合肥大學
HEFEI UNIVERSITY

Frequency Fitness Assignment as a Research Direction

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Outline

1. Introduction
2. Research Field: Optimization
3. Research Direction: Frequency Fitness Assignment
4. Our Results
5. Future Works
6. Summary
7. Advertisement

Introduction

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- My students and I are members of the Optimization Direction.
- Today, I therefore want to first give a short introduction into the field of optimization, before delving into one of our concrete research topics, namely Frequency Fitness Assignment (FFA).

Research Field: Optimization

What is Optimization?

- There are two ways to look at optimization.

What is Optimization?

- The economic view.

Optimization

An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined or required benefit at minimal costs.

What is Optimization?

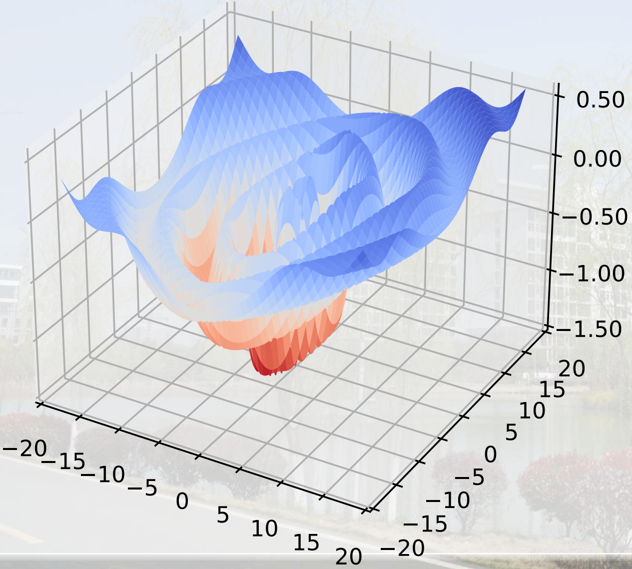
- The mathematical view.

Optimization

An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined or required benefit at minimal costs.

Solving an optimization problem requires finding an input element $x^\star$ within a set $\mathbb{X}$ of allowed elements for which a mathematical function $f: \mathbb{X} \mapsto \mathbb{R}$ takes on the smallest possible value.

Example: Function Optimization



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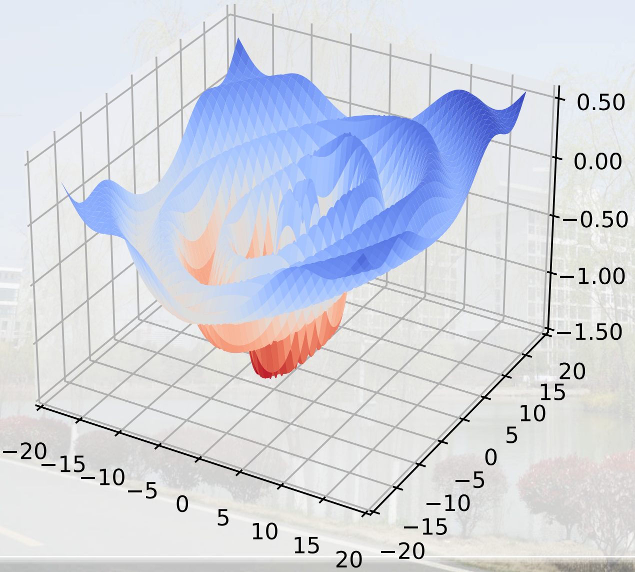

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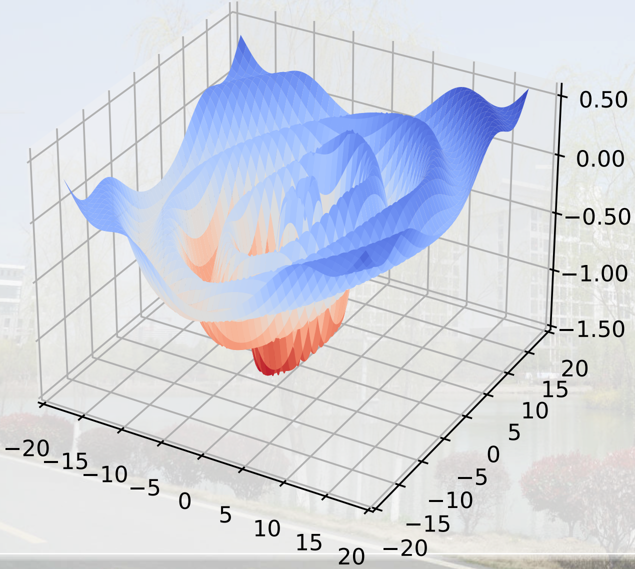

