合肥大學
HEFEI UNIVERSITY

An Introduction to Optimization

Thomas Weise (汤卫思)
tweise@hfuu.edu.cn

Institute of Applied Optimization (IAO)
School of Artificial Intelligence and Big Data
Hefei University
Hefei, Anhui, China

应用优化研究所
人工智能与大数据学院
合肥大学
中国安徽省合肥市

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Outline

Introduction

What is Optimization?

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What is Optimization?

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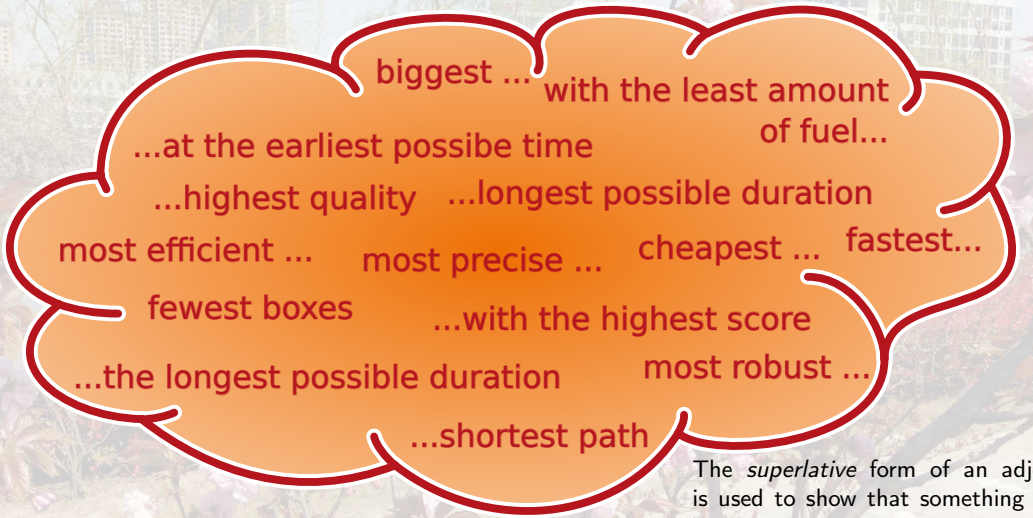
  1. Optimization is an extension of school mathematics into a field where equations are no longer enough and exact solutions cannot always be reached.
  2. Optimization is the art of solving hard problems.
  3. Optimization means searching for superlatives.

Optimization = Search for Superlatives

- Optimization means finding superlatives.



The *superlative* form of an adjective is used to show that something has a quality to the greatest or least degree.

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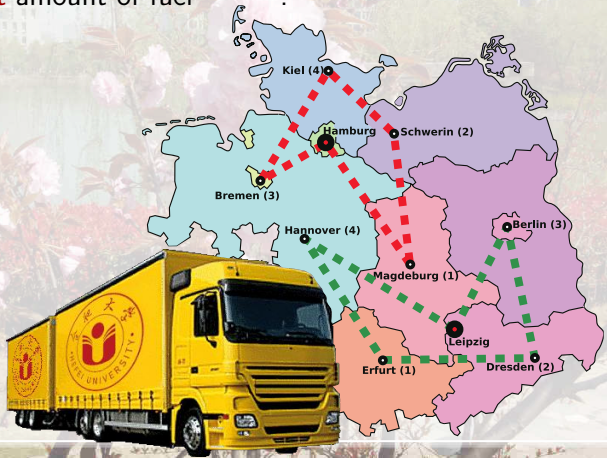

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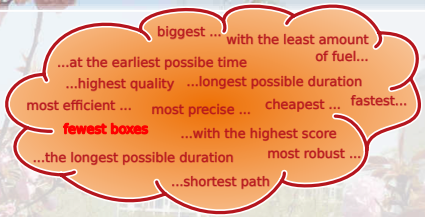

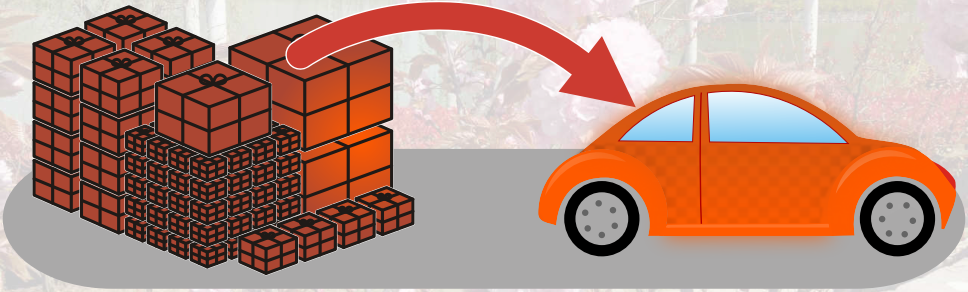

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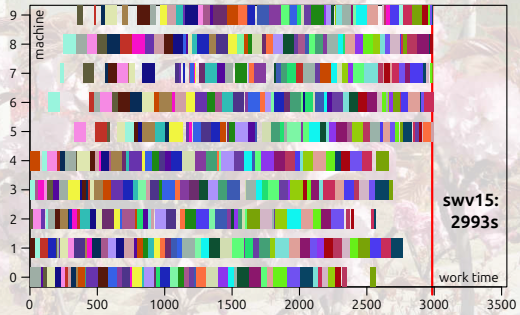

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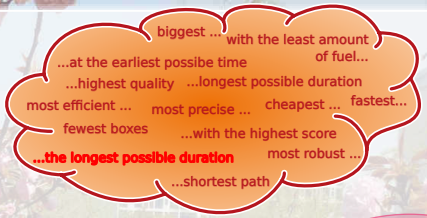

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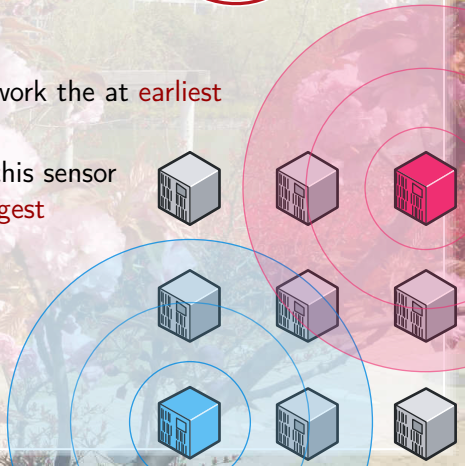

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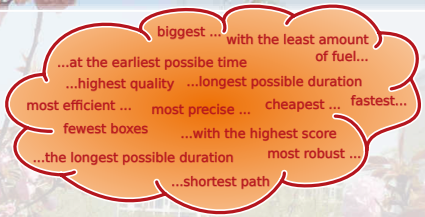

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- And so on.

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- optimized logistics (business-to-customer)
  - planning and scheduling of maintenance visits
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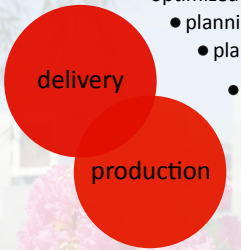

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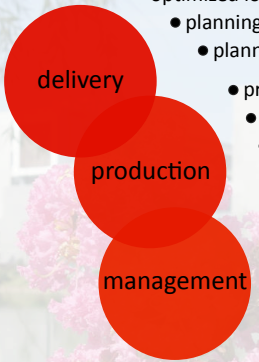

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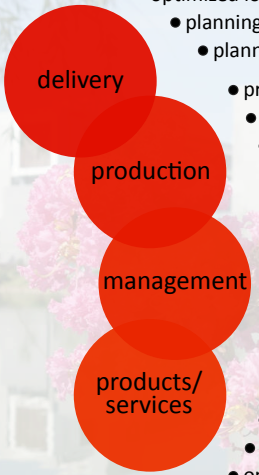

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computational intelligence
machine learning
data mining
delivery
production
management
products/
services
sales
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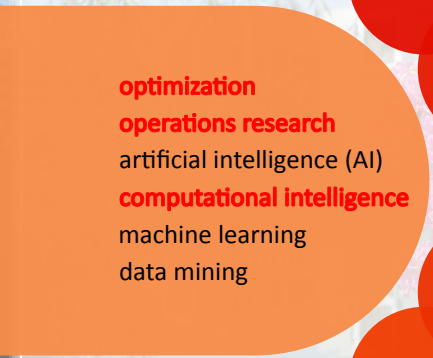

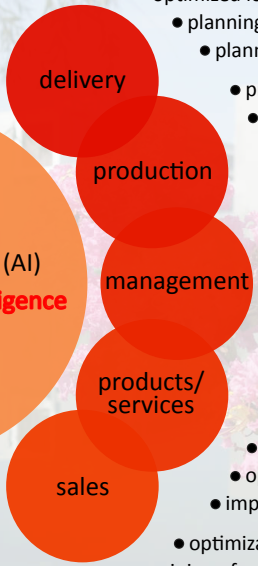

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- There are incredibly many problems from a very wide area where we can use optimization and Operations Research (OR).

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- How is this related to what you already learned?

Problems that we can Solve with Equations

A Problem that we can Solve with (High School Maths) Equations

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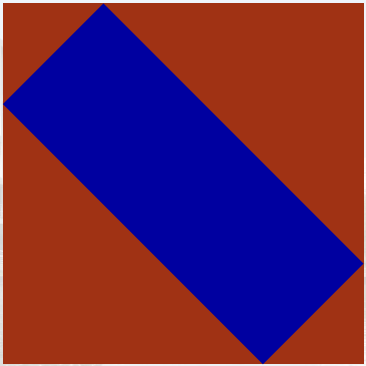

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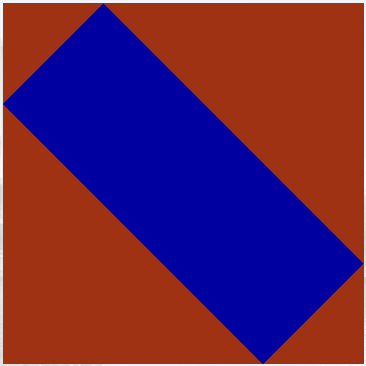

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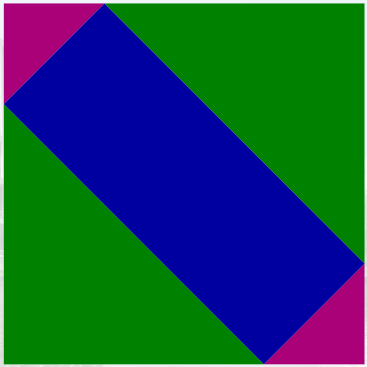

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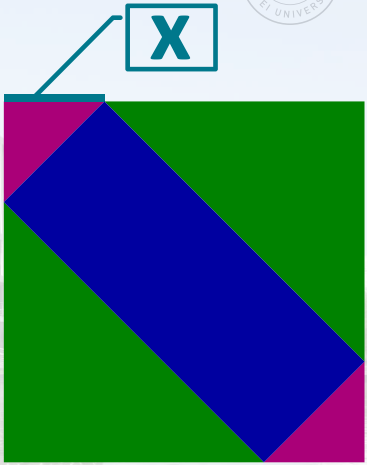

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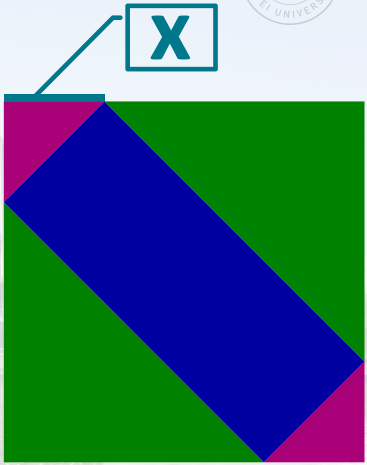

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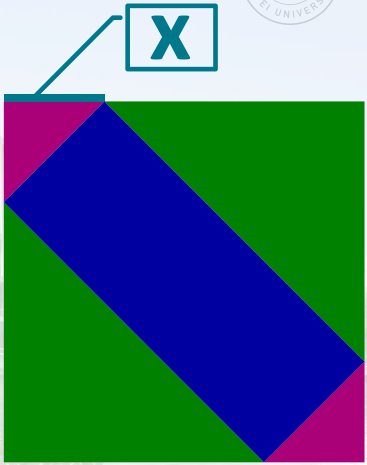

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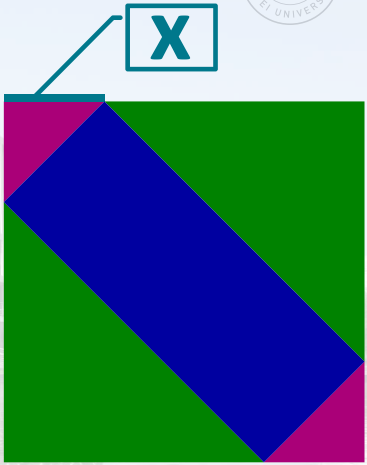

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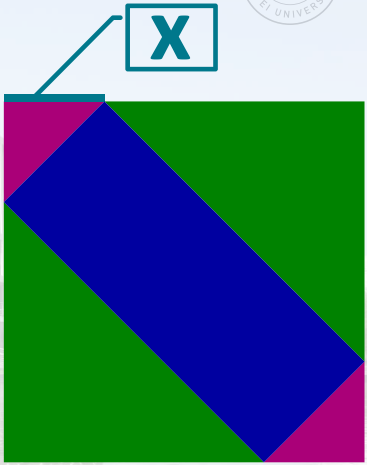

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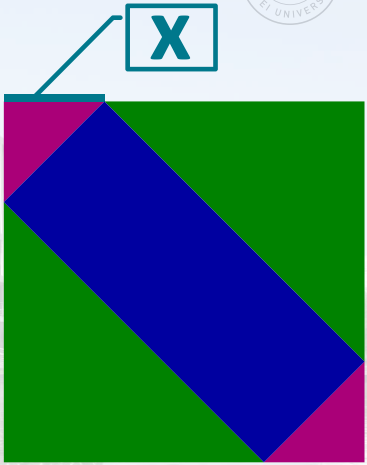

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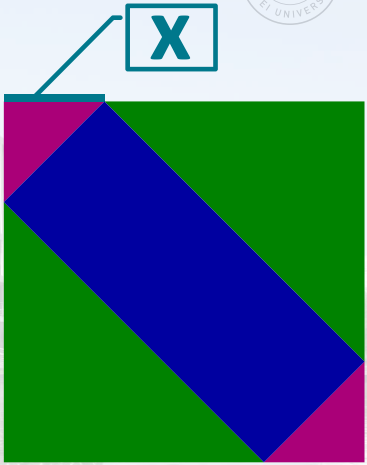

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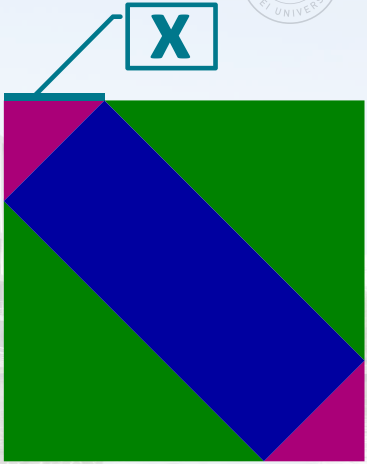

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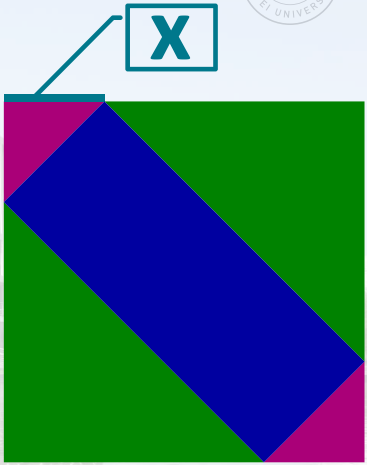

A Problem that we can Solve with (High School Maths) Equations

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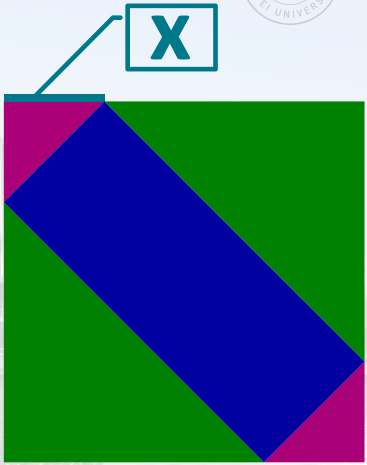

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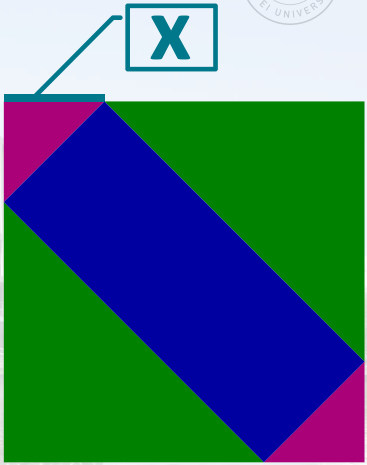

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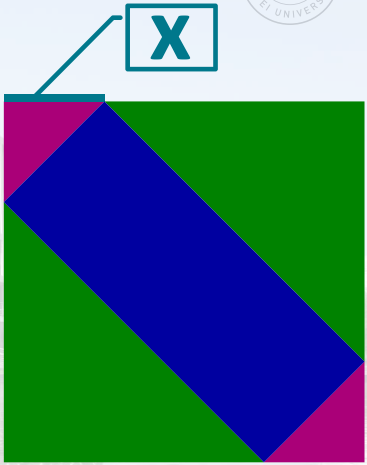

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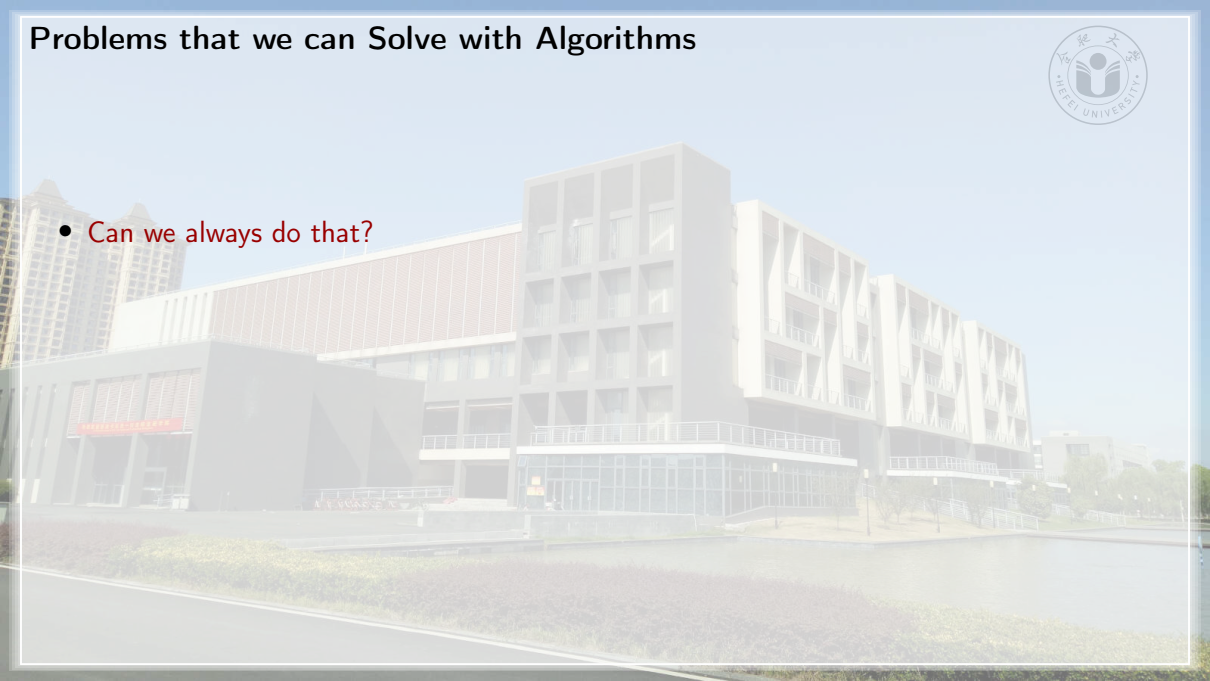

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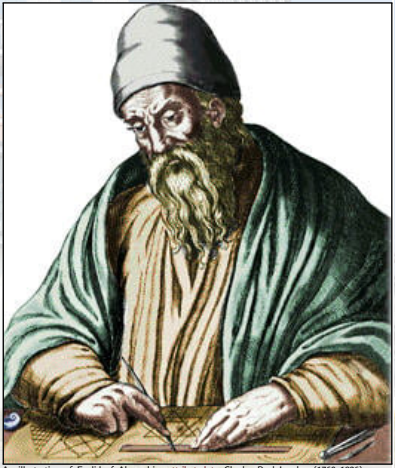

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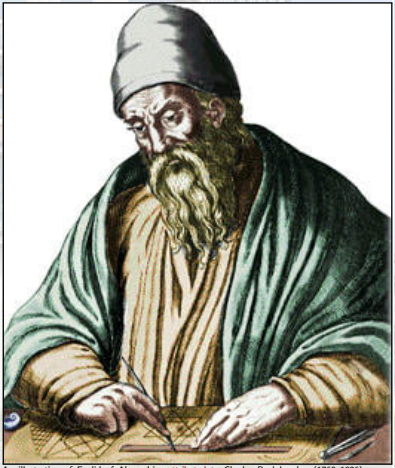

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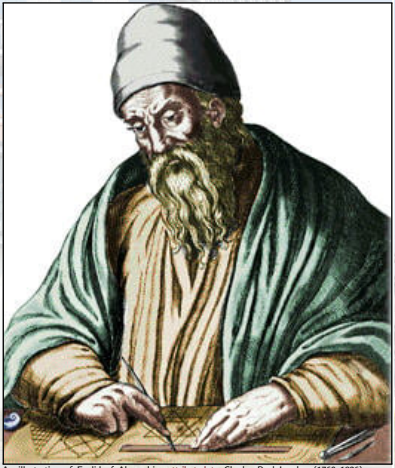

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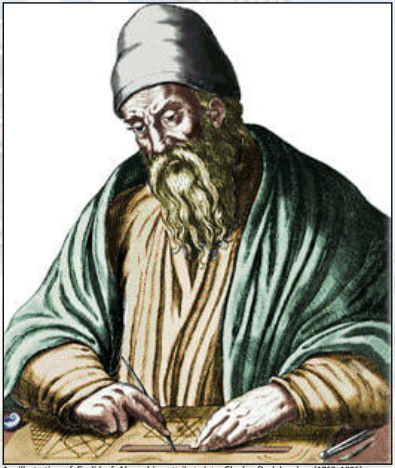

