In a JSSP instance, there are k machines and m jobs. Each job must be processed once by each machine in a job-specific sequence and has a job-specific processing time on each machine. The goal is to find an assignment of jobs to machines that results in an overall shortest makespan, i.e., the schedule which can complete all the jobs in the shortest time. The JSSP is $\mathcal{NP}$-complete[6,18].

Glossary II

MaxSAT The goal of satisfiablity problems is to find an assignment for n Boolean variables that make a given Boolean formula $F:\{0,1\}^n \mapsto \{0,1\}$ become true. In the *Maximum Satisfiability (MaxSAT)* problem[13], F is given in conjunctive normal form, i.e., the variables appear as literals either directly or negated in m "or" clauses, which are all combined into one "and." The objective function $f(x)$, subject to minimization, computes the number of clauses which are false under the variable setting x. If $f(x)=0$, then all clauses of F are true, which solves the problem. The MaxSat problem is $\mathcal{NP}$-complete[7].

ML Machine Learning, see, e.g.,[26]

moptipy is the *Metaheuristic Optimization in Python* library[31]. Learn more at https://thomasweise.github.io/moptipy.

Python The Python programming language[14,20,21,29], i.e., what you will learn about in our book[29]. Learn more at https://python.org.

RLS Randomized local search retains the best solution x_c discovered so far during the search and, in each step, it applies a unary search operator to this best-so-far solution x_c and derives a new solution x_n. If the new solution x_n is *better or equally good* when compared with x_c, i.e., not worse, then it replaces it, i.e., is stored as the new x_c. If the search space are bit strings of length n, then RLS uses a unary search operator that flips exactly one bit. This operator is the main difference to $(1+1)$ EA.

SA Simulated Annealing is a local search that sometimes accepts a worse solution[5,16,17,24]. The probability to do so decreases over time and with the difference in objective values, i.e.,is the lower the worse the new solution is.

SGA The Standard Genetic Algorithm[2,8,10,22,23,28] was the first population EA. It maintains a population of solutions and applies mutation and crossover to generate offspring solutions. It uses fitness proportionate selection to choose which solutions should "survive" into the next generation, which today is considered a very bad design choice, see, e.g.,[32].

Glossary III

TSP In an instance of the *Traveling Salesperson Problem*, also known as *Traveling Salesman Problem*, a set of n cities or locations as well as the distances between them are defined[1,12,19,30]. The goal is to find the shortest round-trip tour that starts at one city, visits all the other cities one time each, and returns to the origin. The TSP is one of the most well-known $\mathcal{NP}$-hard combinatorial optimization problems.

$\mathcal{NP}$ $\mathcal{NP}$ is the class of computational problems that can be solved in polynomial time by a non-deterministic machine and can be verified in polynomial time by a deterministic machine (such as a normal computer)[11].

$\mathcal{NP}$-complete A decision problem is $\mathcal{NP}$-complete if it is in $\mathcal{NP}$ and all problems in $\mathcal{NP}$ are reducible to it in polynomial time[11,25]. A problem is $\mathcal{NP}$-complete if it is $\mathcal{NP}$-hard and if it is in $\mathcal{NP}$.

$\mathcal{NP}$-hard Algorithms that guarantee to find the correct solutions of $\mathcal{NP}$-hard problems[6,7,18] need a runtime that is exponential in the problem scale in the worst case. A problem is $\mathcal{NP}$-hard if all problems in $\mathcal{NP}$ are reducible to it in polynomial time[11].

$\mathbb{R}$ the set of the real numbers.