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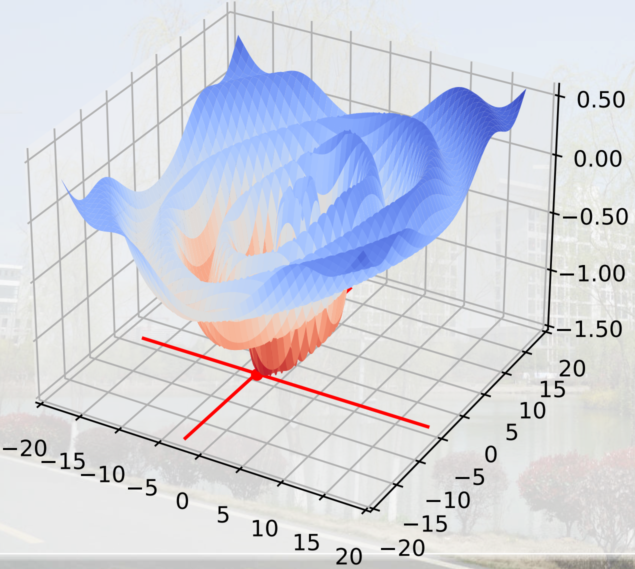

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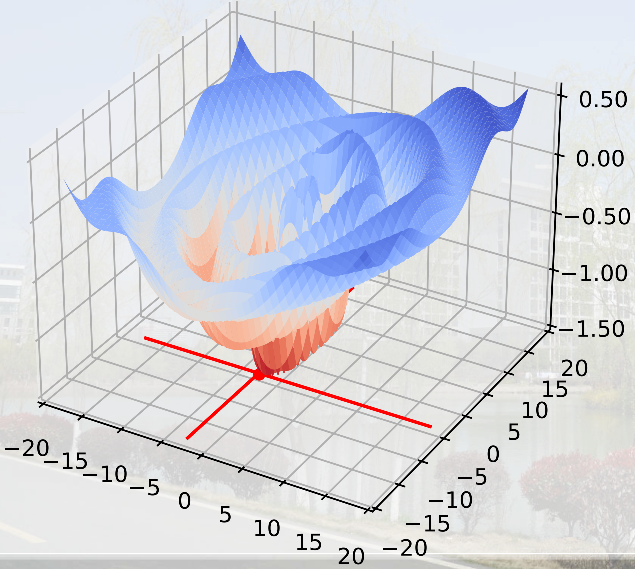

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- This is *not* what I am working on, though.

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- The optimal solution $x^{\star} \in \mathbb{X}$ is the shortest possible tour.

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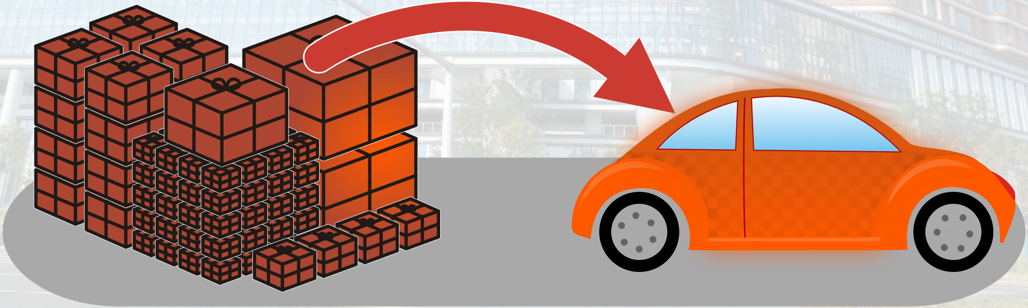
- The goal of the Bin Packing Problem is to pack n objects, each having a specific size, into as few bins (also of a given size) as possible[60,61].