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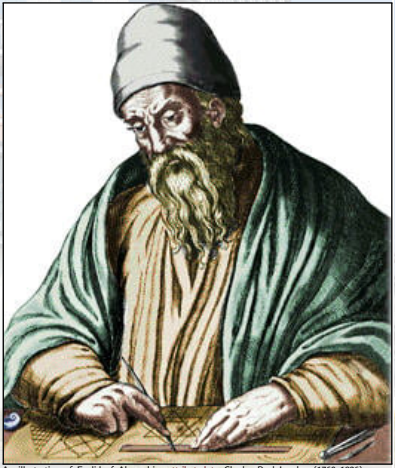

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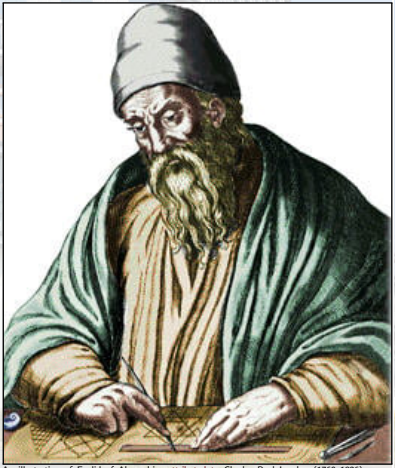

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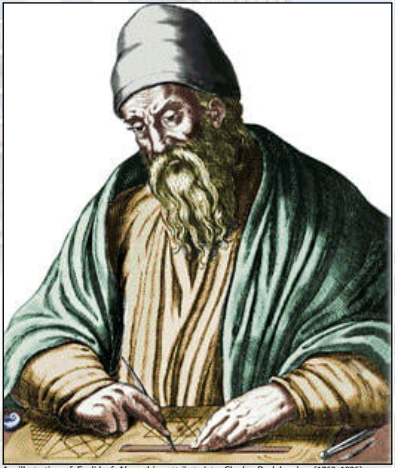

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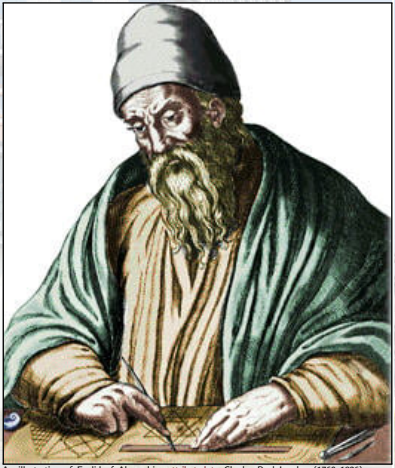

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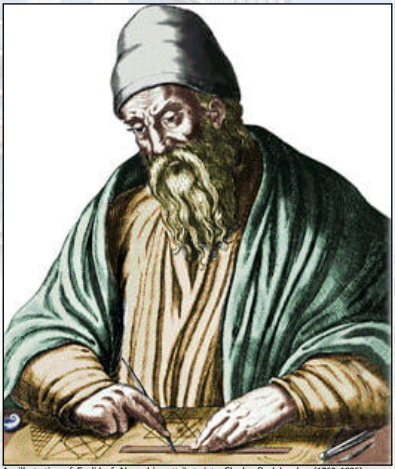

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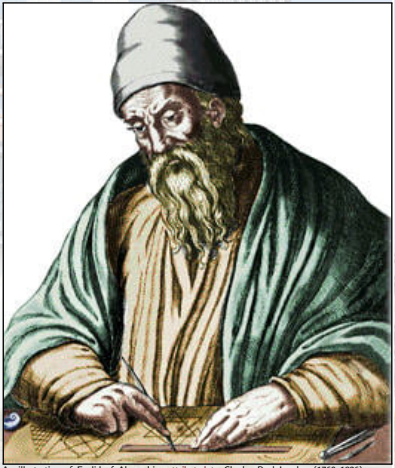

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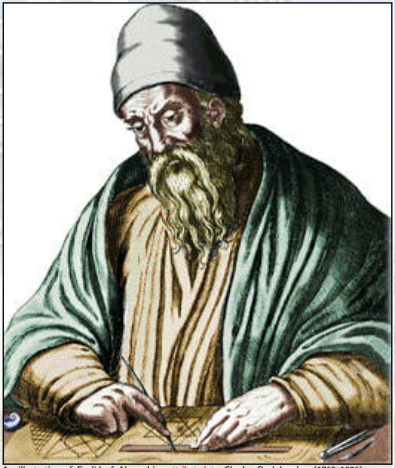

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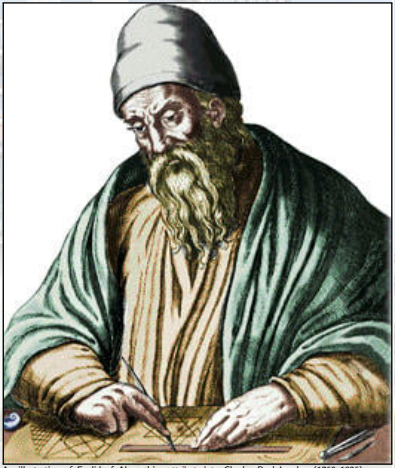

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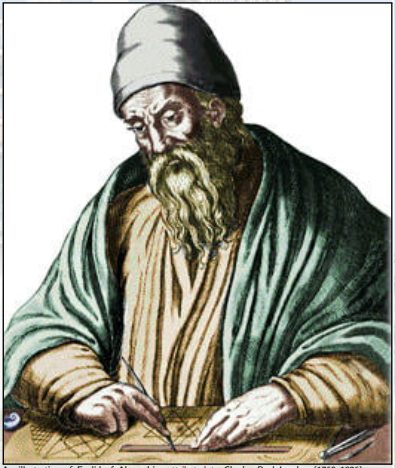

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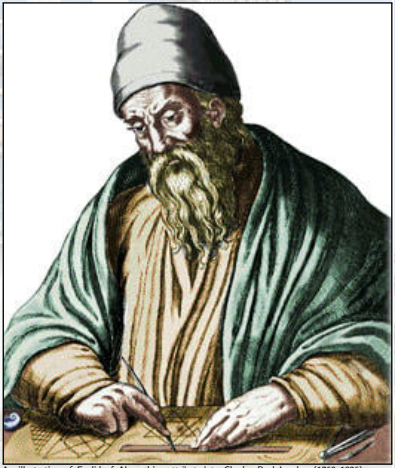

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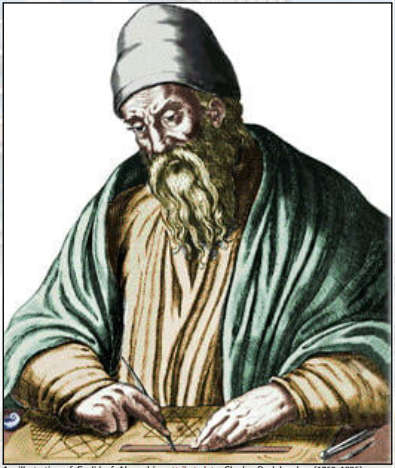

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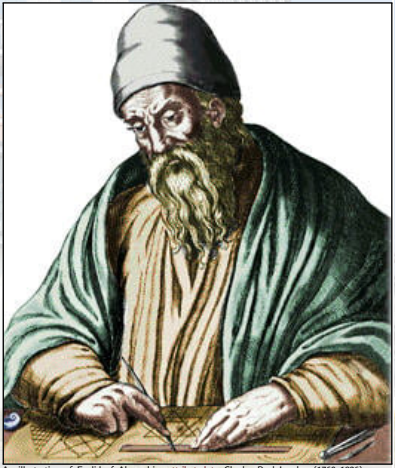


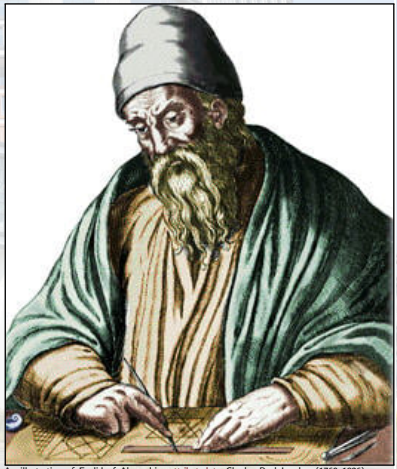
An illustration of Euclid of Alexandria, attributed to Charles Paul Landon (1760–1826). Source: Vividia, where it is listed as *domaine public*.

A Problem that we can Solve with (a High School) Algorithm

- What is the greatest common divisor ($\gcd$) of $a = 257\,9731$ and $b = 164\,1647$?
- We can repeatedly subtract the smaller from the larger number until we "converge."

bigger number: a	**smaller number:** b	$a - b$
257 9731	164 1647	93 8084
164 1647	93 8084	70 3563
93 8084	70 3563	23 4521
70 3563	23 4521	46 9042
46 9042	23 4521	23 4521

- $\gcd(257\,9731, 164\,1647) = 23\,4521$.



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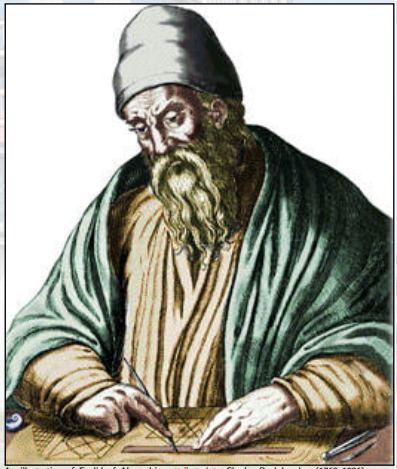
Solved.



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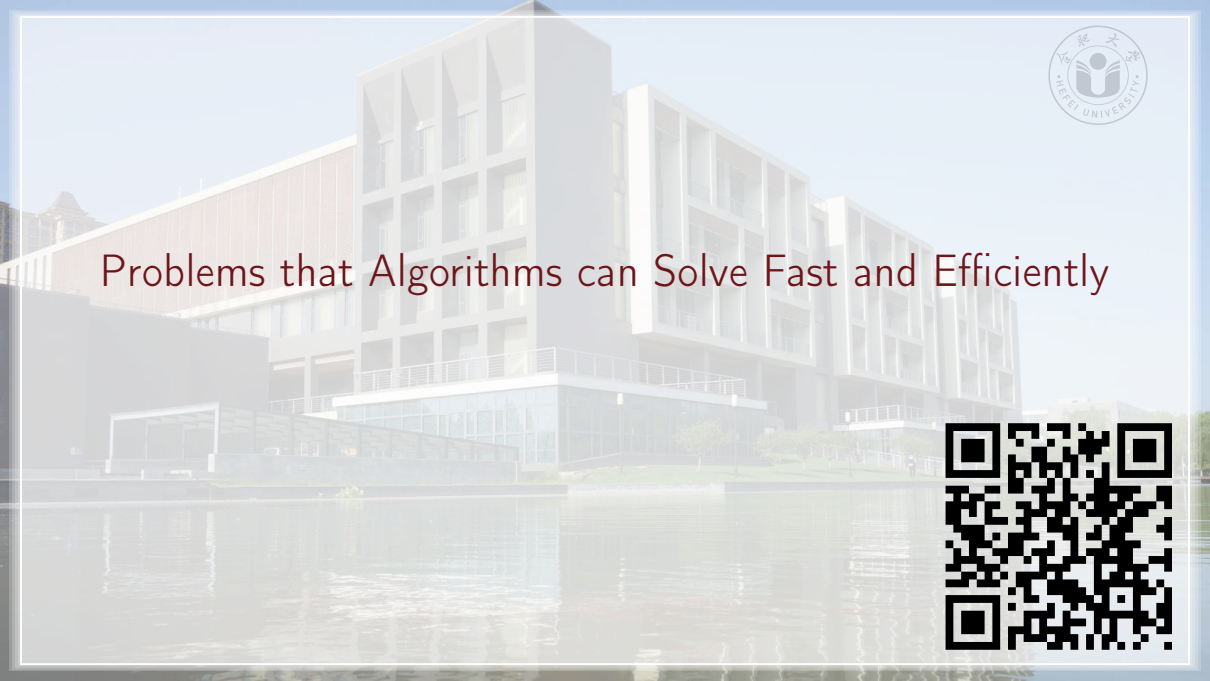
- (Side note: The binary Euclidean algorithm can be computed in $c * (\log a + \log b)$ steps where $c > 0$ is some constance. It needs two memory cells[4,5].)



Problems that Algorithms can Solve Fast and Efficiently

Find the Shortest Path from Start to Goal

Find the Shortest Path from Start to Goal

- I am at the starting point 合肥大学南二区北大门.

Find the Shortest Path from Start to Goal

- I am at the starting point 合肥大学南二区北大门.
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Find the Shortest Path from Start to Goal

- I am at the starting point 合肥大学南二区北大门.
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Find the Shortest Path from Start to Goal

- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- How do I get there the fastest?
- We know the campus map.

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- We know the campus map.
- We want to compute the shortest path (before actually walking it).

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- We know the campus map.
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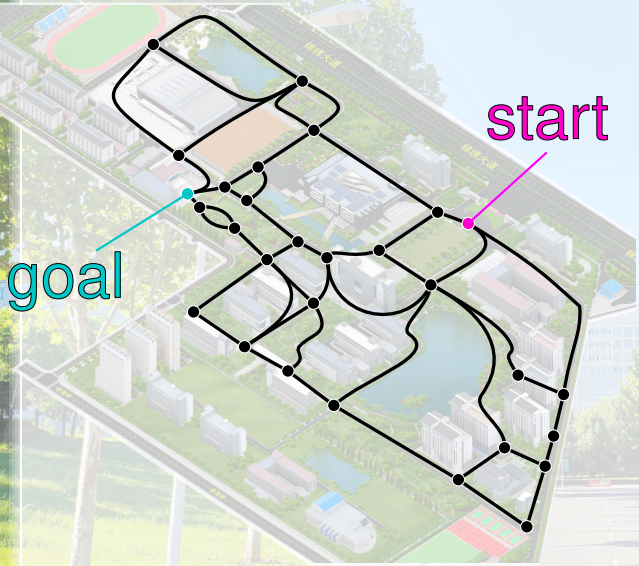
- We know all the intersections where I could make turns.

Find the Shortest Path from Start to Goal



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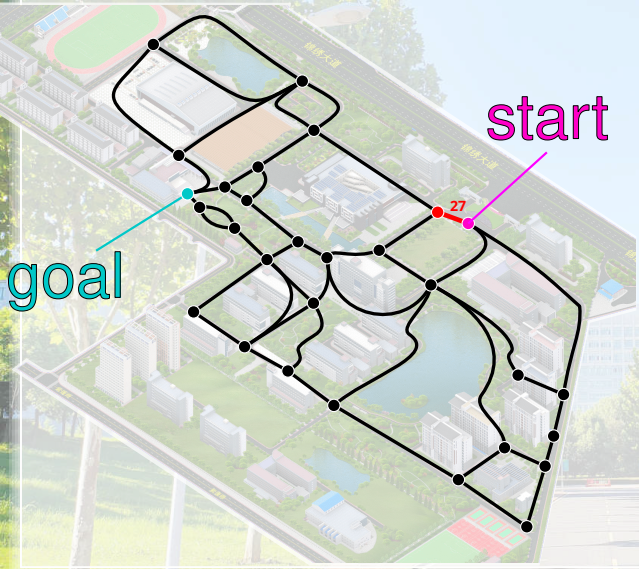
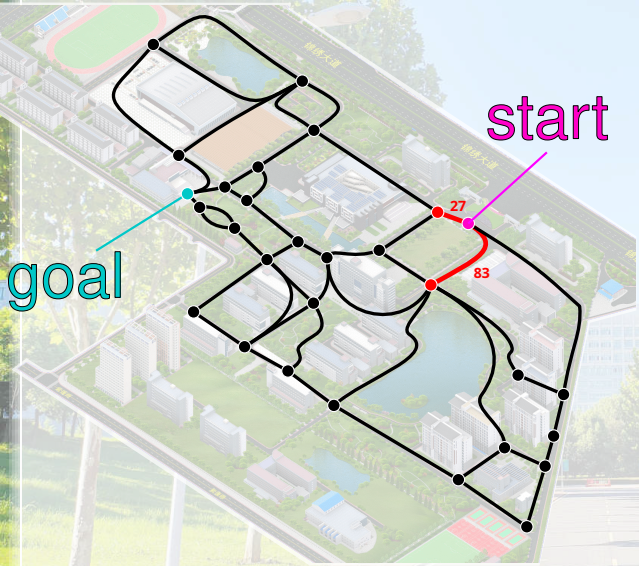


- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- For example, I could walk for 27s to this intersection.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- For example, I could walk for $27s$ to this intersection.
- Or for $83s$ to that one.

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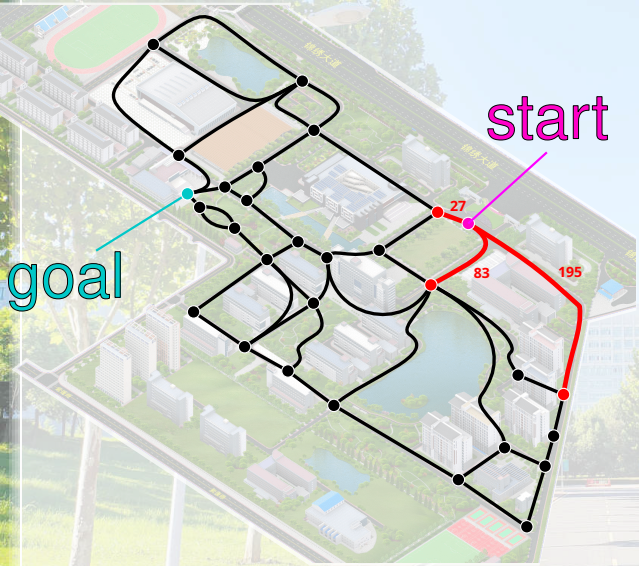

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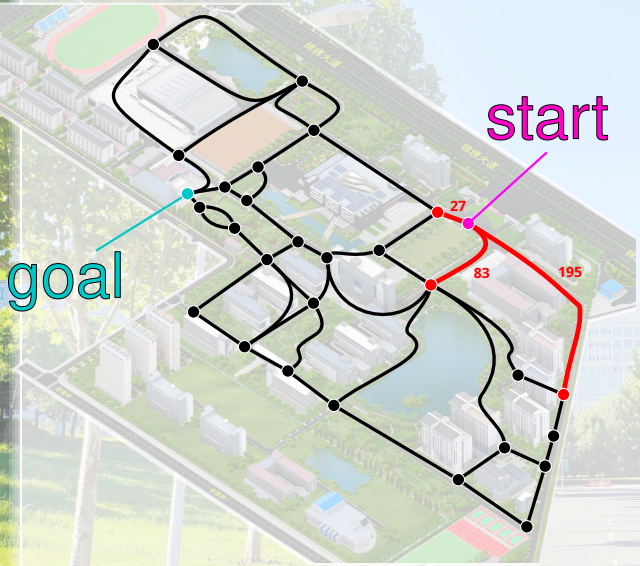

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- Which one should we pick?

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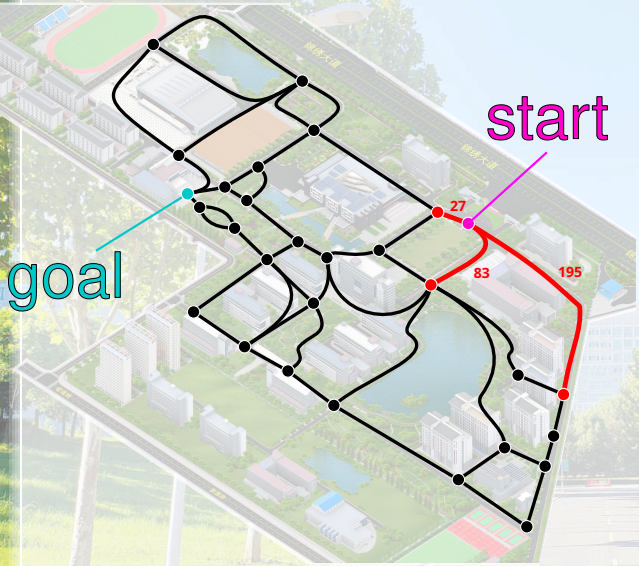

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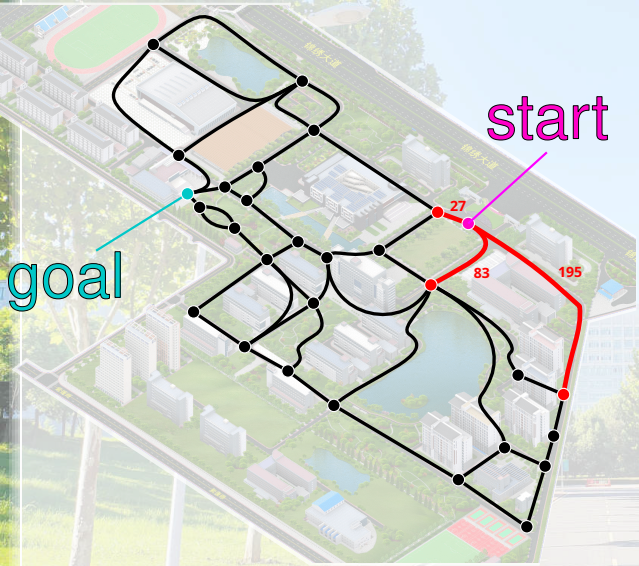

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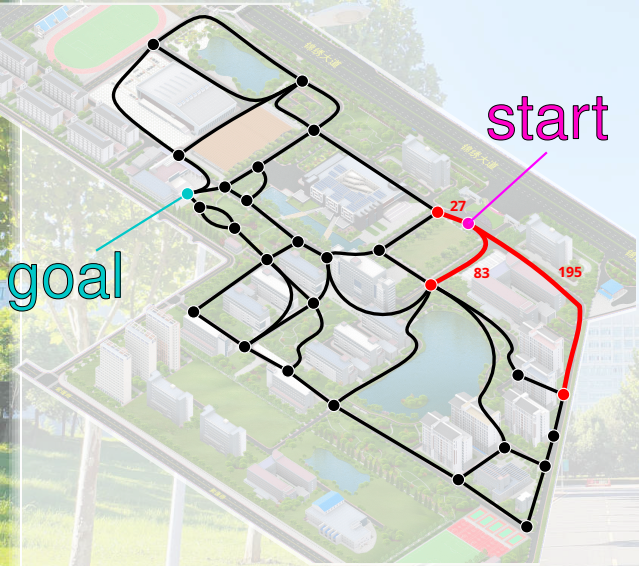

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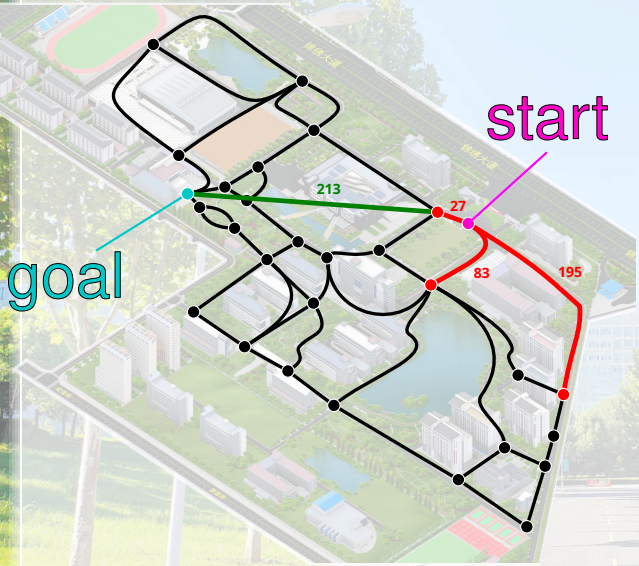

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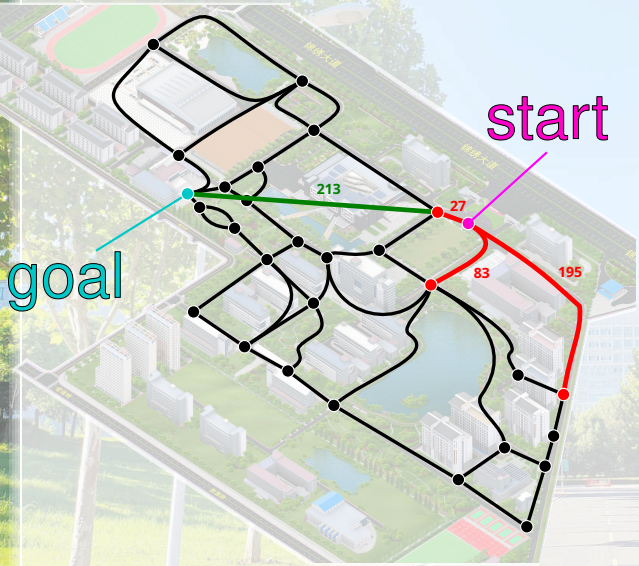

- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We have 3 choices: (a) $27s + 213s$, (b) $83s$, (c) $195s$.
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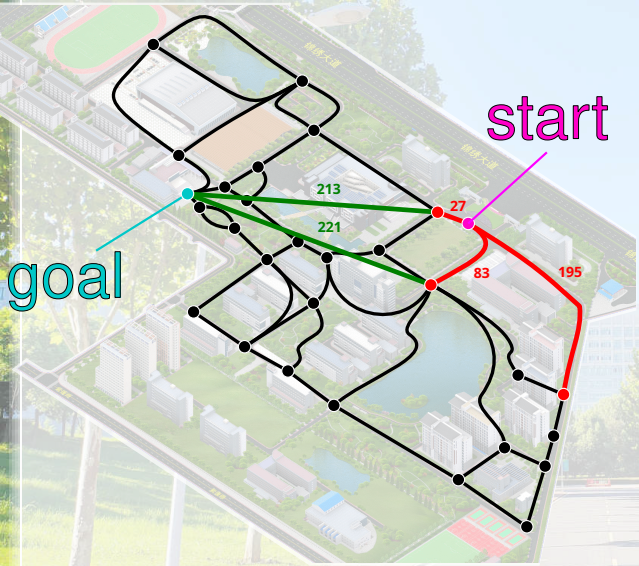
Find the Shortest Path from Start to Goal



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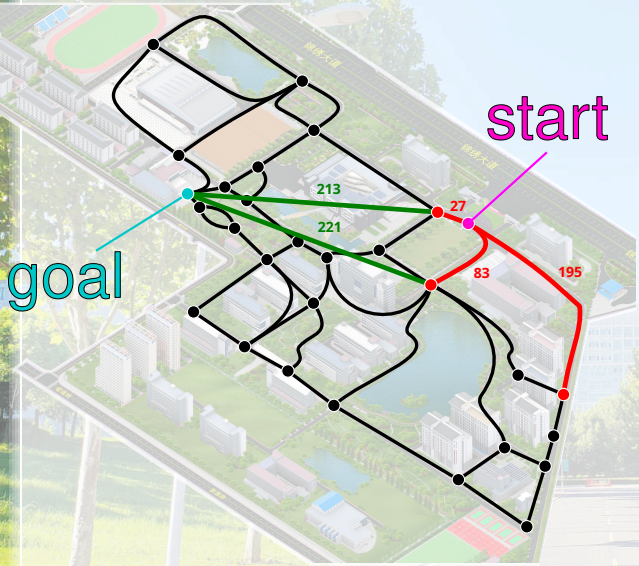

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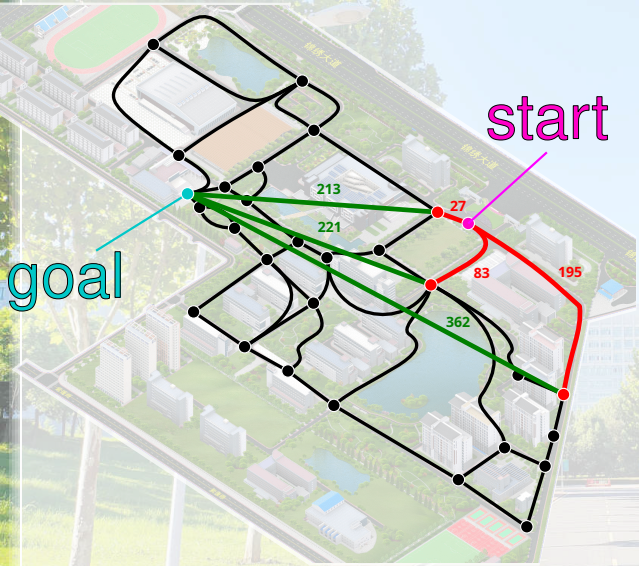

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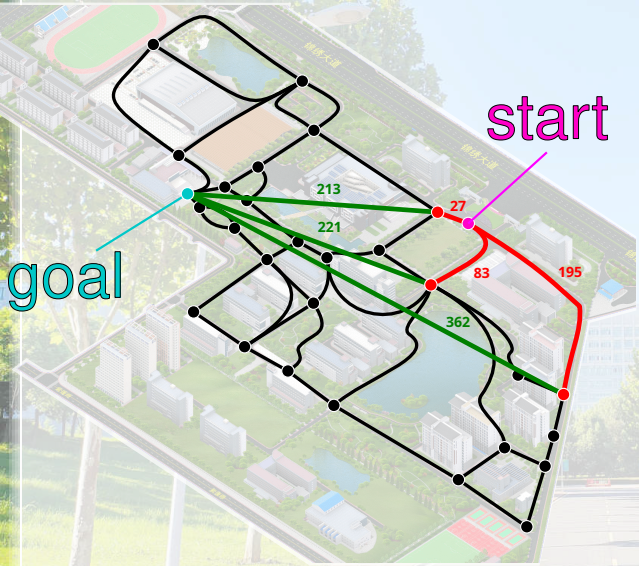

- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We have 3 choices: (a) $27s + 213s$, (b) $83s + 221s$, (c) $195s + 362s$.
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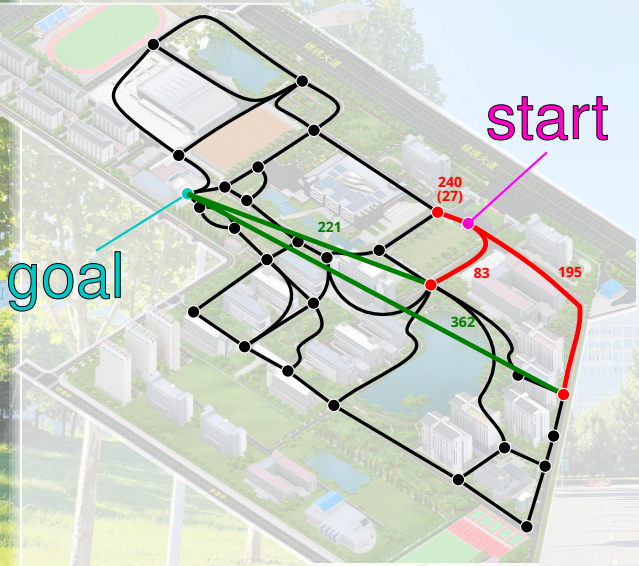
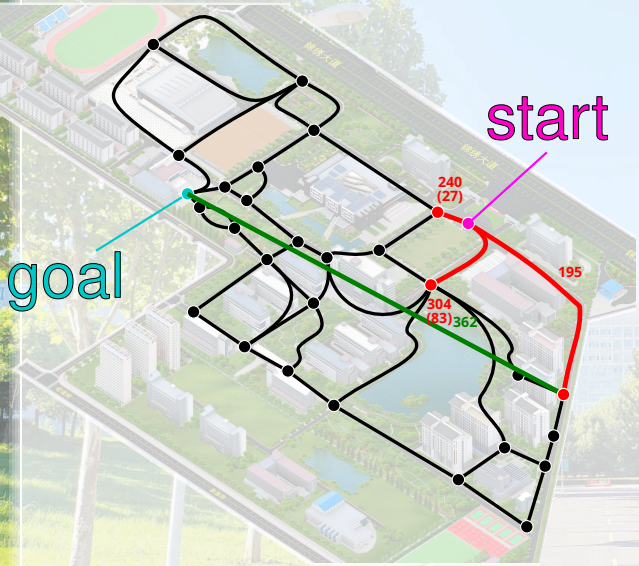
Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We have 3 choices:
  (a) $240s = 27s + 213s$, (b) $83s + 221s$,
  (c) $195s + 362s$.

Find the Shortest Path from Start to Goal



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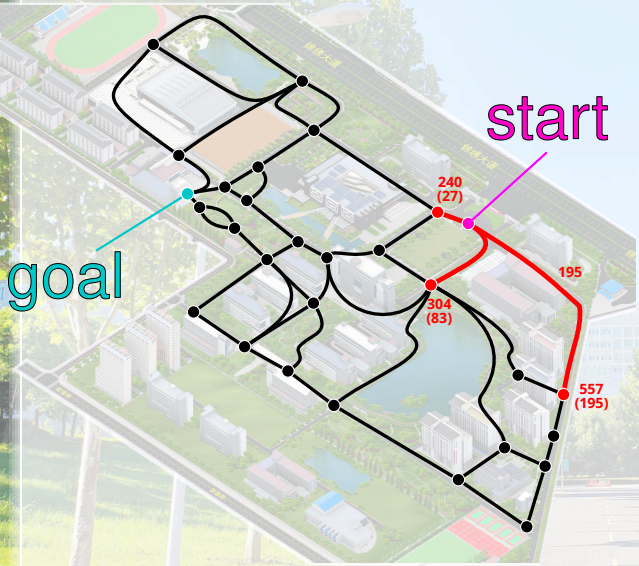
Find the Shortest Path from Start to Goal



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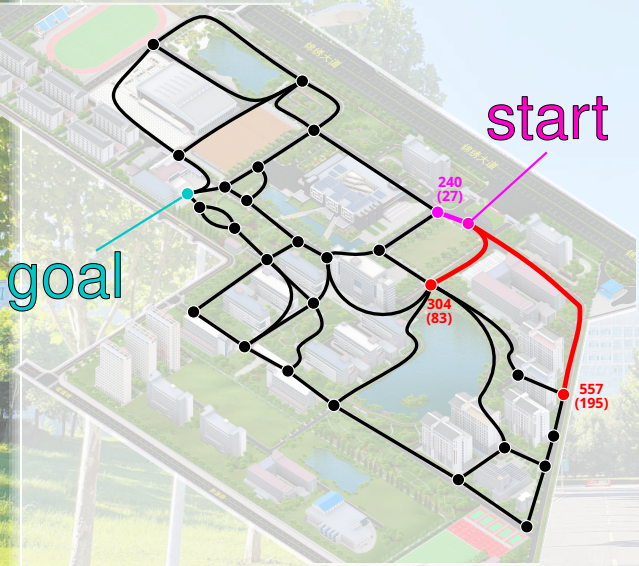

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- We have 3 choices:
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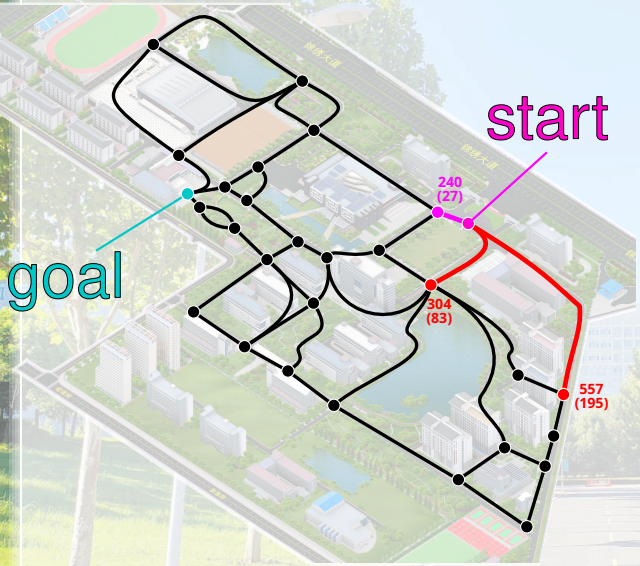
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$.
- The most interesting candidate for the first step is clearly (a).

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- The most interesting candidate for the first step is clearly (a).
- We have 2 remaining unexplored choices:
  (b) $304s = 83s + 221s$,
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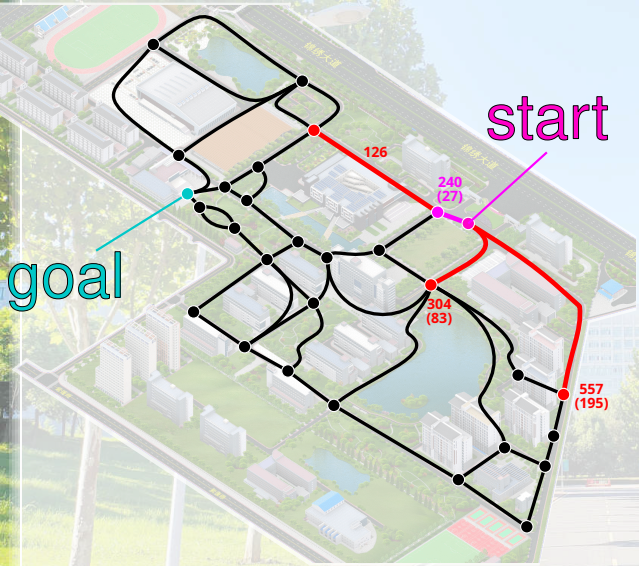
Find the Shortest Path from Start to Goal



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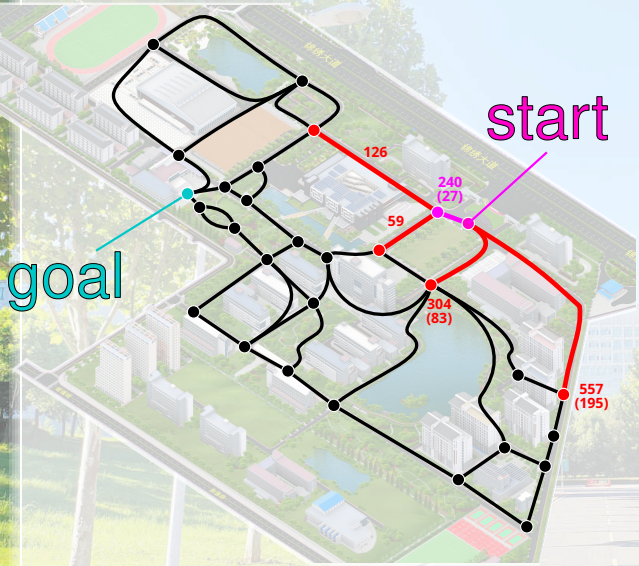
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$.
- From (a), we can go to (d) by walking for $126s$.

Find the Shortest Path from Start to Goal



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- The most interesting candidate for the first step is clearly (a).
- We have 2 remaining unexplored choices:
  (b) $304s = 83s + 221s$,
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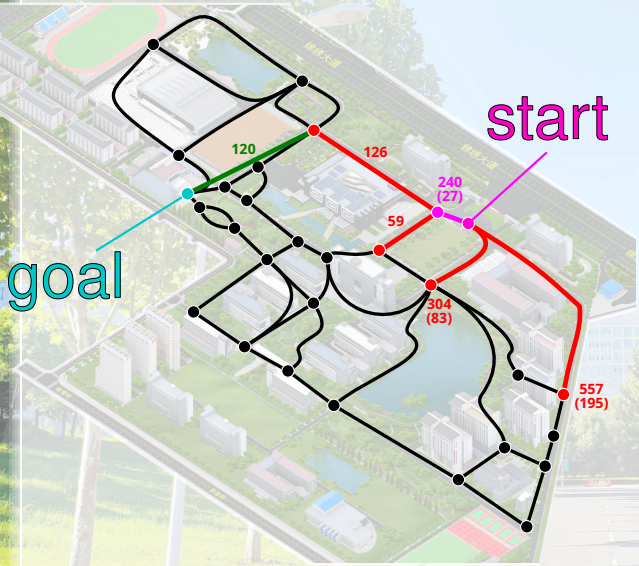
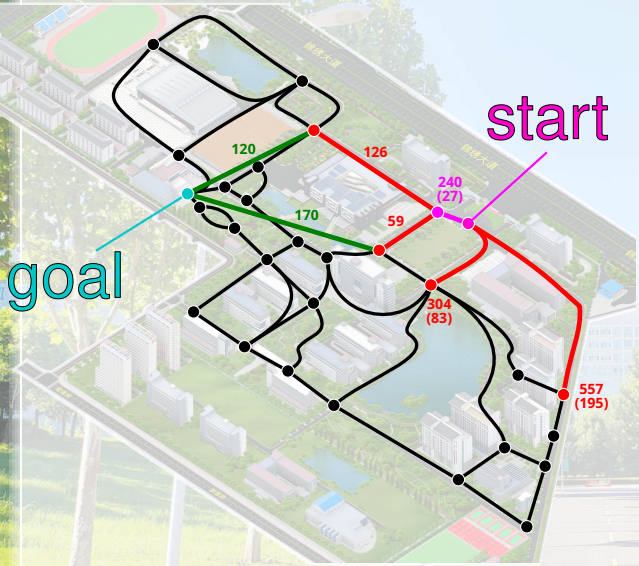
- From (a), we can go to (d) by walking for $126s$ or to (e) by walking for $59s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- The most interesting candidate for the first step is clearly (a).
- We have 2 remaining unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$.
- From (a), we can go to (d) by walking for $126s$ or to (e) by walking for $59s$.
- For both we compute the distance as the crow flies.

Find the Shortest Path from Start to Goal



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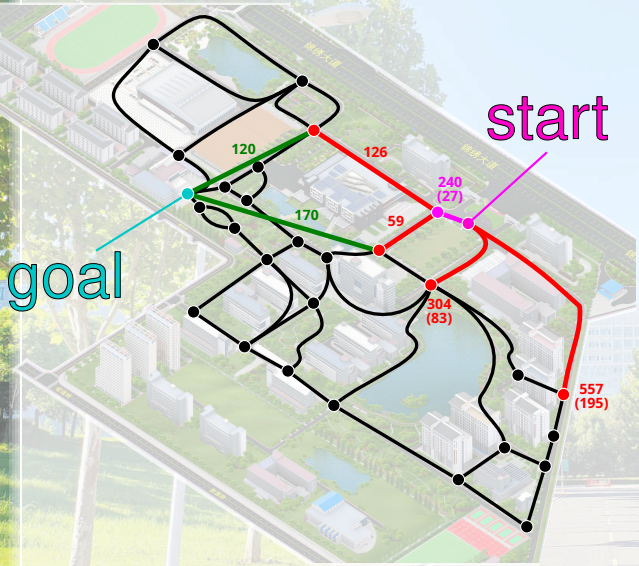
Find the Shortest Path from Start to Goal



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  (b) $304s = 83s + 221s$,
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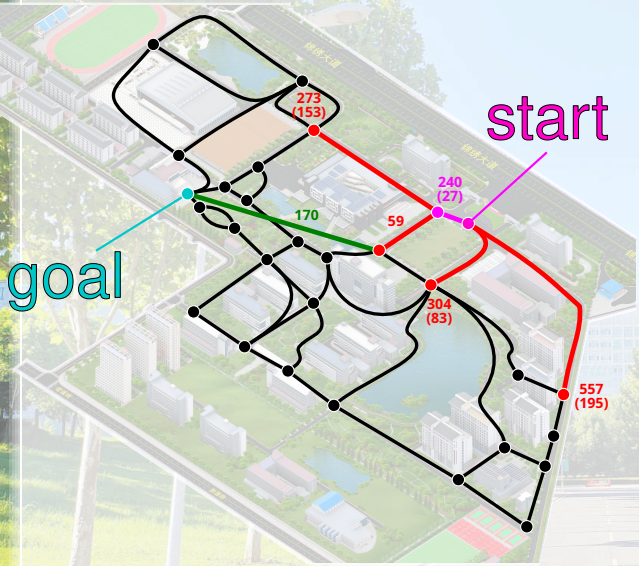
Find the Shortest Path from Start to Goal



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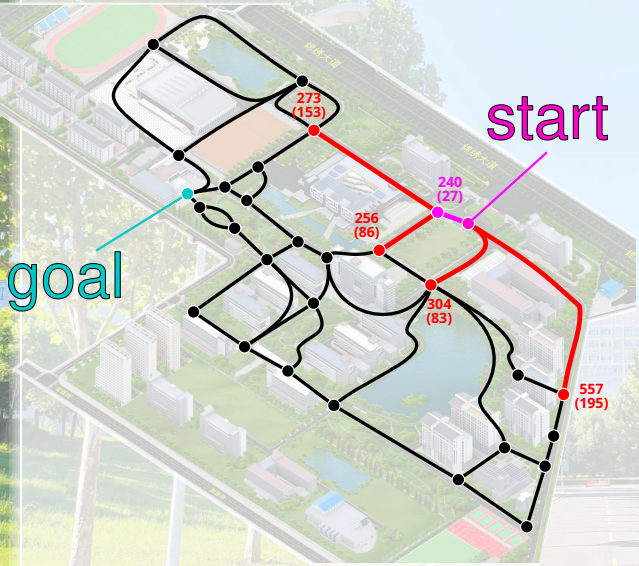
- I want to go to the goal: 食堂.
- The most interesting candidate for the first step is clearly (a).
- From (a), we can go to (d) by walking for $126s$ or to (e) by walking for $59s$.
- For both we compute the distance as the crow flies.
- We now have 3 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$.