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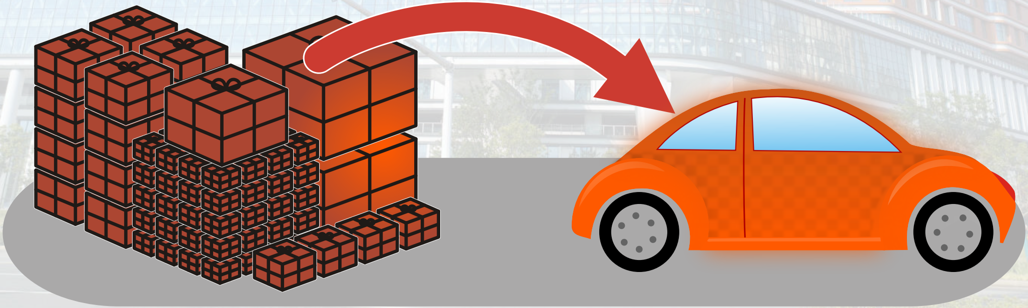

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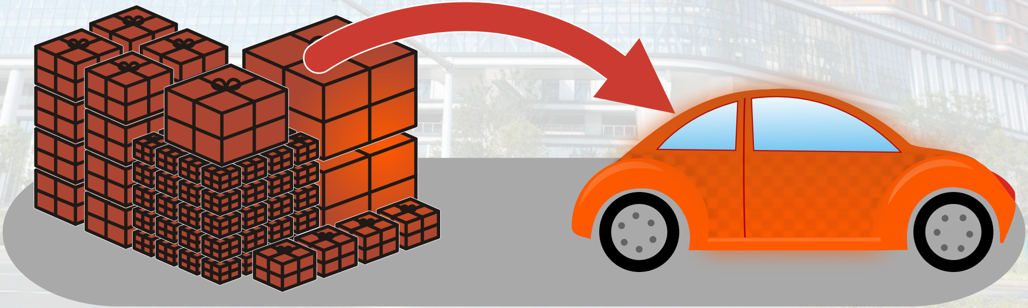

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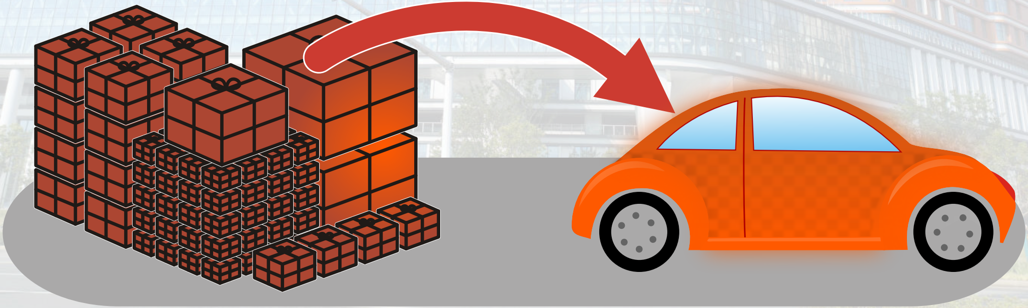

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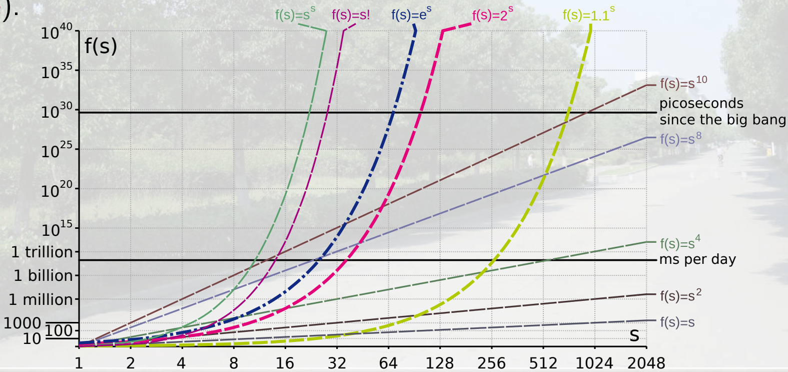

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- In other words, if we want to guarantee to find the best possible solution $x^\star$ for all possible instances of a problem, we often cannot really be much faster than testing all possible candidate solutions $x \in \mathbb{X}$ in the worst case.

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Begin with a set $S_0 \subset \mathbb{X}$ of one or multiple randomly sampled solutions

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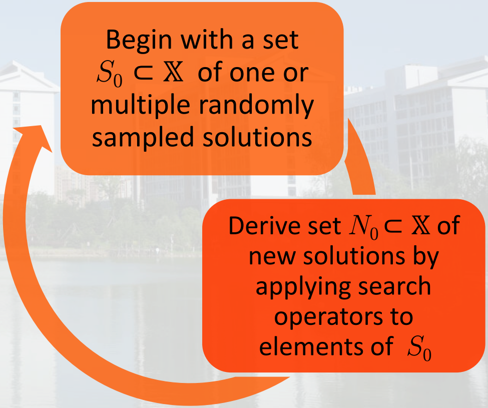

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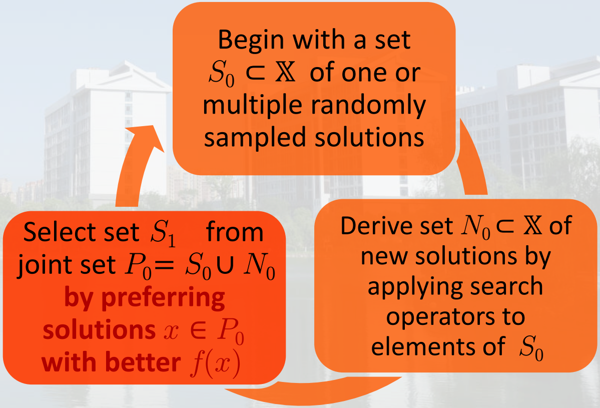

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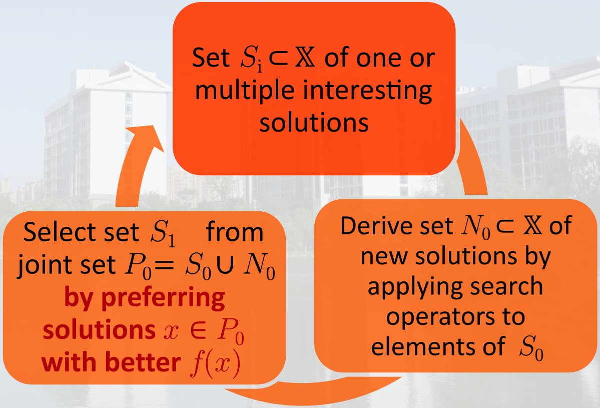

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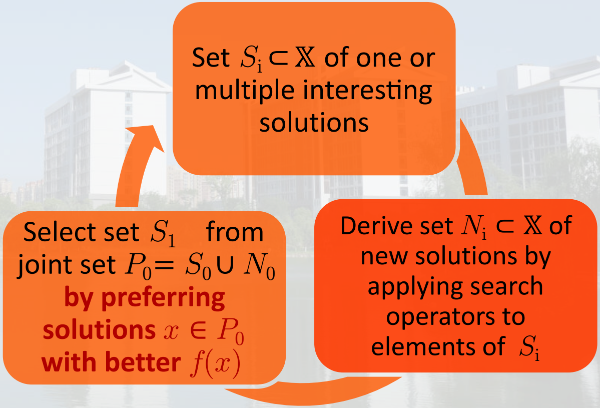

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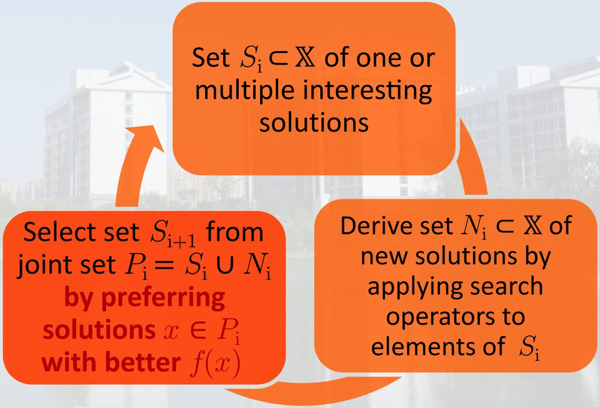

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- We investigate classical hard problems that have discrete search spaces, such as bit strings or permutations.
- Here, the objective functions take on only natural numbers in $\mathbb{N}_0$.
- We try to enable metaheuristic algorithms to find better solutions for these problems.

Research Direction: Frequency Fitness Assignment

Metaheuristic Optimization Algorithms

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- Algorithms like random sampling or exhaustive enumeration that do not at least statistically prefer better solutions have extremely bad performance.
- We challenge this principle.
- Our Frequency Fitness Assignment (FFA) does **not** prefer better solutions ... yet it can improve the performance of existing algorithms in several cases!