Find the Shortest Path from Start to Goal



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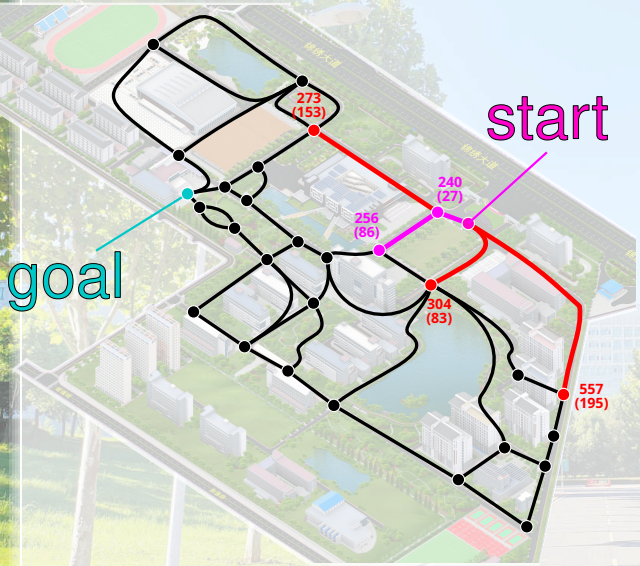
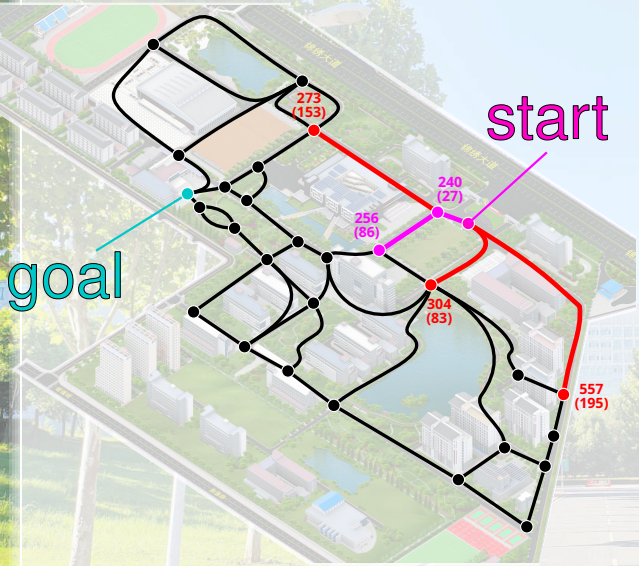
- I want to go to the goal: 食堂.
- The most interesting candidate for the first step is clearly (a).
- From (a), we can go to (d) by walking for $126s$ or to (e) by walking for $59s$.
- For both we compute the distance as the crow flies.
- We now have 4 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (e) $256s = 86s + 170s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 4 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (e) $256s = 86s + 170s$.
- (e) it is. . .

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- (e) it is...
- We have 3 remaining unexplored choices:
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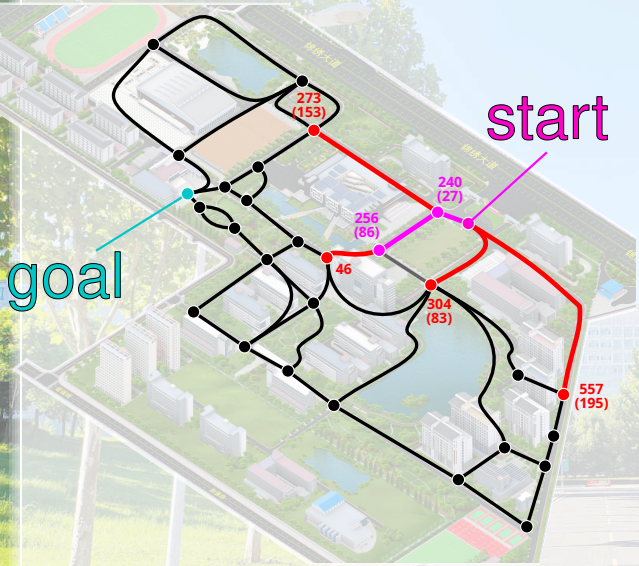
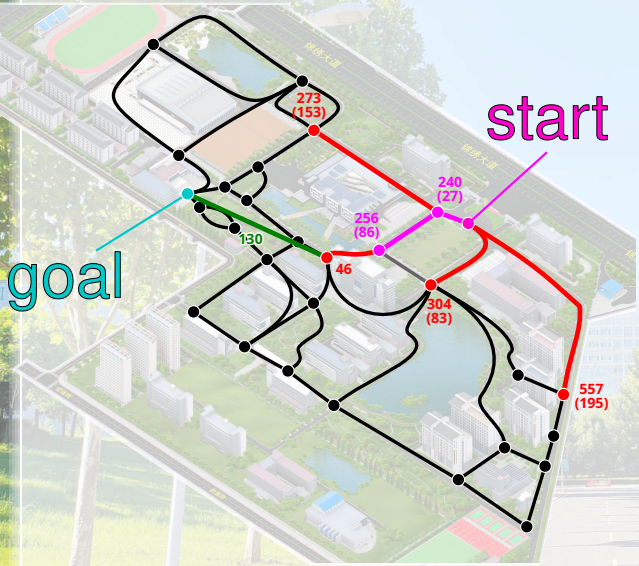
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We have 3 remaining unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$.
- From this point (e), we only have one reasonable choice where to go next.

Find the Shortest Path from Start to Goal



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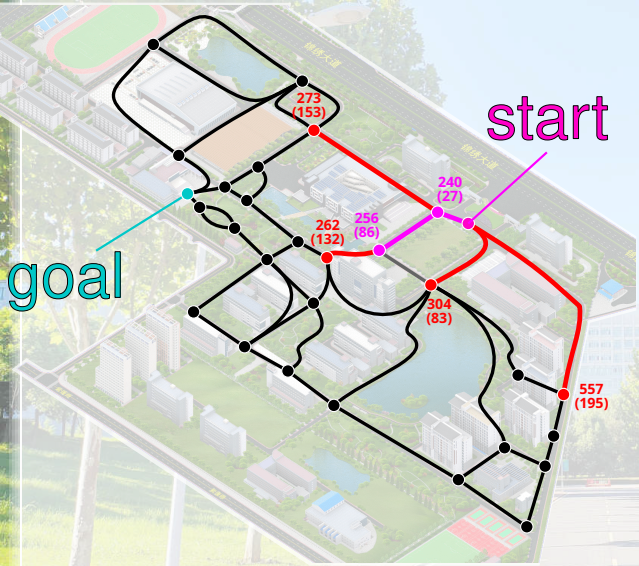
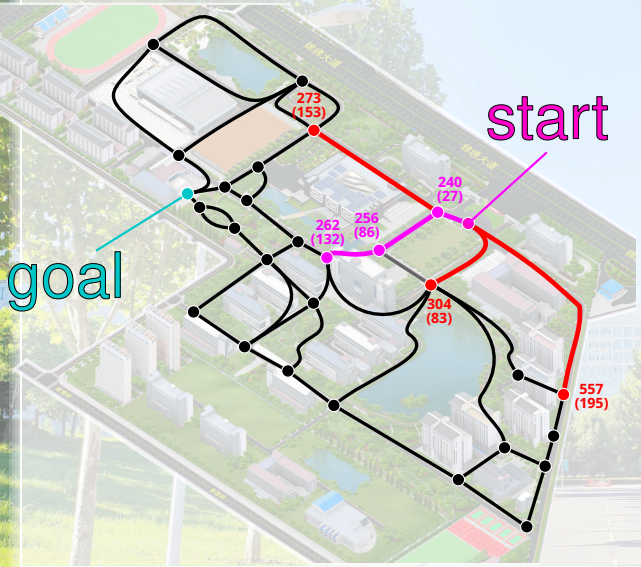
- I want to go to the goal: 食堂.
- We have 3 remaining unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$.
- From this point (e), we only have one reasonable choice where to go next.
- And we again compute the airline distance to the goal.

Find the Shortest Path from Start to Goal



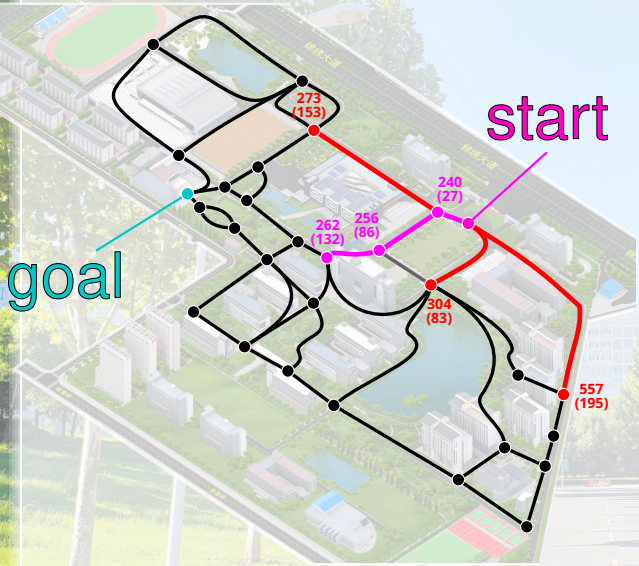
- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- From this point (e), we only have one reasonable choice where to go next.
- And we again compute the airline distance to the goal.
- We now have 4 choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (f) $262s = 132s + 130s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- And we again compute the airline distance to the goal.
- We now have 4 choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (f) $262s = 132s + 130s$.
- (f) it is. . .

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
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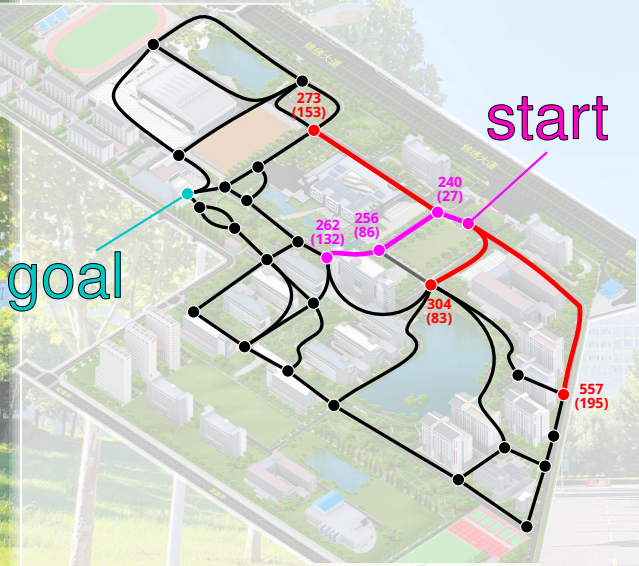

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- And we again compute the airline distance to the goal.
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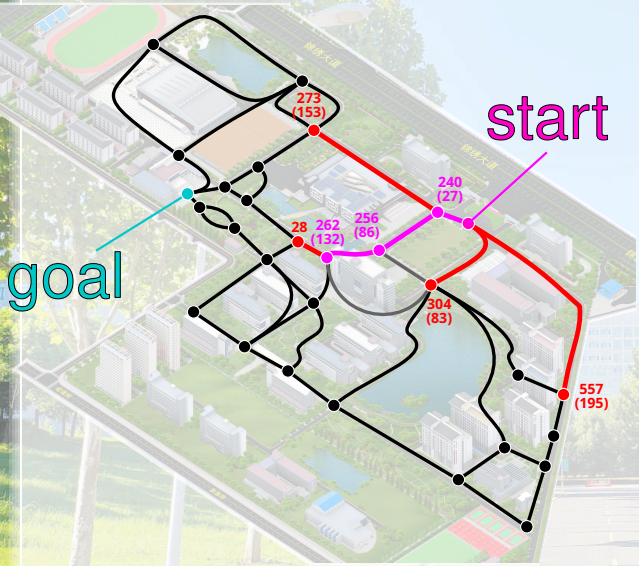
- From (f), we have two possible choices to continue.

Find the Shortest Path from Start to Goal



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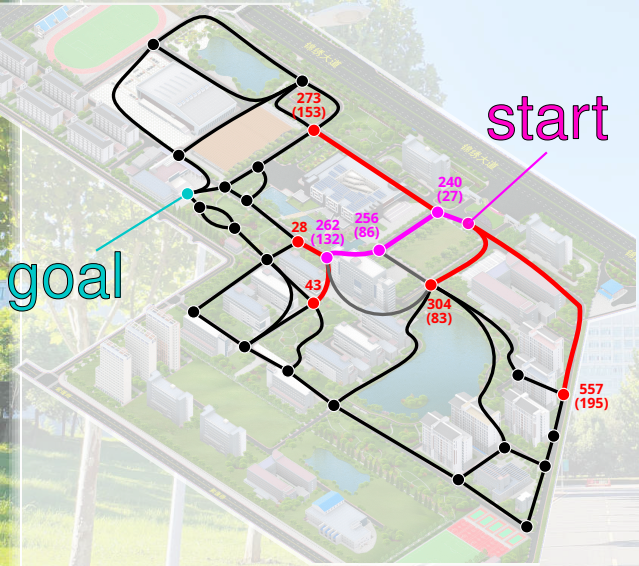

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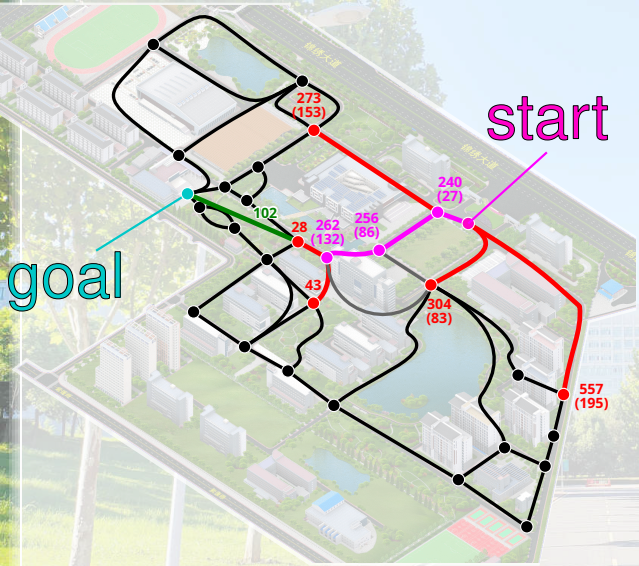

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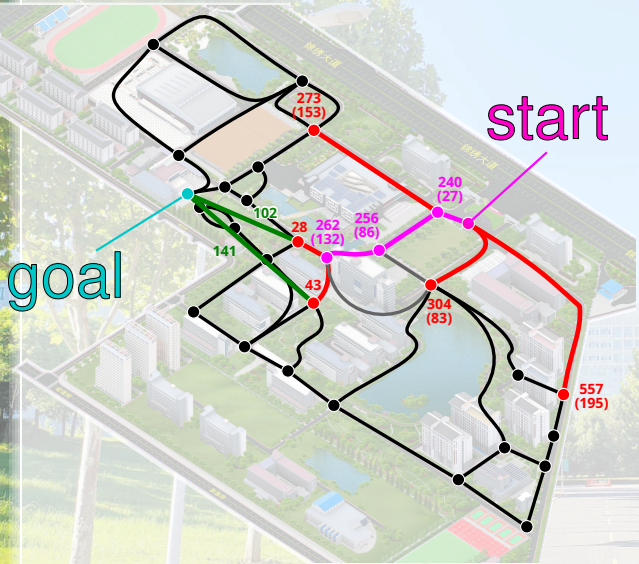

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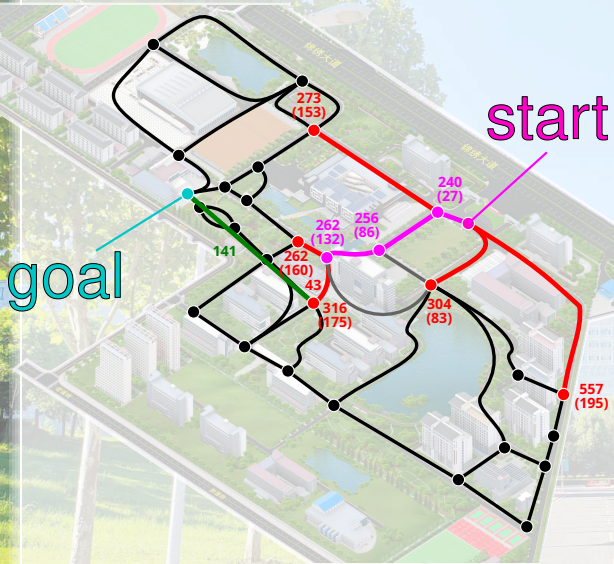

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- And we again compute the airline distance to the goal.
- (f) it is...
- From (f), we have two possible choices to continue.
- We now have 4 unexplored choices:
  (b) $304s = 83s + 221s$,
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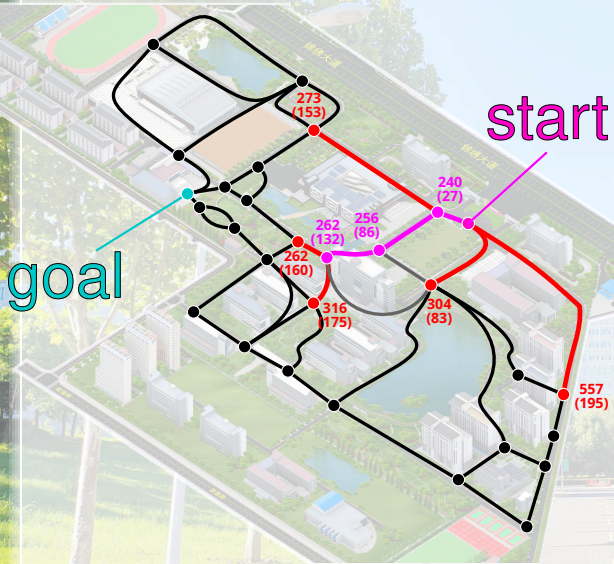
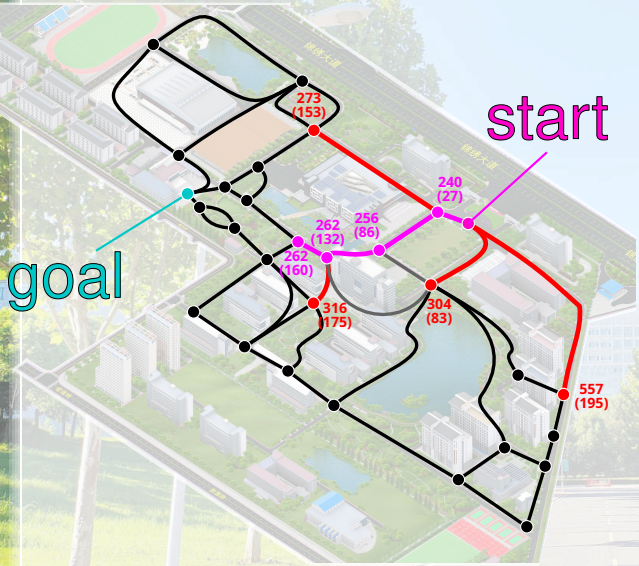
  (d) $273s = 153s + 120s$,
  (g) $262s = 160s + 102s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- And we again compute the airline distance to the goal.
- (f) it is...
- From (f), we have two possible choices to continue.
- We now have 5 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (g) $262s = 160s + 102s$,
  (h) $316s = 175s + 141s$.

Find the Shortest Path from Start to Goal



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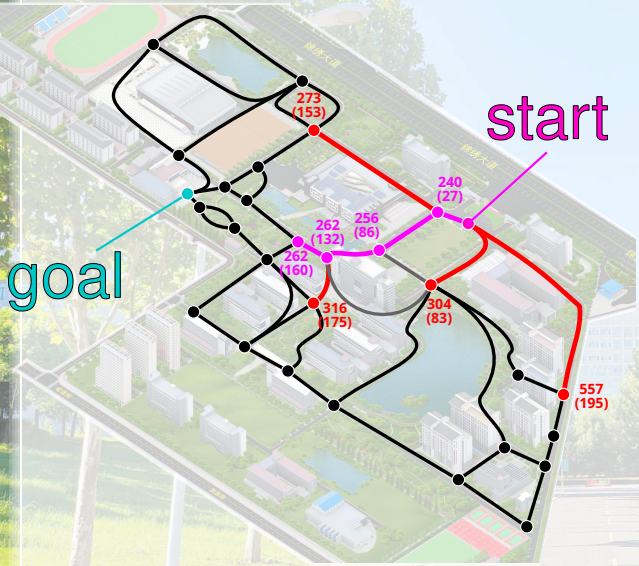
- And we again compute the airline distance to the goal.
- From (f), we have two possible choices to continue.
- We now have 5 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (g) $262s = 160s + 102s$,
  (h) $316s = 175s + 141s$.
- (g) it is...

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- (g) it is...
- We now have 4 unexplored choices:
  (b) $304s = 83s + 221s$,
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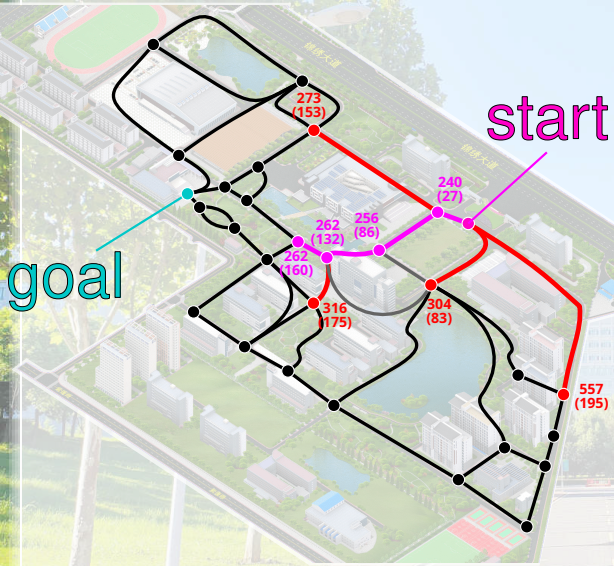
Find the Shortest Path from Start to Goal



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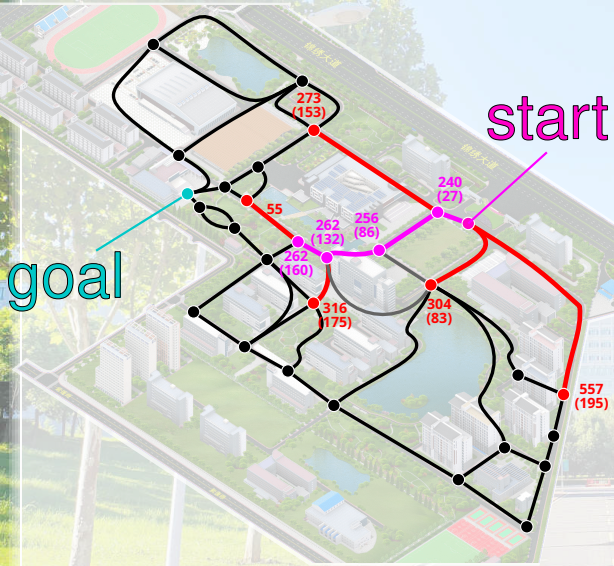
- I want to go to the goal: 食堂.
- (g) it is...
- We now have 4 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (h) $316s = 175s + 141s$.
- And we get two new choices.

Find the Shortest Path from Start to Goal



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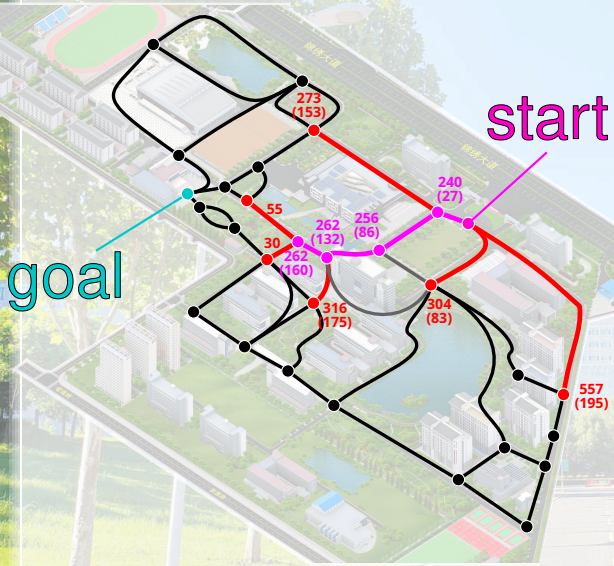

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- (g) it is...
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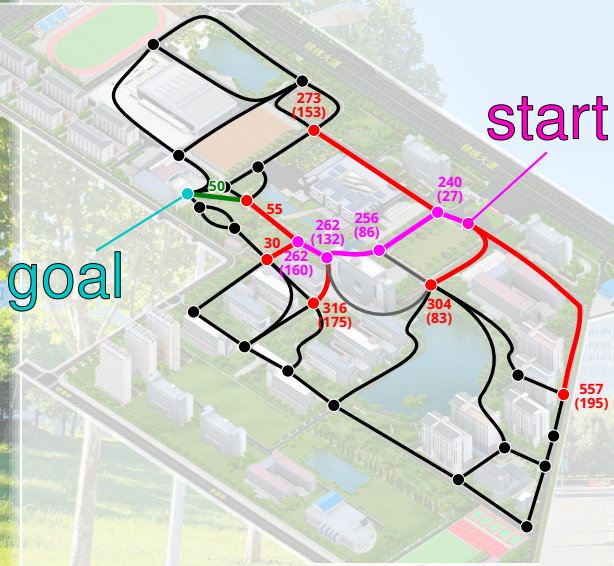

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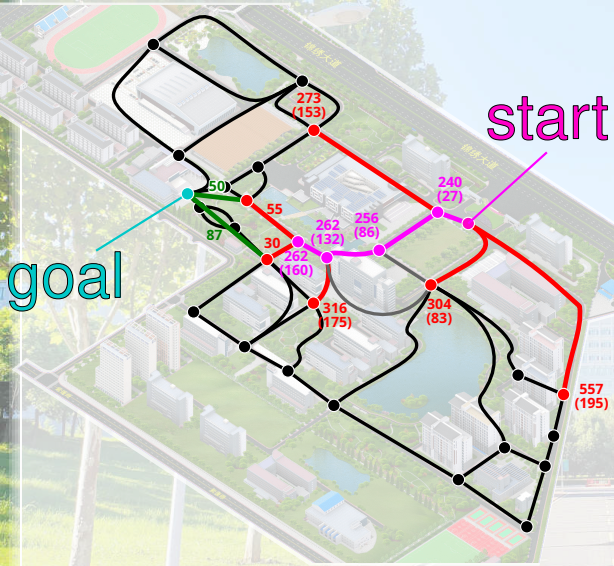

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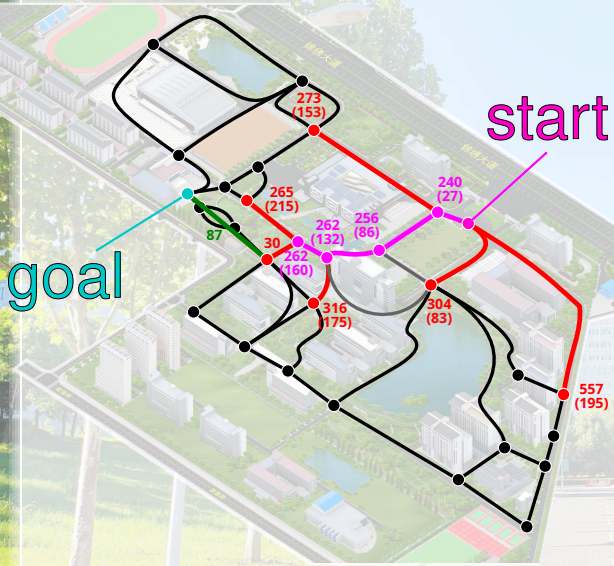
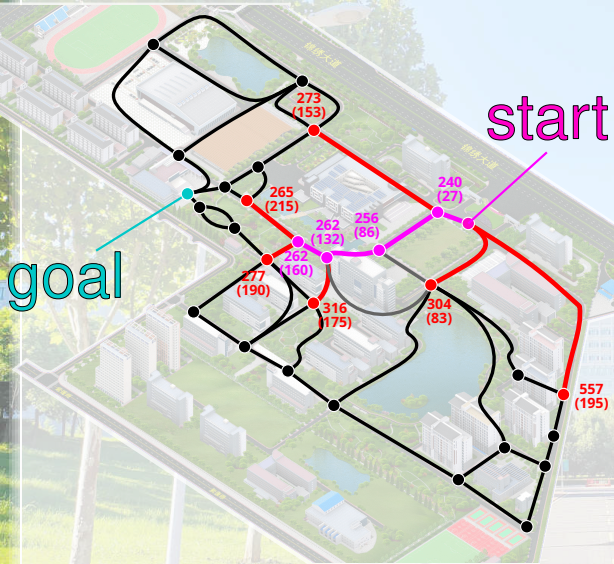
- And we get two new choices.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- And we get two new choices.
- We now have 5 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (h) $316s = 175s + 141s$,
  (i) $265s = 215s + 50s$.

Find the Shortest Path from Start to Goal



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- And we get two new choices.
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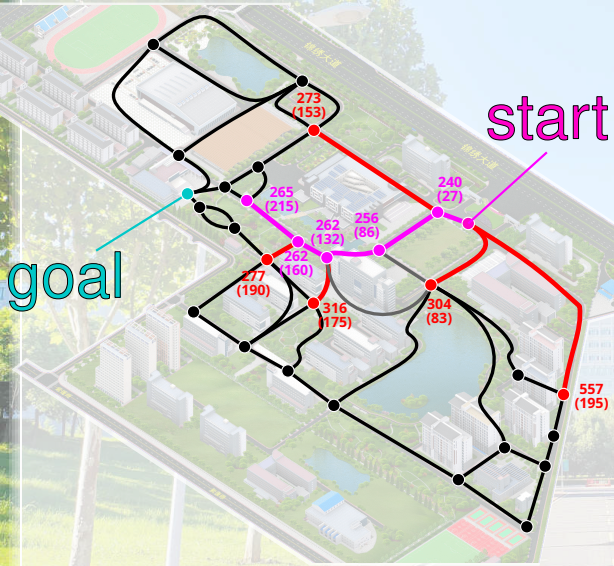
  - (i) $265s = 215s + 50s$,
  - (j) $277s = 190s + 87s$.

Find the Shortest Path from Start to Goal



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- I want to go to the goal: 食堂.
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  - (b) $304s = 83s + 221s$,
  - (c) $557s = 195s + 362s$,
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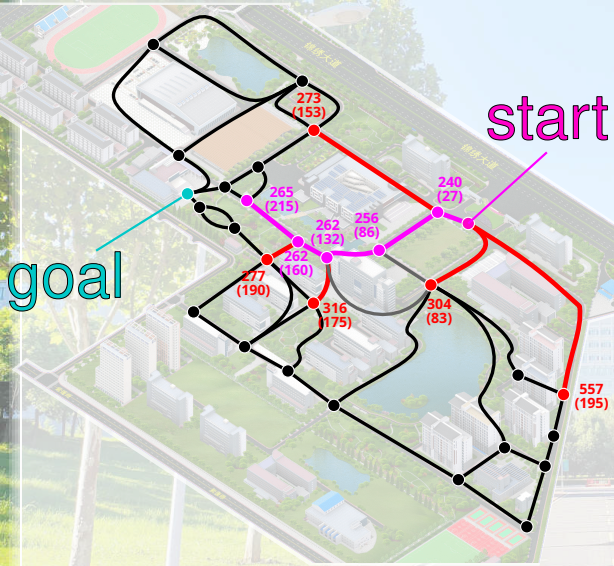
  - (h) $316s = 175s + 141s$,
  - (i) $265s = 215s + 50s$,
  - (j) $277s = 190s + 87s$.
- (i) it is. . .

Find the Shortest Path from Start to Goal



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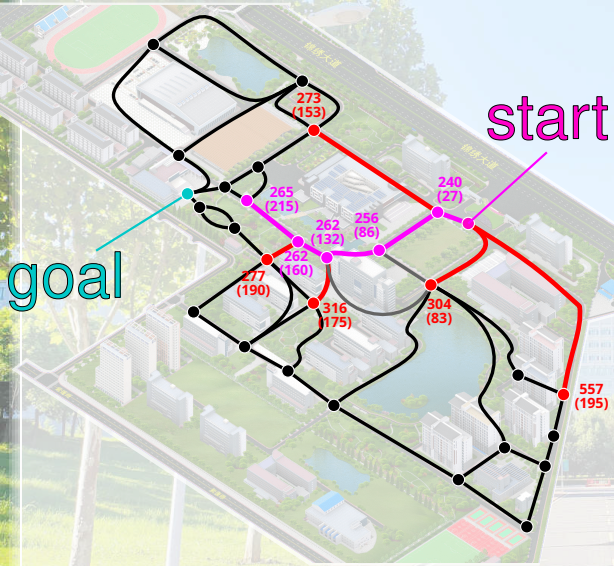
Find the Shortest Path from Start to Goal



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- From (i), we again got two new choices.

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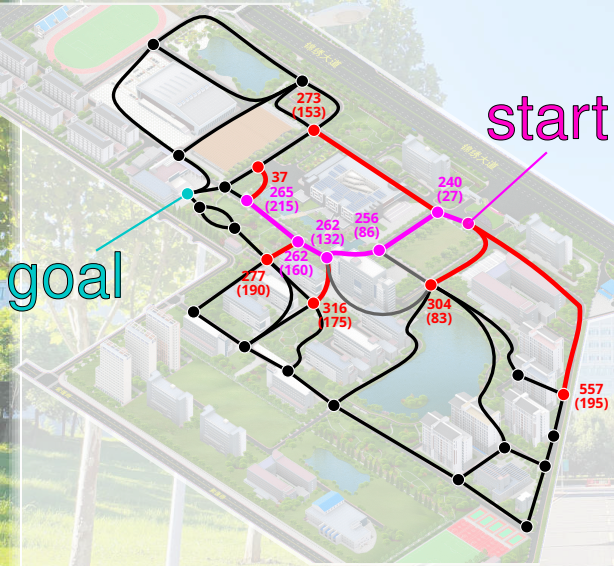

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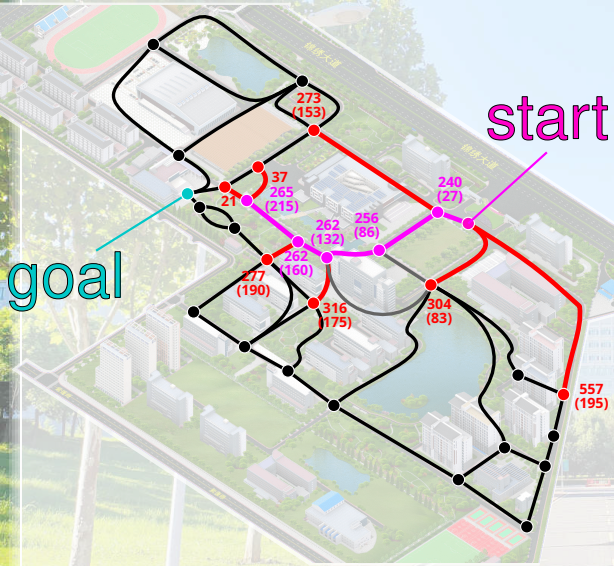

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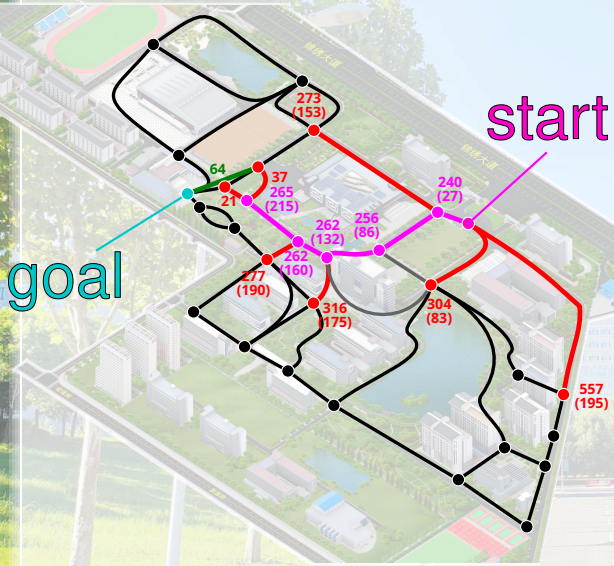

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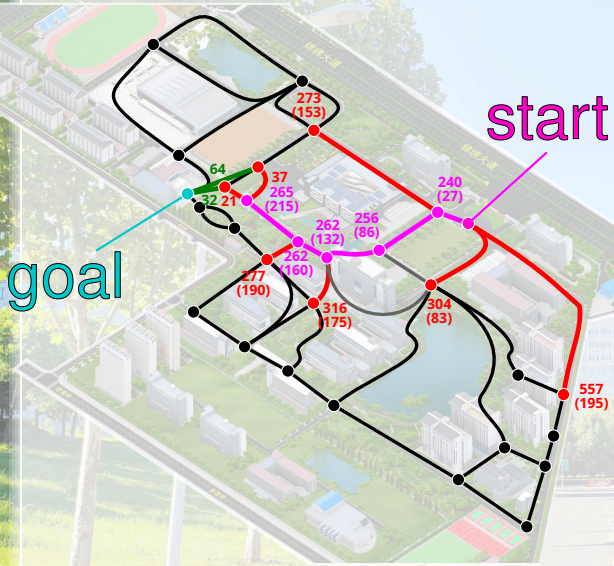

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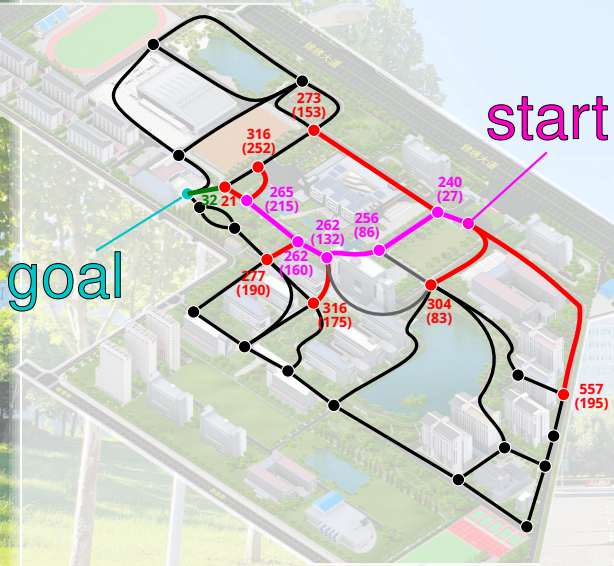
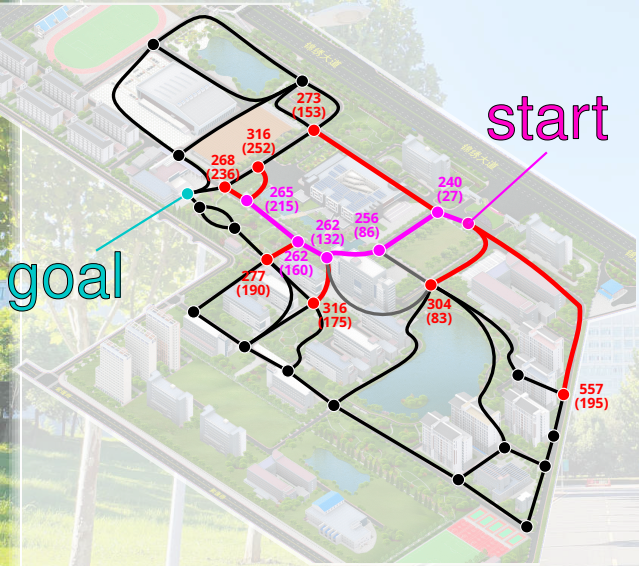


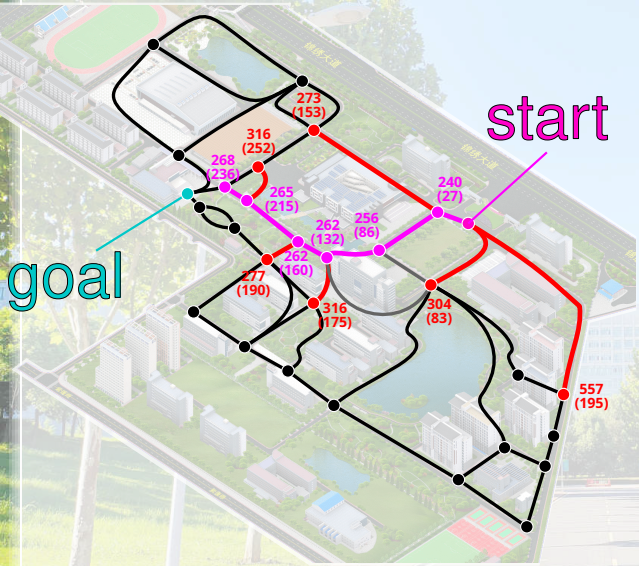
- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- From (i), we again got two new choices.
- We now have 6 unexplored choices:
  - (b) $304s = 83s + 221s$,
  - (c) $557s = 195s + 362s$,
  - (d) $273s = 153s + 120s$,
  - (h) $316s = 175s + 141s$,
  - (j) $277s = 190s + 87s$,
  - (k) $316s = 252s + 64s$.

Find the Shortest Path from Start to Goal



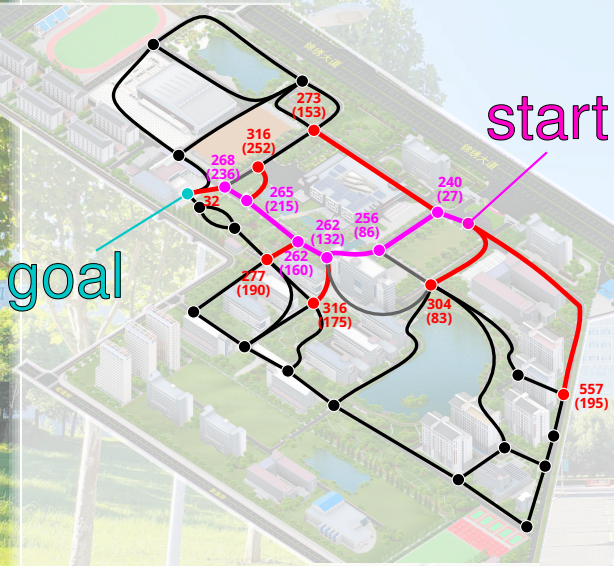
- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- From (i), we again got two new choices.
- We now have 7 unexplored choices:
  - (b) $304s = 83s + 221s$,
  - (c) $557s = 195s + 362s$,
  - (d) $273s = 153s + 120s$,
  - (h) $316s = 175s + 141s$,
  - (j) $277s = 190s + 87s$,
  - (k) $316s = 252s + 64s$,
  - (l) $268s = 236s + 32s$.

Find the Shortest Path from Start to Goal



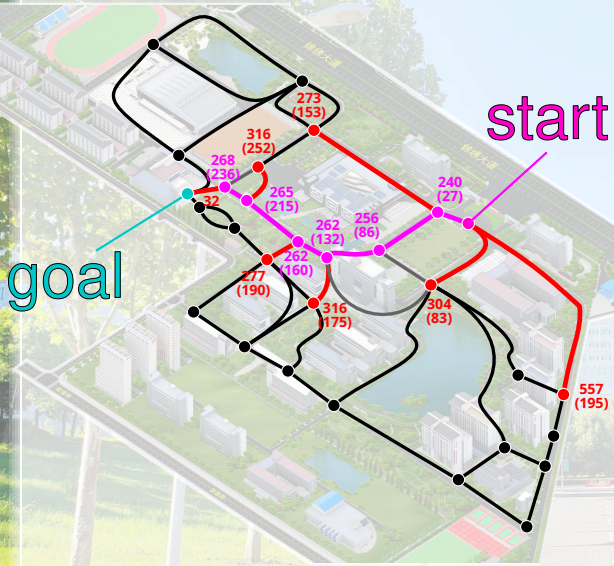
- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- From (i), we again got two new choices.
- We now have 7 unexplored choices:
  - (b) $304s = 83s + 221s$,
  - (c) $557s = 195s + 362s$,
  - (d) $273s = 153s + 120s$,
  - (h) $316s = 175s + 141s$,
  - (j) $277s = 190s + 87s$,
  - (k) $316s = 252s + 64s$,
  - (l) $268s = 236s + 32s$.
- (l) it is...

Find the Shortest Path from Start to Goal



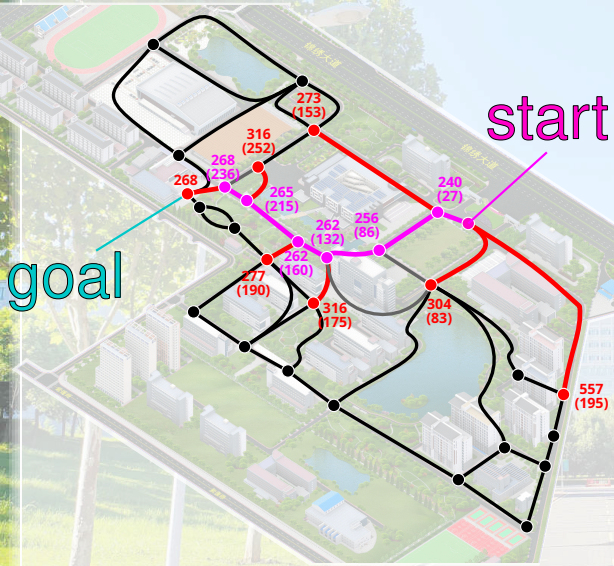
- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 6 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (h) $316s = 175s + 141s$,
  (j) $277s = 190s + 87s$,
  (k) $316s = 252s + 64s$.
- (l) it is...
- There is only one choice to continue: go to the goal!