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- The table H is initially filled with zeros.
- Before the selection step of the algorithm, $H[f(P_i[j])]$ for all $j \in 1..|P_i|$ is incremented by 1.
- Then, the frequencies $H[f(P_i[j])]$ replace the objective values $f(P_i[j])$ in the actual selection decisions.

FFA: (1+1) EA and (1+1) FEA

- Let's plug FFA into the $(1+1)$ EA and obtain the $(1+1)$ EA with FFA ($(1+1)$ FEA).

procedure $(1+1)$ EA$(f : \mathbb{X} \mapsto \mathbb{R})$
  randomly sample x_c from $\mathbb{X}$; $y_c \leftarrow f(x_c)$;
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- We start with the $(1+1)$ EA.

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FFA: (1+1) EA and (1+1) FEA

- We begin by initializing the frequency table H with all zeros.

procedure (1 + 1) EA($f : \mathbb{X} \mapsto \mathbb{R}$)
  randomly sample x_c from $\mathbb{X}$; $y_c \leftarrow f(x_c)$;
  while $\neg$ terminate **do**
    $x_n \leftarrow \texttt{move}(x_c)$; $y_n \leftarrow f(x_n)$;
    if $\boldsymbol{y_n \leq y_c}$ **then** $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
  return x_c, y_c

procedure (1 + 1) FEA($f : \mathbb{X} \mapsto \mathbb{N}$)
  $H \leftarrow (0, 0, \cdots, 0)$;
  randomly sample x_c from $\mathbb{X}$; $y_c \leftarrow f(x_c)$;

  while $\neg$ terminate **do**
    $x_n \leftarrow \texttt{move}(x_c)$; $y_n \leftarrow f(x_n)$;

    if $\boldsymbol{y_n \leq y_c}$ **then**
      $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;

  return x_c, y_c

FFA: (1+1) EA and (1+1) FEA

- *Before* the selection decision, we increment the frequency values of the objective values of all current solutions.

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  randomly sample x_c from $\mathbb{X}$; $y_c \leftarrow f(x_c)$;

  while $\neg$ terminate **do**
    $x_n \leftarrow \texttt{move}(x_c)$; $y_n \leftarrow f(x_n)$;
    $H[y_c] \leftarrow H[y_c] + 1$; $H[y_n] \leftarrow H[y_n] + 1$;
    if $y_n \leq y_c$ **then**
      $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;

  return x_c, y_c

FFA: (1+1) EA and (1+1) FEA

- Now the frequency values replace the objective values in the selection decisions.

procedure (1 + 1) EA($f : \mathbb{X} \mapsto \mathbb{R}$)
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    $x_{\mathrm{B}} \leftarrow x_c$; $y_{\mathrm{B}} \leftarrow y_c$;
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    if $H[y_n] \leq H[y_c]$ **then**
      $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
      if $y_c < y_\mathrm{B}$ **then** $x_\mathrm{B} \leftarrow x_c$; $y_\mathrm{B} \leftarrow y_c$;
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- . . . which are then the return values of the $(1+1)$ FEA.

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        if $H[y_n] \leq H[y_c]$ **then**
            $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
            if $y_c < y_{\mathrm{B}}$ **then** $x_{\mathrm{B}} \leftarrow x_c$; $y_{\mathrm{B}} \leftarrow y_c$;
    return x_{B}, y_{B}

FFA: What does this do?

- The rating $H[f(x)]$ of a solution x depends only on how often solutions x' with $f(x') = f(x)$ have previously been seen in the optimization process.

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- Instead, solutions with less-frequent objective values are preferred.
- Algorithms using FFA are invariant under all injective transformations of the objective function value.
- They are less likely to get stuck at local optima, which is a problem for, e.g., the $(1+1)$ EA.

Our Results

Three Core Works

snippets of [53, 55, 56]

Frequency Fitness Assignment

Thomas Weise, *Member, IEEE,* Mingxu Wan, Pu Wang, *Member, IEEE,* Ke Tang, *Senior Member, IEEE,* Alexandre Devert, and Xin Yao, *Fellow, IEEE*