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 6 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (h) $316s = 175s + 141s$,
  (j) $277s = 190s + 87s$,
  (k) $316s = 252s + 64s$.
- (l) it is...
- There is only one choice to continue: go to the goal!
- We found a first complete path from start to goal.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 6 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
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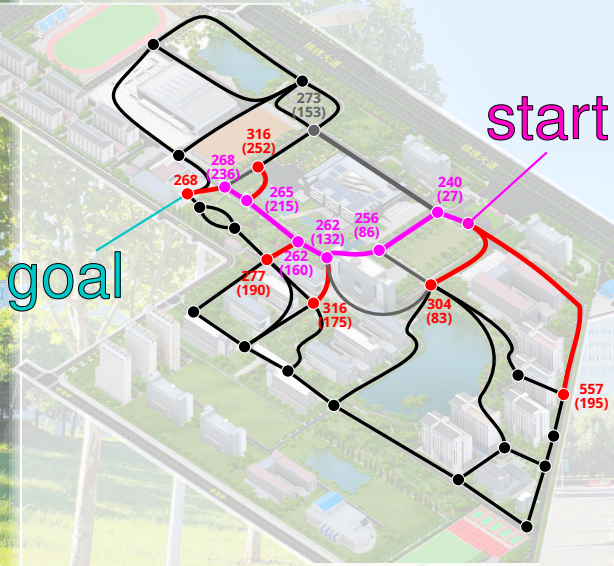
  (h) $316s = 175s + 141s$,
  (j) $277s = 190s + 87s$,
  (k) $316s = 252s + 64s$.
- There is only one choice to continue: go to the goal!
- We found a first complete path from start to goal.
- It has the total length $268s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 56 unexplored choices:
  (b) $304s = 83s + 221s$,
  (c) $557s = 195s + 362s$,
  (d) $273s = 153s + 120s$,
  (h) $316s = 175s + 141s$,
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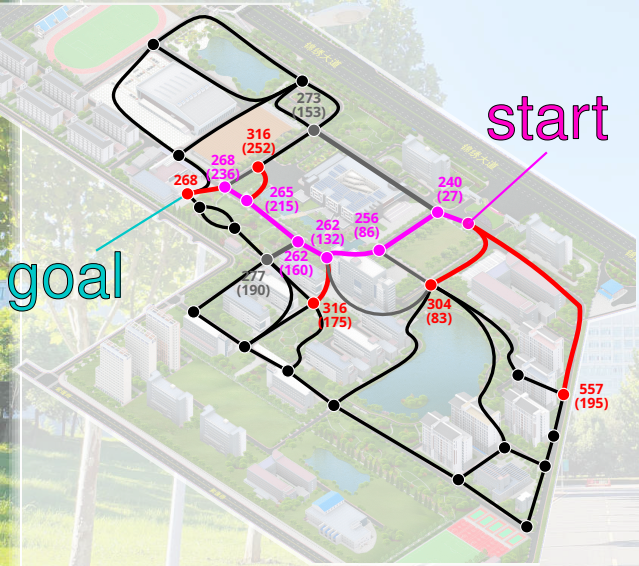
  (k) $316s = 252s + 64s$.
- (d) definitely will need more than $268s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 4 unexplored choices:
  - (b) $304s = 83s + 221s$,
  - (c) $557s = 195s + 362s$,
  - (d) $273s = 153s + 120s$,
  - (h) $316s = 175s + 141s$,
  - (j) $277s = 190s + 87s$,
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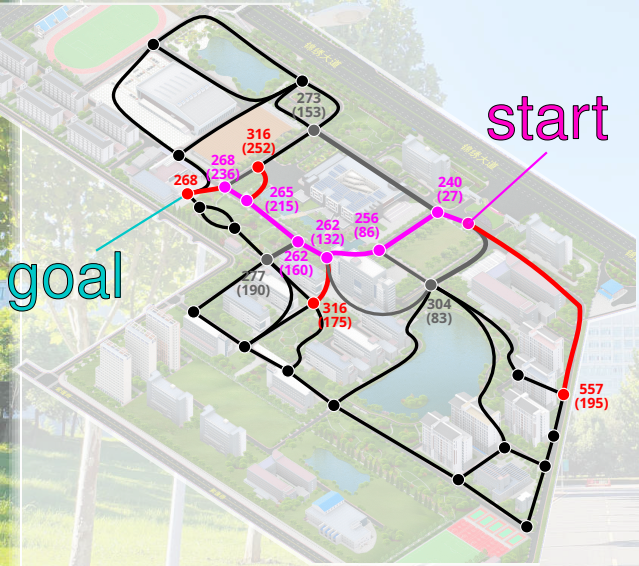
- (j) definitely will need more than $268s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 3 unexplored choices:
  - (b) $304s = 83s + 221s$,
  - (c) $557s = 195s + 362s$,
  - (d) $273s = 153s + 120s$,
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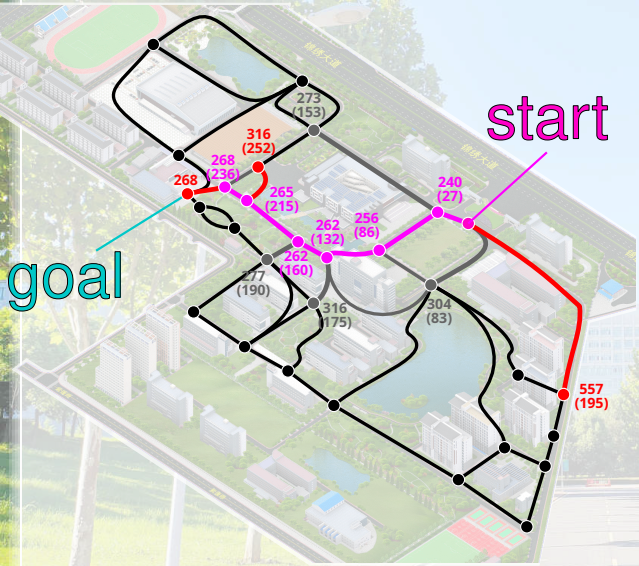
- (b) definitely will need more than $268s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 2 unexplored choices:
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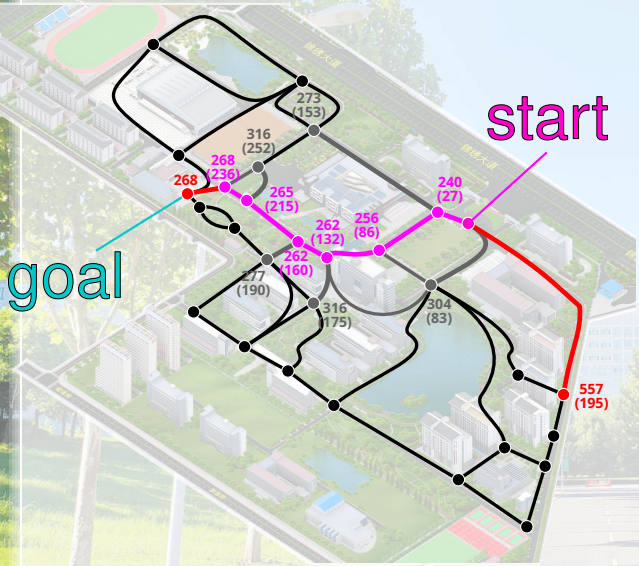
Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 1 unexplored choices:
  - (b) $304s = 83s + 221s$,
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  - (d) $273s = 153s + 120s$,
  - (h) $316s = 175s + 141s$,
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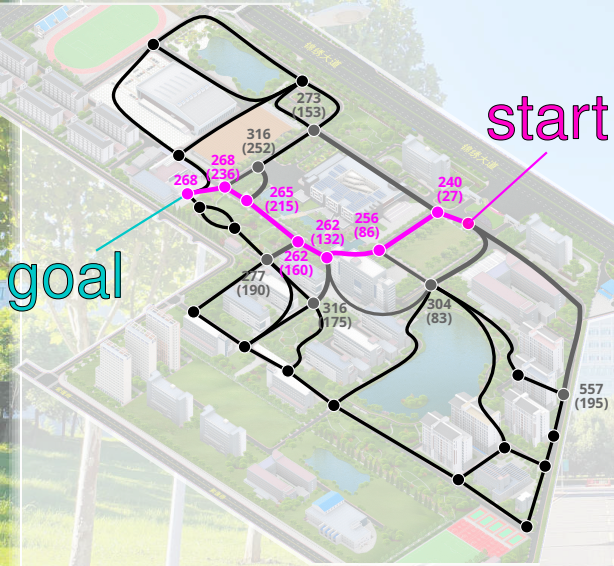
Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 0 unexplored choices:
  - (b) $304s = 83s + 221s$,
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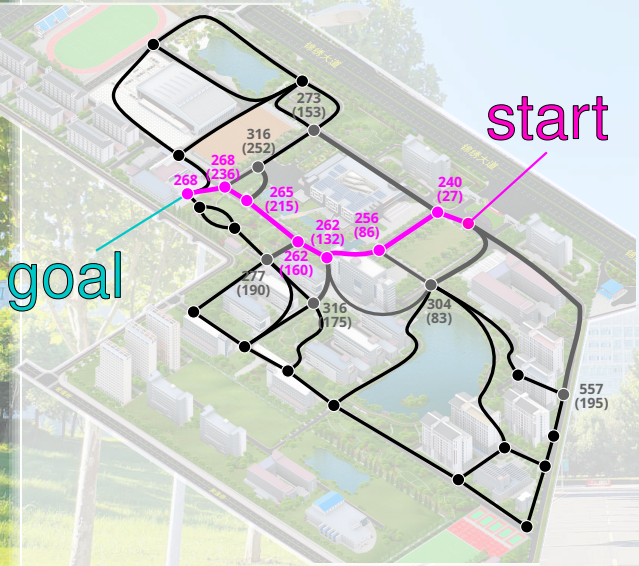
Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 0 unexplored choices:
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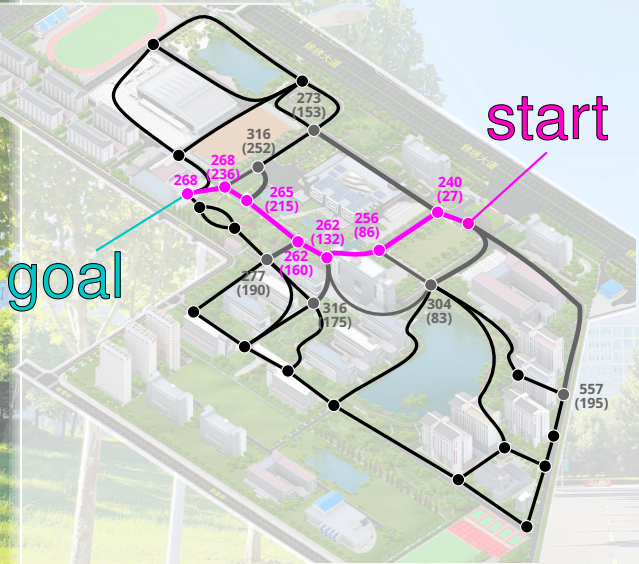
- We found the shortest possible path and it takes $268s$.

Find the Shortest Path from Start to Goal



- I am at the starting point 合肥大学南二区北大门.
- I want to go to the goal: 食堂.
- We now have 0 unexplored choices:
  - (b) $304s = 83s + 221s$,
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- **Solved.**

The A* Algorithmfor Finding Shortest Paths

- We just applied the A* Algorithm[23,39].

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- In the worst case, it needs to "look" at all intersections (nodes) and streets (edges) once.
- Usually, it is quite efficient to find shortest paths on maps.
- (If we look for shortest paths that do not visit any node twice, where the edge distances are not negative, where the number of choices per node is limited, and where the graph and path are not too big – then this algorithm is very efficient.)

Problems that Algorithms cannot Solve Efficiently *and* Exactly

Hard Problems (1)

- There exists a group of problems that are *hard*.

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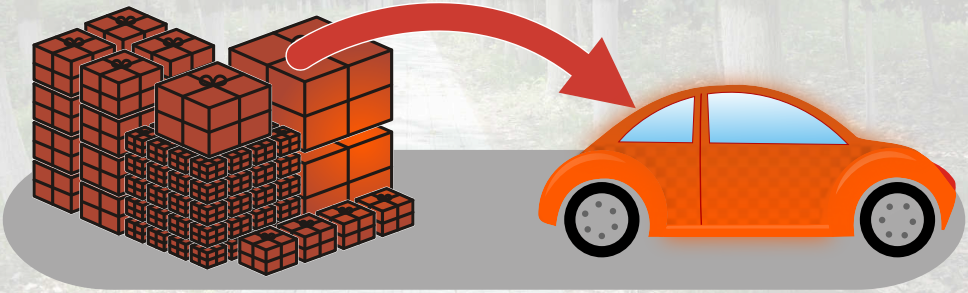
- Let‘s look at two quick examples.

Bin Packing

- My car can carry Tkg of weight.



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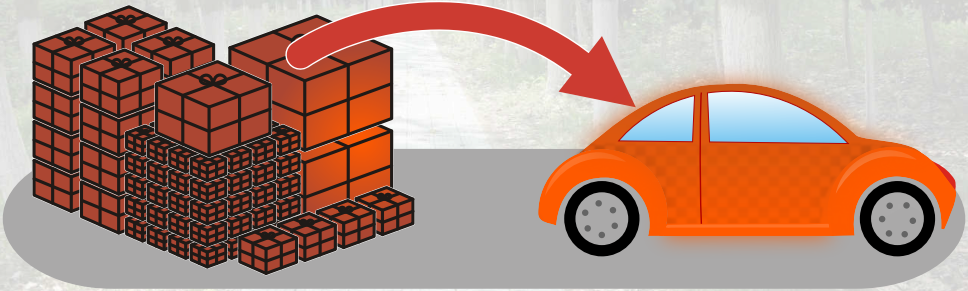
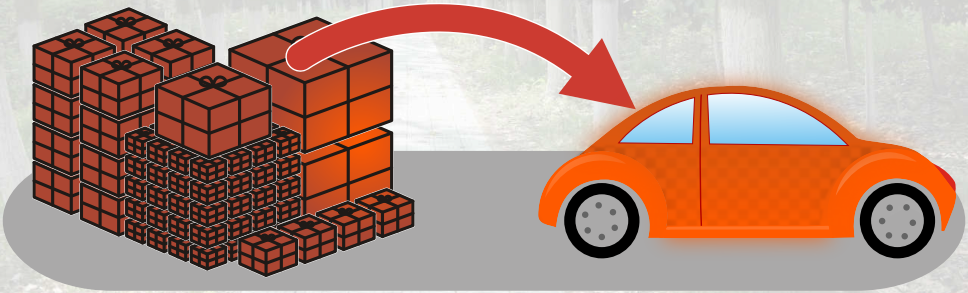


Bin Packing

- My car can carry Tkg of weight.
- I have n objects, each with weight wi for i in 1 to n.
- How can I pack my car so that I can carry them from A to B with the fewest possible hauls?[56,57]



Traveling Salesperson Problem

- Find the shortest path that visits n locations and returns back to its origin.[1,29]

Hard Problems (2)

- These are just two examples from a huge family of problems called $\mathcal{NP}$-hard.

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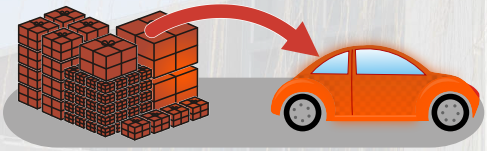

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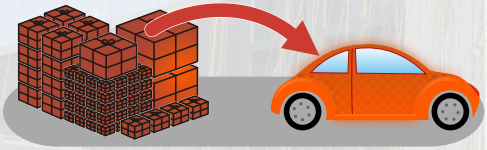

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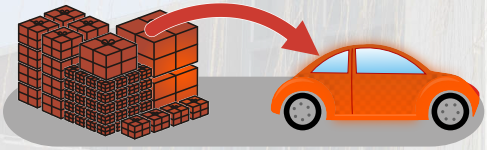

packing with fewest hauls

shortest round-trip tour

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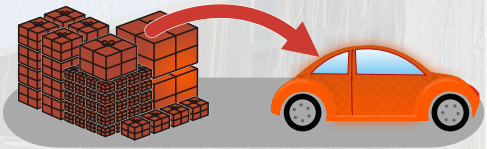

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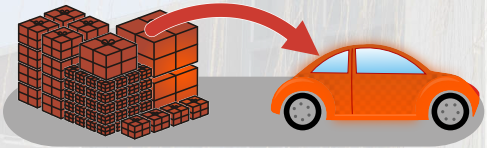

number of items to pack

number of cities to visit

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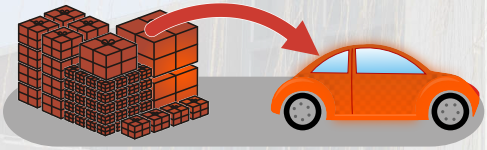

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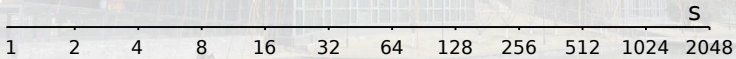

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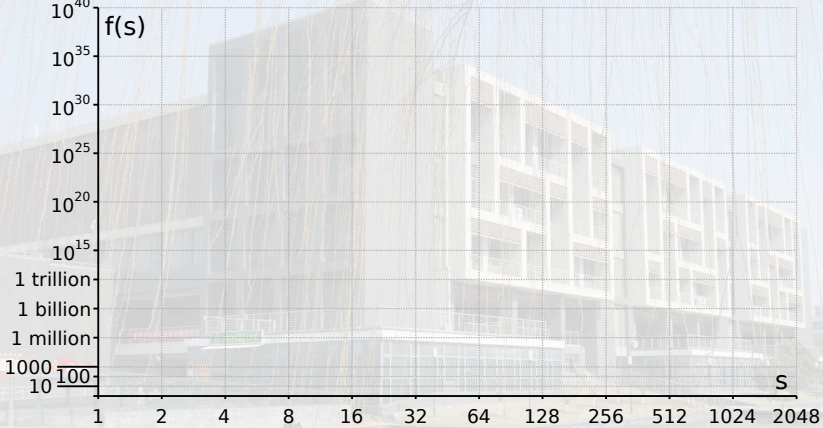

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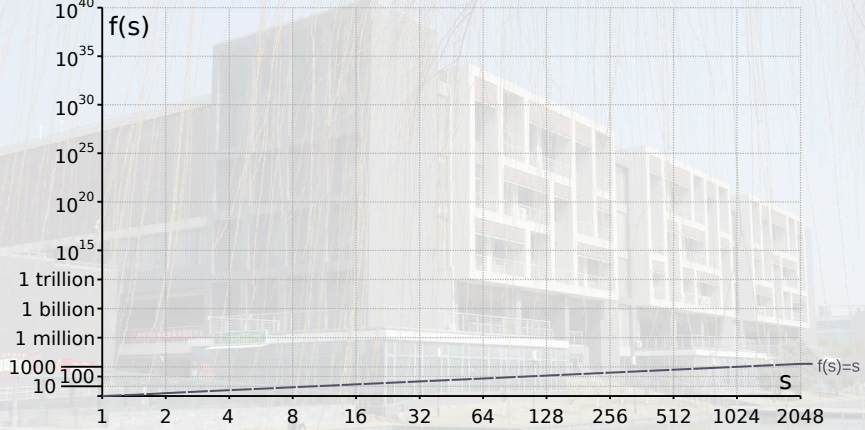

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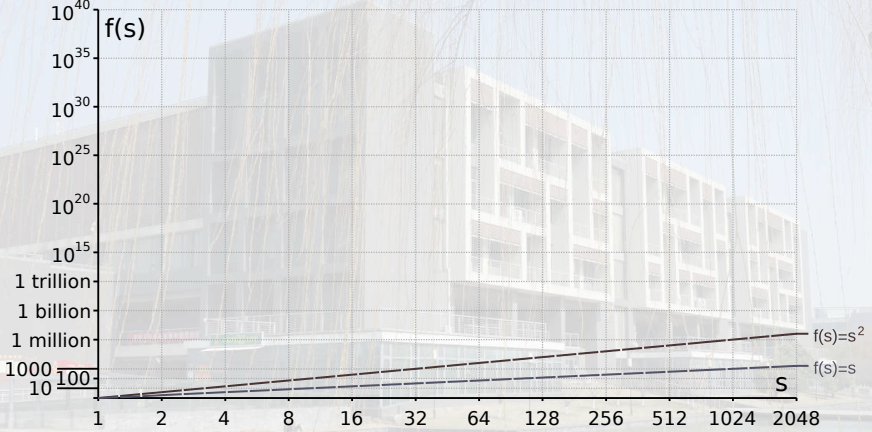

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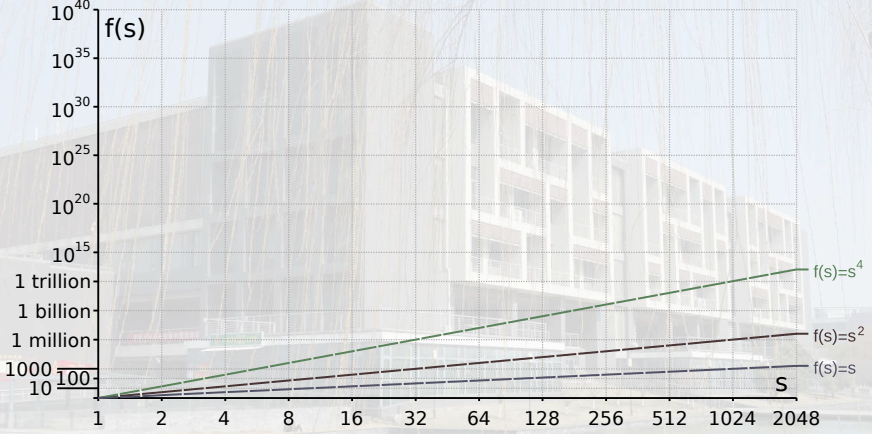

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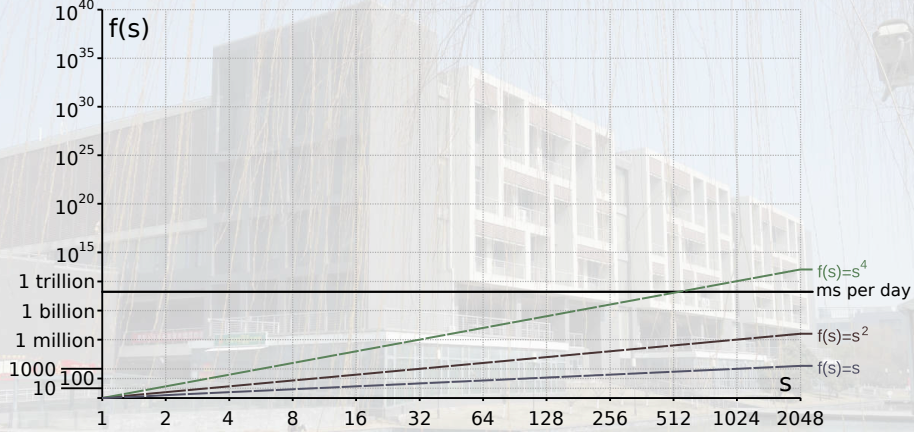

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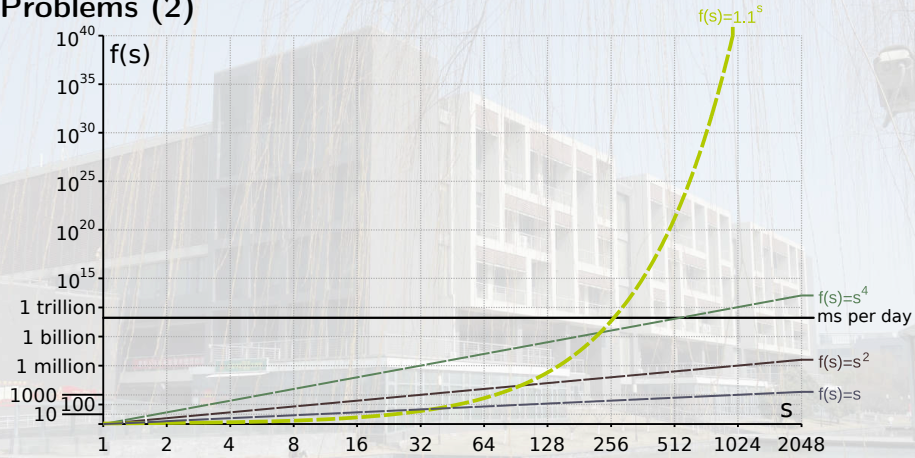

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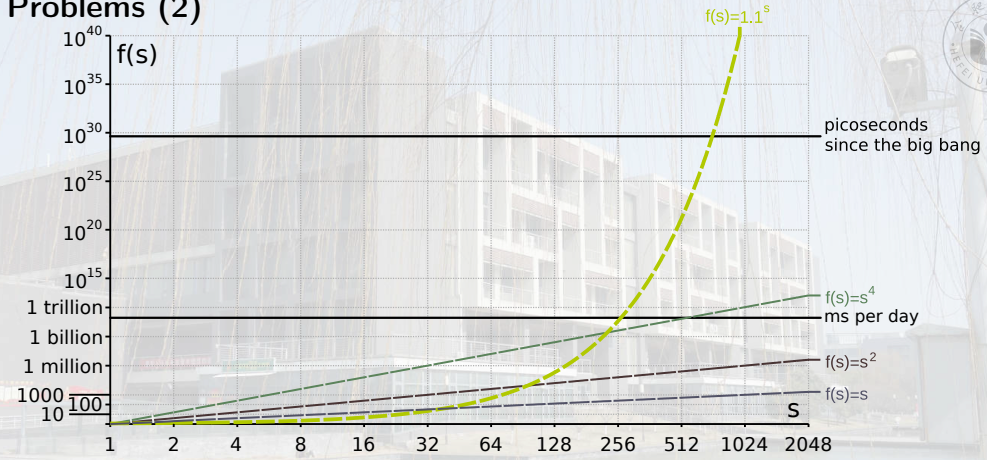

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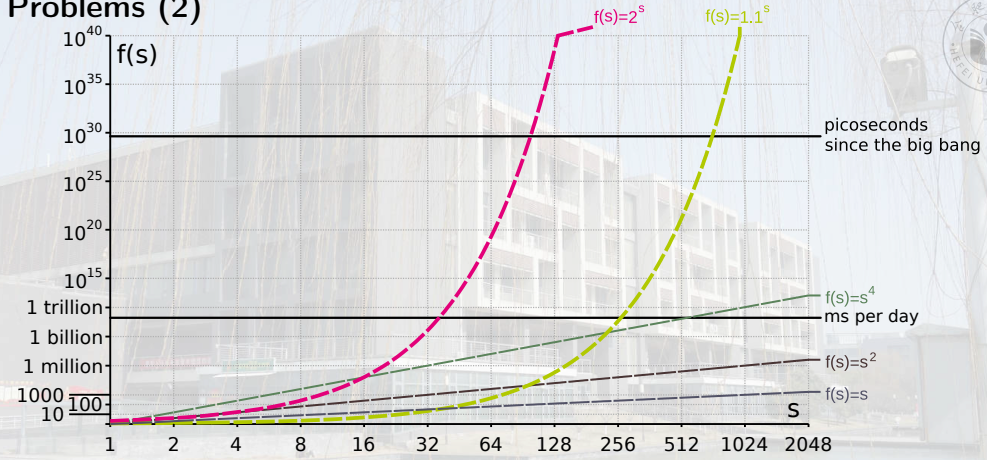

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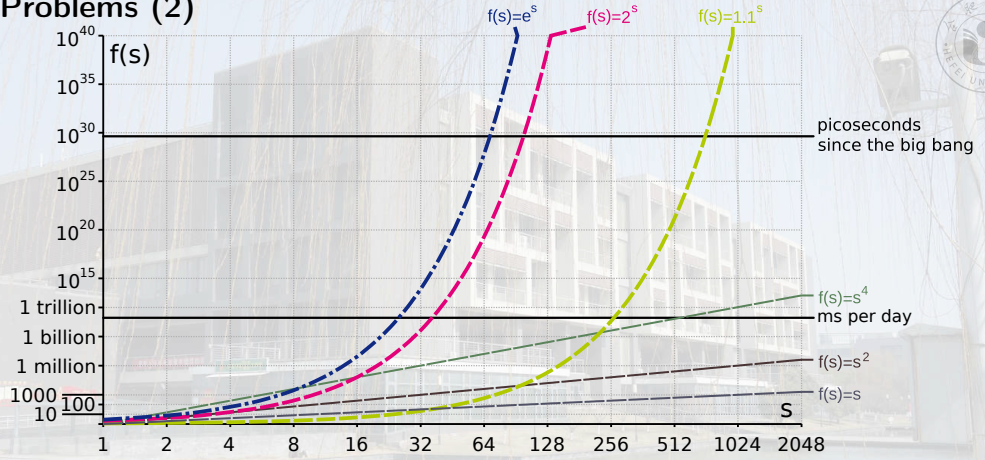

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If we want to get the optimal solution,
it will sometimes/often/always take too long.

If an algorithm guarantees to always find the best-possible solution for an $\mathcal{NP}$-hard problem, then its runtime will grow exponential with the input size s in the **worst case**.

Traveling Salesperson Problem

- In an Traveling Salesperson Problem (TSP), we want to find the shortest path through n locations and back to the start[1,22,29,31,47,53].

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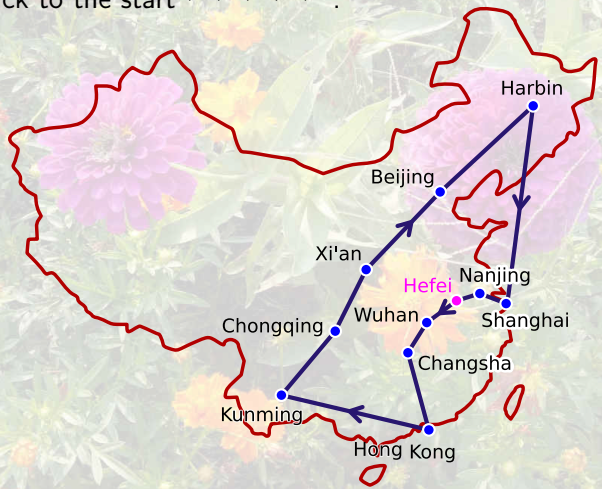

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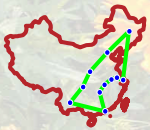

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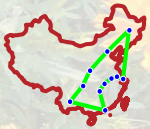

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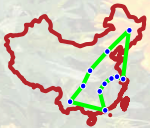

very little / fast — consumed runtime — very much / too (?) long

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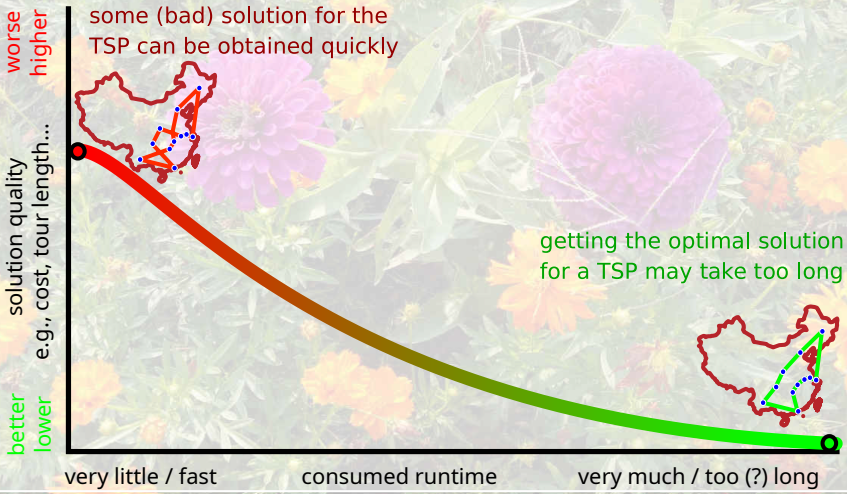

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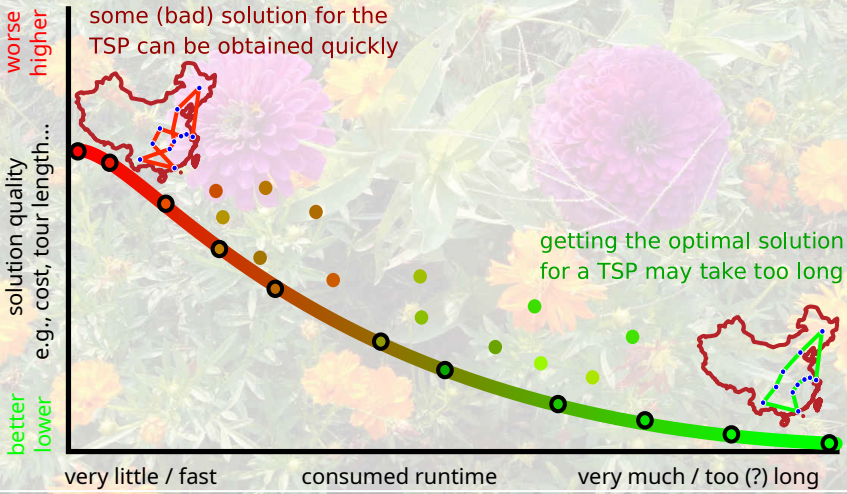

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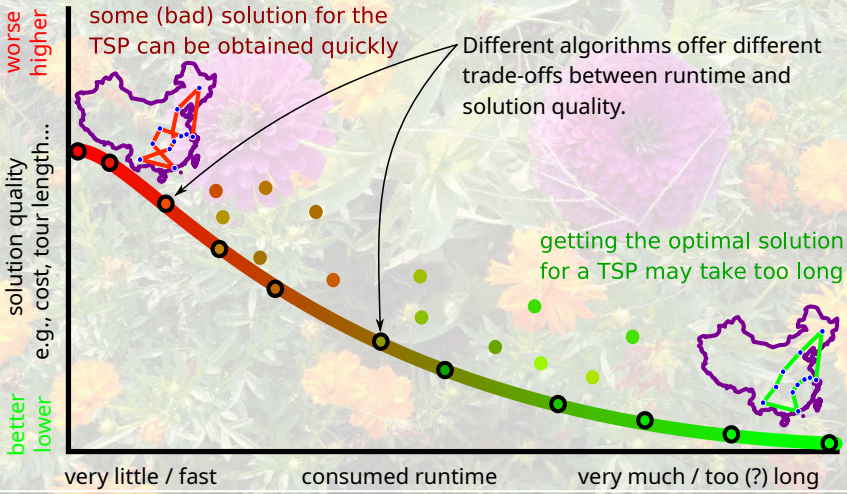

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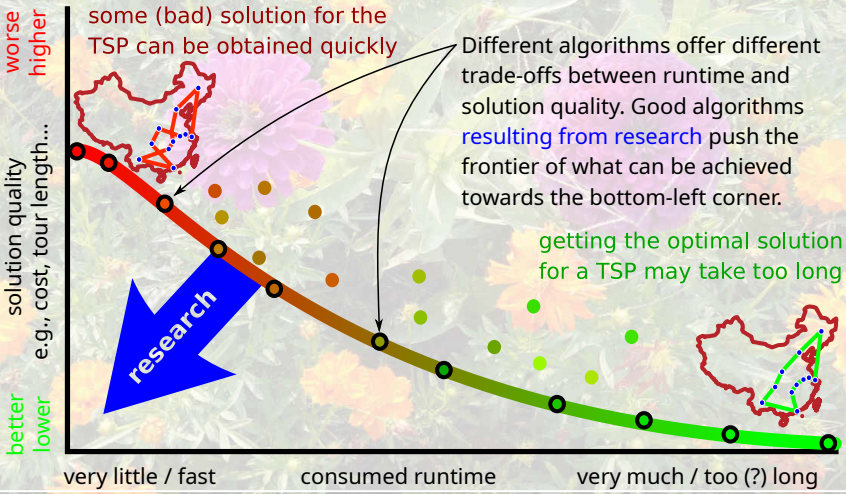

Randomly Guessing Solutions

Almost Solving Hard Problems

- For many problems, it is very easy to "guess" a solution.

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Almost Solving Hard Problems

- For many problems, it is very easy to "guess" a solution.
- This solution will (very likely) not be optimal.
- It will very likely be very bad.
- But it will be better than nothing.

Random Sampling for the TSP

- Back to the Traveling Salesperson Problem (TSP).

Random Sampling for the TSP

- Back to the Traveling Salesperson Problem (TSP).
- How do we "guess" a solution?

Random Sampling for the TSP

- Back to the Traveling Salesperson Problem (TSP).
- How do we "guess" a solution?
- We could write down the cities in a random order.

Random Sampling for the TSP

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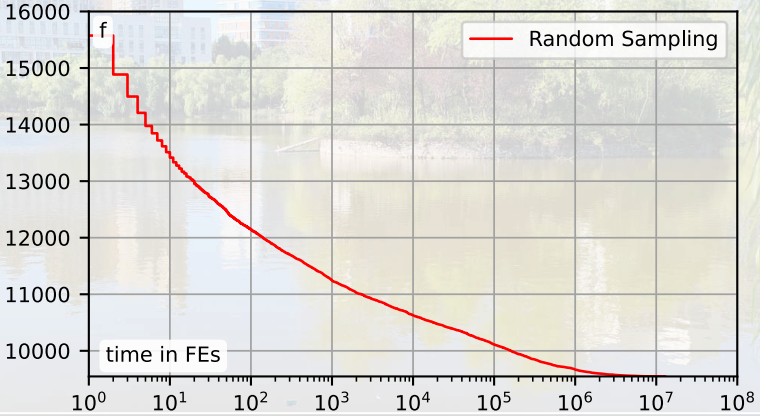
- And we can do this often.
- For this small problem, my computer can do this *millions* of times in a few seconds.
- We can let the algorithm run for as much time as we can wait ... and then simply take the best random tour we found.

Random Sampling for the TSP

- We could write down the cities in a random order.
- This is the improvement of the best-so-far solution of this so-called Random Sampling algorithm over the number of solutions it has generated and tested (the so-called objective function evaluations (FEs)).



Random Sampling for the TSP

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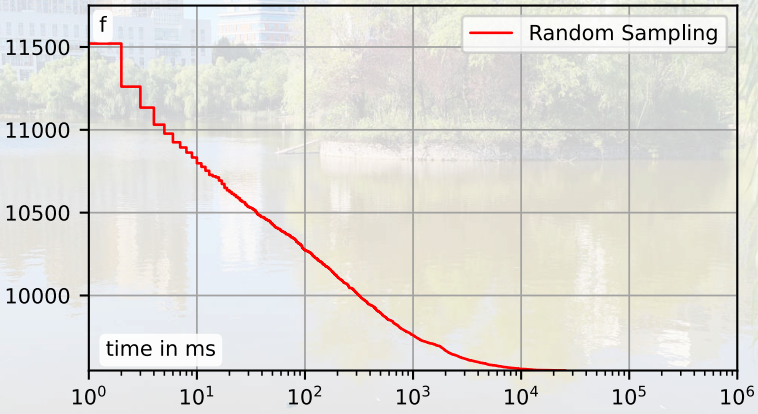

Random Sampling for the TSP

- We could write down the cities in a random order.
- And for this small problem, this random sampling eventually finds the optimal tour.

Principle 1: We can randomly construct solutions.

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Problem 1: Guessing an optimal (or even just good) solution randomly is very unlikely.

Why don't we use the A* Algorithm for the TSP?

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1. The A* Algorithm needs memory exponential in the number of nodes in the final solution[25].

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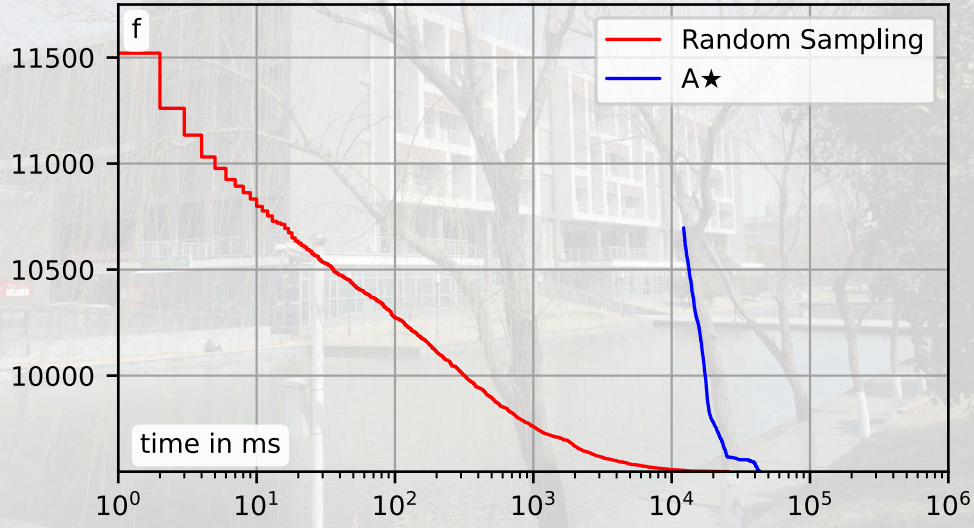

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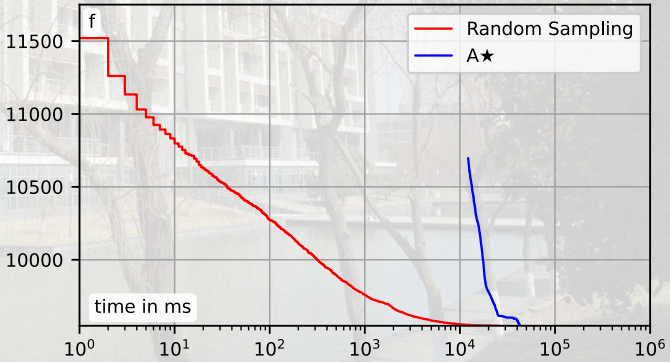
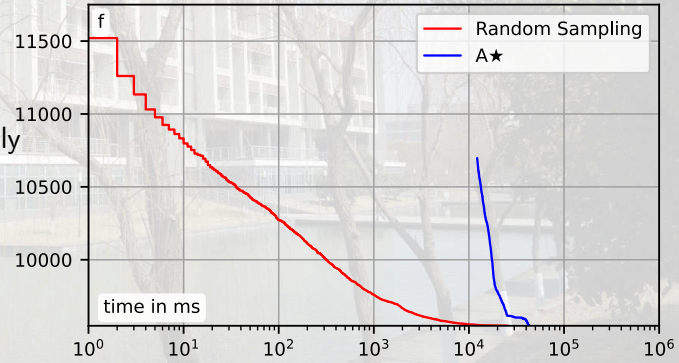


Why don't we use the A* Algorithm for the TSP?

- Because not only will it use too much memory for larger instances...
- ...it is also slower than randomly guessing on smaller instances.



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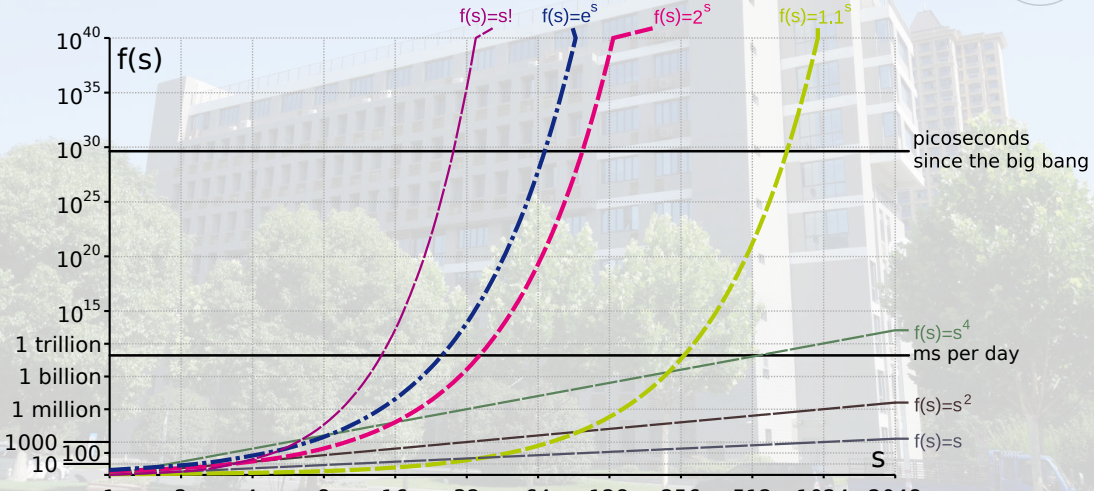

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- If we have n cities, then there are $1 * 2 * 3 * \cdots * (n-1) * n = n!$ tours.
- Factorial growth ($n!$) is even worse than exponential growth...
- So if we try to randomly sample the best possible tour, our chance is extremely small...

Local Search: Using Information

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- In each step, random sampling creates a completely random tour.
- It does not gather any information.
- It keeps the best tour, but does not use any information inside this tour.
- Just guessing answers randomly is not a good method.
- Clearly, seeing millions of tours, we should be able learn *something* and somehow use that to find better tours???

Randomized Local Search

- But what could we do?

Randomized Local Search

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- Then we do the same with a better tour that we find, and with yet a better tour, and so on.
- This method is called randomized local search (RLS).

Randomized Local Search for the TSP

15983 km

合肥, 北京, 西安, 南京, 哈尔滨,
香港, 长沙, 昆明, 武汉, 上海, 重庆

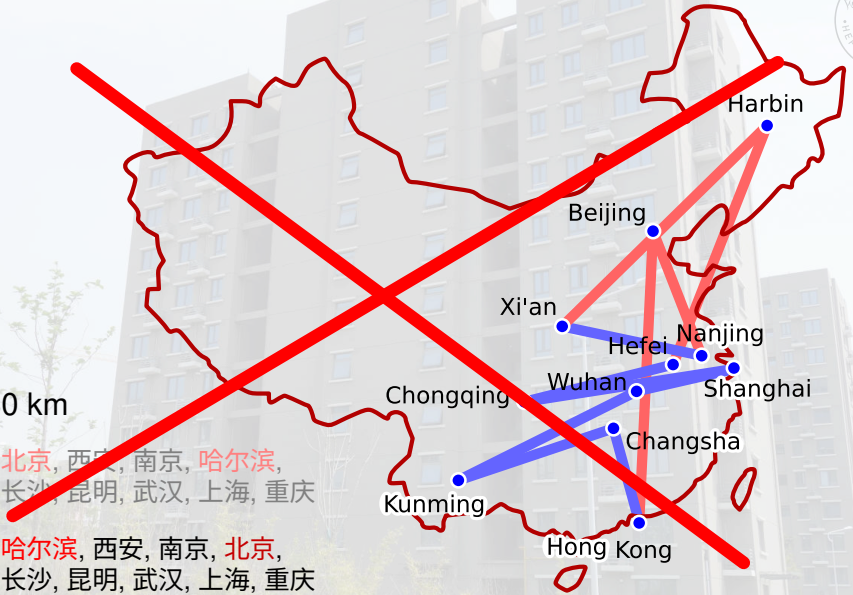
Randomized Local Search for the TSP

16080 km

合肥, 北京, 西安, 南京, 哈尔滨,
香港, 长沙, 昆明, 武汉, 上海, 重庆

合肥, 哈尔滨, 西安, 南京, 北京,
香港, 长沙, 昆明, 武汉, 上海, 重庆

Randomized Local Search for the TSP



16080 km

合肥, 北京, 西安, 南京, 哈尔滨,
香港, 长沙, 昆明, 武汉, 上海, 重庆

合肥, 哈尔滨, 西安, 南京, 北京,
香港, 长沙, 昆明, 武汉, 上海, 重庆

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合肥, 北京, 西安, 南京, 哈尔滨,
香港, 长沙, 昆明, 武汉, 上海, 重庆

Randomized Local Search for the TSP

15128 km

合肥, 北京, 西安, 南京, 哈尔滨,
香港, 长沙, 昆明, 武汉, 上海, 重庆

合肥, 北京, 西安, 南京, 上海,
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合肥, 北京, 西安, 南京, 上海,
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Randomized Local Search for the TSP

16435 km

合肥, 北京, 西安, 南京, 上海,
香港, 长沙, 昆明, 武汉, 哈尔滨, 重庆

合肥, 北京, 西安, 南京, 香港,
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Randomized Local Search for the TSP

12432 km

合肥, 北京, 西安, 南京, 上海,
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合肥, 北京, 哈尔滨, 南京, 上海,
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Randomized Local Search for the TSP

14036 km

合肥, 北京, 哈尔滨, 南京, 上海,
香港, 长沙, 昆明, 武汉, 西安, 重庆

合肥, 北京, 哈尔滨, 武汉, 上海,
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Randomized Local Search for the TSP

10857 km

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Randomized Local Search for the TSP

10620 km

合肥, 北京, 哈尔滨, 南京, 上海,
香港, 长沙, 昆明, 重庆, 西安, 武汉

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Randomized Local Search for the TSP

10567 km

合肥, 北京, 哈尔滨, 南京, 上海,
长沙, 香港, 昆明, 重庆, 西安, 武汉

合肥, 北京, 哈尔滨, 上海, 南京,
长沙, 香港, 昆明, 重庆, 西安, 武汉

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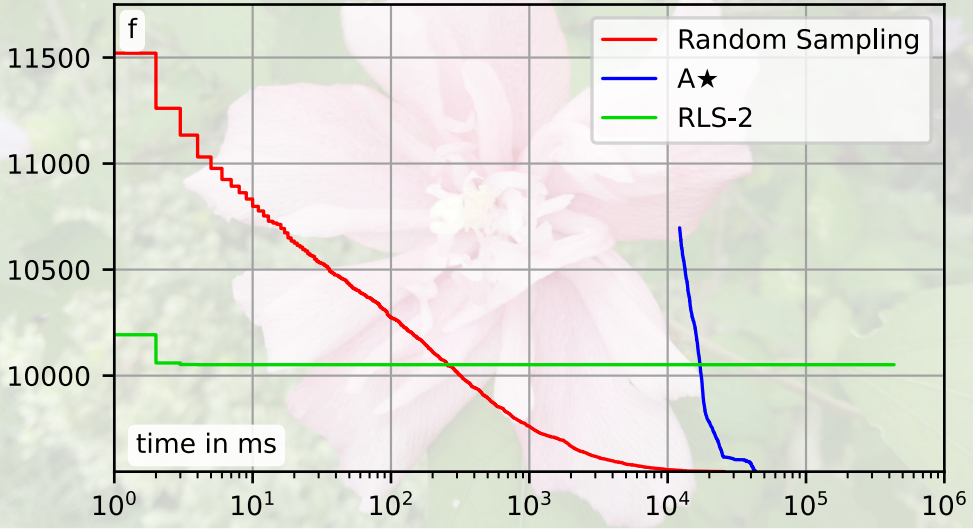

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- However, if our algorithm is at a tour x, then it can only find other tours by swapping two cities.
- There are some tours that it cannot reach in one step.
- If all tours that we can reach in one step are worse than x, we will never leave x.
- If x is not the optimum, then we are stuck and won't ever find the optimal/best tour.

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Problem 2: Sometimes, a small change is not enough. We can get stuck at a so-called *local* optimum.

Randomized Local Search for the TSP

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- There are many things that we can do, for example:
  1. We could restart the algorithm at a new starting point after some time.
  2. Or we could sometimes allow it swap more cities in the current tour.

Randomized Local Search with a Larger Neighborhood

15151 km

合肥, 重庆, 武汉, 长沙, 北京, 上海,
南京, 昆明, 香港, 西安, 哈尔滨

Randomized Local Search with a Larger Neighborhood

15195 km

合肥, 重庆, 武汉, 长沙, 北京, 上海, 南京, 昆明, 香港, 西安, 哈尔滨

合肥, 重庆, 武汉, 长沙, 北京, 南京, 上海, 昆明, 香港, 西安, 哈尔滨

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Randomized Local Search with a Larger Neighborhood

14539 km

合肥, 重庆, 武汉, 长沙, 北京, 上海,
南京, 昆明, 香港, 西安, 哈尔滨

合肥, 南京, 武汉, 长沙, 北京, 上海,
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Randomized Local Search with a Larger Neighborhood

16624 km

合肥, 南京, 武汉, 长沙, 北京, 上海,
昆明, 重庆, 香港, 西安, 哈尔滨

昆明, 南京, 武汉, 长沙, 北京, 上海,
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昆明, 重庆, 香港, 西安, 哈尔滨

Randomized Local Search with a Larger Neighborhood

13882 km

合肥, 南京, 武汉, 长沙, 北京, 上海, 昆明, 重庆, 香港, 西安, 哈尔滨

合肥, 南京, 武汉, 长沙, 北京, 上海, 重庆, 昆明, 香港, 西安, 哈尔滨

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重庆, 昆明, 香港, 西安, 哈尔滨

Randomized Local Search with a Larger Neighborhood

10688 km

合肥, 南京, 武汉, 长沙, 北京, 上海,
重庆, 昆明, 香港, 西安, 哈尔滨

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重庆, 昆明, 香港, 长沙, 上海

Randomized Local Search with a Larger Neighborhood

10688 km

合肥, 南京, 武汉, 哈尔滨, 北京, 西安,
重庆, 昆明, 香港, 长沙, 上海

Randomized Local Search with a Larger Neighborhood

12111 km

合肥, 南京, 武汉, 哈尔滨, 北京, 西安, 重庆, 昆明, 香港, 长沙, 上海

合肥, 南京, 哈尔滨, 武汉, 北京, 西安, 重庆, 昆明, 香港, 长沙, 上海

Randomized Local Search with a Larger Neighborhood

Randomized Local Search with a Larger Neighborhood

9547 km

合肥, 南京, 武汉, 哈尔滨, 北京, 西安,
重庆, 昆明, 香港, 长沙, 上海

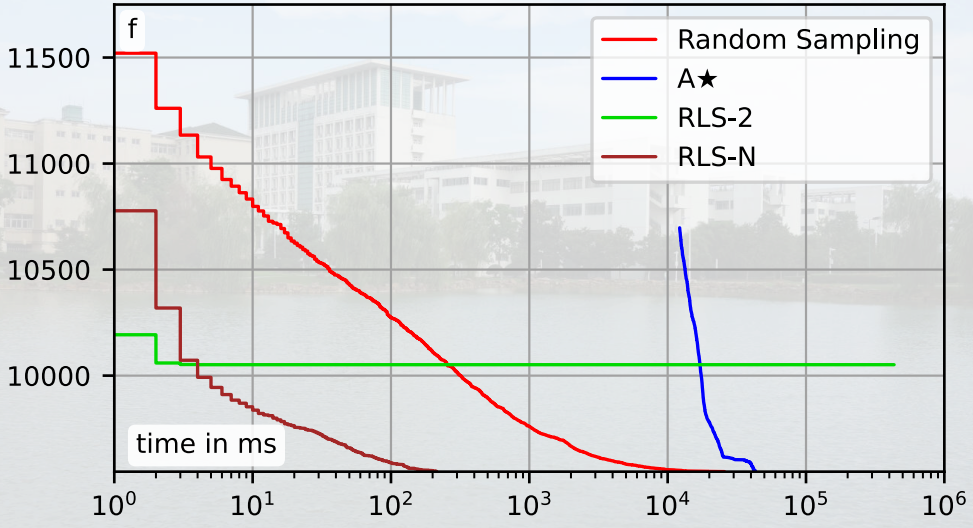
合肥, 南京, 上海, 哈尔滨, 北京, 西安,
重庆, 昆明, 香港, 长沙, 武汉

Randomized Local Search with a Larger Neighborhood

9547 km

合肥, 南京, 上海, 哈尔滨, 北京, 西安,
重庆, 昆明, 香港, 长沙, 武汉

Randomized Local Search with a Larger Neighborhood



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- If all tours that we can reach in one step are worse than the current tour x, then our RLS will never leave x.
- There are many things that we can do, for example:
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  1. We could restart the algorithm at a new starting point after some time.
  2. Or we could sometimes allow it swap more cities in the current tour.
- The larger neighborhood of solutions that can be reached in one step makes the search slower, but allows it to escape from local optima.

Principle 3: Larger changes can help us escape from local optima.

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Problem 3: Larger changes are less likely to yield improvements (because the new solution is more different), so they slow down the search.

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But we will leave it at what we have seen so far.

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- This requires carefully balancing the step-size of algorithms and to develop methods against getting stuck at local optima.

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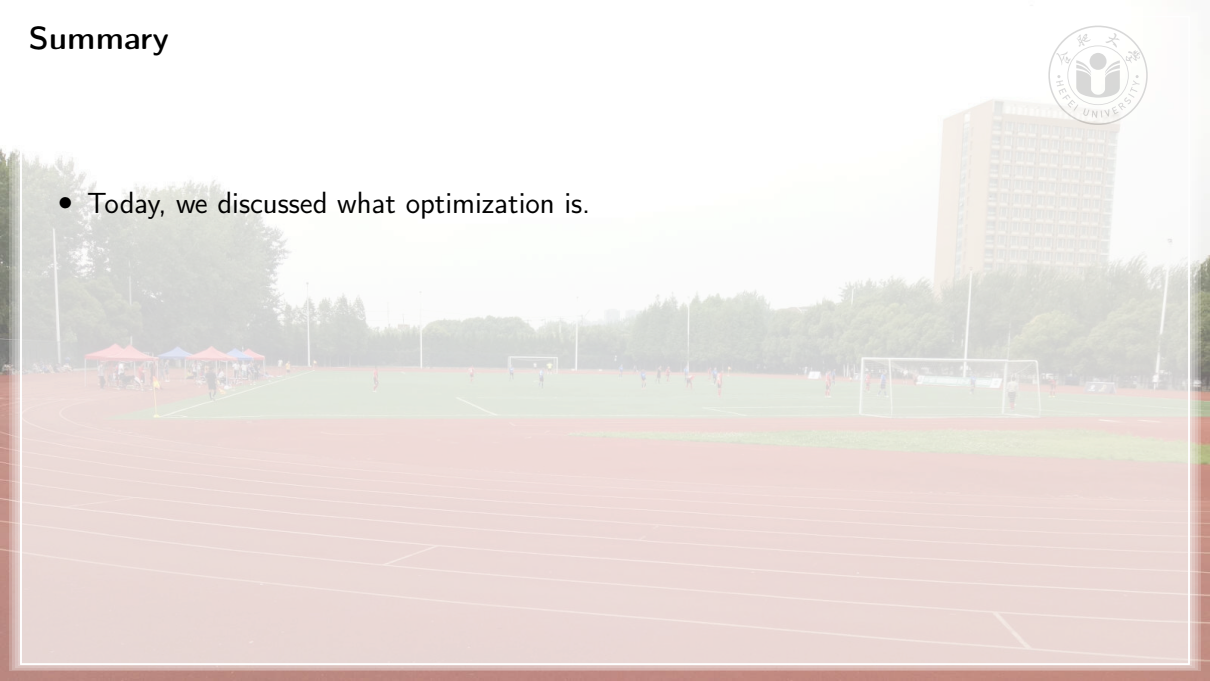

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- For example: As long as we can randomly construct and randomly modify a solution, we can attack the problem with randomized local search (RLS).
- Understanding the basic principles of optimization is not very hard.

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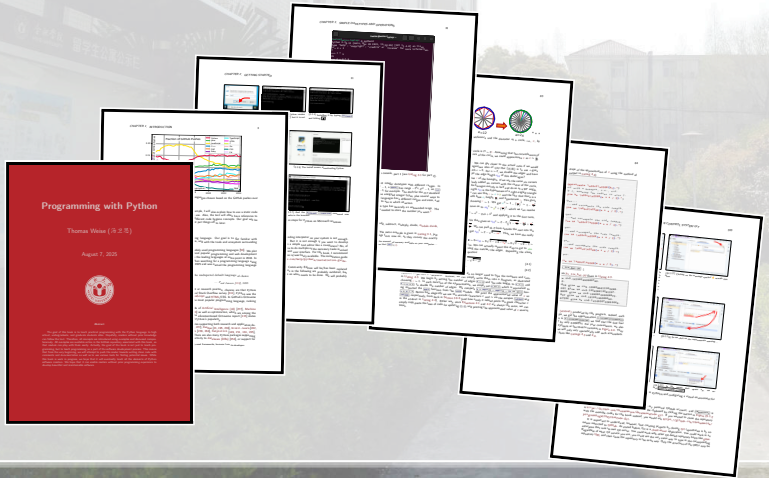
Programming with Python

We have a freely available course book on *Programming with Python* at `https://thomasweise.github.io/programmingWithPython`, with focus on practical software development using the Python ecosystem of tools[46].

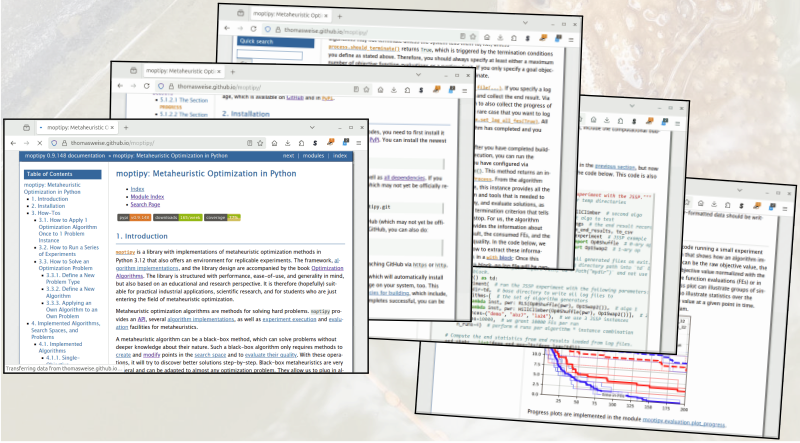


Databases

We have a freely available course book on *Databases* at `https://thomasweise.github.io/databases`, with actual practical examples using a real database management system (DBMS)[44].

Metaheuristic Optimization in Python: moptipy

We offer moptipy[52] a mature open source Python package for metaheuristic optimization, which implements several algorithms, can run self-documenting experiments in parallel and in a distributed fashion, and offers statistical evaluation tools.



谢谢您们！

Thank you!

Vielen Dank!

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Glossary I

$(1+1)$ EA The $(1+1)$ EA is a local search algorithm that retains the best solution x_c discovered so far during the search[6,17]. In each step, it applies a unary search operator to this best-so-far solution x_c and derives a new solution x_n. If the new solution x_n is *better or equally good* when compared with x_c, i.e., not worse, then it replaces it, i.e., is stored as the new x_c. If the search space are bit strings of length n, then the $(1+1)$ EA uses a unary search operator that flips each bit independently with probability m/n, where usually $m = 1$. This operator is the main difference to randomized local search (RLS). The $(1+1)$ EA is a special case of the $(\mu+\lambda)$ evolutionary algorithm ($(\mu+\lambda)$ EA) where $\mu = \lambda = 1$.

EA An *evolutionary algorithm* is a metaheuristic optimization method that maintains a population of candidate solutions, which undergo selection (where better solutions are chosen with higher probability) and reproduction (where mutation and recombination create a new candidate solution from one or two existing ones, repectively)[2,45].

$(\mu+\lambda)$ EA The $(\mu+\lambda)$ EA is an evolutionary algorithm (EA) where, in each generation, λ offspring solutions are generated from the current population of μ parent solutions. The offspring and parent populations are merged, yielding $\mu+\lambda$ solutions, from which then the best μ solutions are ratained to form the parent population of the next generation. If the search space is the bit strings of length n, then this algorithm usually applies a mutation operator flipping each bit independently with probability $1/n$.

A* Algorithm The A* Algorithm is an a greedy best-first-first search for finding the shortest path between two locations[23,39]. The algorithm iteratively constructs the shortest path. It maintains a list of current candidate paths. For choosing the next candidate path whose end node should be expanded, it computes a value $f(p)$ of these paths p. $f(p)$ is the sum of the cost $g(p)$ so far incurred by the path as well as a heuristic $h(p)$ predicting the cost from the current end of the path to the goal. $h(p)$ must never overestimate that cost.

BCE The time notation *before Common Era* is a non-religious but chronological equivalent alternative to the traditional *Before Christ (BC)* notation, which refers to the years *before* the birth of Jesus Christ[7]. The years BCE are counted down, i.e., the larger the year, the farther in the past. The year 1 BCE comes directly before the year 1 CE[41,58].

Glossary II

CE The time notation *Common Era* is a non-religious but chronological equivalent alternative to the traditional *Anno Domini (AD)* notation, which refers to the years *after* the birth of Jesus Christ[7]. The years CE are counted upwards, i.e., the smaller they are, the farther they are in the past. The year 1 CE comes directly after the year 1 BCE[41,58].

DB A *database* is an organized collection of structured information or data, typically stored electronically in a computer system. Databases are discussed in our book *Databases*[44].

DBMS A *database management system* is the software layer located between the user or application and the database (DB). The DBMS allows the user/application to create, read, write, update, delete, and otherwise manipulate the data in the DB[55].

FE *Objective function evaluations* are an implementation-independent measure of runtime for optimization algorithms[3,47,48,52]. 1 FE equals to one evaluated candidate solution during the optimization process.

moptipy is the *Metaheuristic Optimization in Python* library[52]. It has been used in several different research works, including[10,31–33,43,54,56,57]. Learn more at https://thomasweise.github.io/moptipy and https://thomasweise.github.io/moptipyapps.

OR Operations Research (or Operational Research) is the application of sciences such as mathematics and computer science to the management and organization of systems, organizations, enterprises, factories, or projects. It encompasses the development and application of problem-solving methods and techniques (such as mathematical optimization, simulation, queueing theory and other stochastic models) with the goal to improve decision-making and efficiency[16].

Python The Python programming language[26,30,36,46], i.e., what you will learn about in our book[46]. Learn more at https://python.org.

Glossary III

RLS Randomized local search retains the best solution x_c discovered so far during the search and, in each step, it applies a unary search operator to this best-so-far solution x_c and derives a new solution x_n. If the new solution x_n is *better or equally good* when compared with x_c, i.e., not worse, then it replaces it, i.e., is stored as the new x_c. If the search space are bit strings of length n, then RLS uses a unary search operator that flips exactly one bit. This operator is the main difference to $(1+1)$ evolutionary algorithm ($(1+1)$ EA).

TSP In an instance of the *Traveling Salesperson Problem*, also known as *Traveling Salesman Problem*, a set of n cities or locations as well as the distances between them are defined[1,22,29,31,47,53]. The goal is to find the shortest round-trip tour that starts at one city, visits all the other cities one time each, and returns to the origin. The TSP is one of the most well-known $\mathcal{NP}$-hard combinatorial optimization problems[22].

$i!$ The factorial $a!$ of a natural number $a \in \mathbb{N}_1$ is the product of all positive natural numbers less than or equal to a, i.e., $a! = 1 * 2 * 3 * 4 * \cdots * (a-1) * a$[8,18,35].

$\mathbb{N}_1$ the set of the natural numbers *excluding* 0, i.e., 1, 2, 3, 4, and so on. It holds that $\mathbb{N}_1 \subset \mathbb{Z}$.

$\mathcal{NP}$ is the class of computational problems that can be solved in polynomial time by a non-deterministic machine and can be verified in polynomial time by a deterministic machine (such as a normal computer)[21].

$\mathcal{NP}$-hard Algorithms that guarantee to find the correct solutions of $\mathcal{NP}$-hard problems[9,11,28] need a runtime that is exponential in the problem scale in the worst case. A problem is $\mathcal{NP}$-hard if all problems in $\mathcal{NP}$ are reducible to it in polynomial time[21].

$\mathbb{R}$ the set of the real numbers.

$\mathbb{Z}$ the set of the integers numbers including positive and negative numbers and 0, i.e., . . . , -3, -2, -1, 0, 1, 2, 3, . . . , and so on. It holds that $\mathbb{Z} \subset \mathbb{R}$.