***Abstract*—Metaheuristic optimization procedures such as evolutionary algorithms are usually driven by an objective function that rates the quality of a candidate solution. However, it is not clear in practice whether an objective function adequately rewards intermediate solutions on the path to the global optimum and it may exhibit deceptiveness, epistasis, neutrality, ruggedness, and a lack of causality. In this paper, we introduce the frequency fitness H, subject to minimization, which rates how often solutions with the same objective value have been discovered so far. The ideas behind this method are that good solutions are difficult to find and that if an algorithm gets stuck at a local optimum, the frequency of the objective values of the surrounding solutions will increase over time, which will eventually allow it to leave that region again. We substitute a frequency fitness assignment process (FFA) for the objective function into several different optimization algorithms. We conduct a comprehensive set of experiments: the synthesis of algorithms with genetic programming (GP), the solution of MAX-3SAT problems with genetic algorithms, classification with Memetic Genetic Programming, and numerical optimization with a (1+1) Evolution Strategy, to verify the utility of FFA. Given that they have no access to the original objective function at all, it is surprising that for some problems (e.g., the algorithm synthesis task) the FFA-based algorithm variants perform significantly better. However, this cannot be guaranteed for all tested problems. Thus, we also analyze scenarios where algorithms using FFA do not perform better or perform even worse than with the original objective functions.**

***Index Terms*—Combinatorial optimization, diversity, fitness assignment, frequency, genetic programming (GP), numerical optimization.**

I. Introduction

SINGLE-OBJE… VE optimi… ion is … process with the goal of findi… (id… all… i.e. … nal) sol… tions x from within a space $\mathbb{X}$ of possible solutions. An objective function f serves as quality measure guiding the search. Black-box metaheuristic approaches are methods that only require such an objective function and search operations to solve an optimization problem without any further insight into their structure. The most prominent family of these methods are evolutionary algorithms, which have wide applications ranging from engineering, planning and scheduling, numerical optimization, to even program synthesis.

Most metaheuristic optimization methods start with a randomly generated set of candidate solutions. New points in the search space are derived by modifying or combining promising existing solutions. Promising here means having a better objective value than the other points visited so far, maybe combined with some considerations about diversity. The rationale is that in the ideal case, solutions that have better objective values should be closer to the global optimum or, at least, may have even better solutions in their vicinity.

The principle of tending to choose areas of the solution space for sampling where points with better objective values have previously been discovered is one of the most universally applied ideas in black-box optimization. Lehman and Stanley [1] argued that increasing fitness does not always reveal the best path through the search space. Building on their work, we believe that there is at least one other fun… ental principle inherent to non-trivial optimiz… tion problems … be exploited to solve them: good solut… ons (and henc… objective … lues) are hard to find.

If … e … nsi… er optimi… ation as sa… of th… pace … v… uld exp… freq…

Three Core Works

snippets of [53, 55, 56]

226 IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, VOL. 18, NO. 2, APRIL 2014

Frequency Fitness A...

Thomas Weise, *Member, IEEE,* Mingxu Wan, Pu Wang, *Me...*
Alexandre Devert, and Xin Ya...

***Abstract*—Metaheuristic optimization procedures such as evolutionary algorithms are usually driven by an objective function that rates the quality of a candidate solution. However, it is not clear in practice whether an objective function adequately rewards intermediate solutions on the path to the global optimum and it may exhibit deceptiveness, epistasis, neutrality, ruggedness, and a lack of causality. In this paper, we introduce the frequency fitness H, subject to minimization, which rates how often solutions with the same objective value have been discovered so far. The ideas behind this method are that good solutions are difficult to find and that if an algorithm gets stuck at a local optimum, the frequency of the objective values of the surrounding solutions will increase over time, which will eventually allow it to leave that region again. We substitute a frequency fitness assignment process (FFA) for the objective function into several different optimization algorithms. We conduct a comprehensive set of experiments: the synthesis of algorithms with genetic programming (GP), the solution of MAX-3SAT problems with genetic algorithms, classification with Memetic Genetic Programming, and numerical optimization with a (1+1) Evolution Strategy, to verify the utility of FFA. Given that they have no access to the original objective function at all, it is surprising that for some problems (e.g., the algorithm synthesis task) the FFA-based algorithm variants perform significantly better. However, this cannot be guaranteed for all tested problems. Thus, we also analyze scenarios where algorithms using FFA do not perform better or perform even worse than with the original objective functions.**

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I. INTRODUCTION

IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, VOL. 25, NO. 2, APRIL 2021 307

Frequency Fitness Assignment: Making Optimization Algorithms Invariant Under Bijective Transformations of the Objective Function Value

Thomas Weise, *Member, IEEE*, Zhize Wu, Xinlu Li, and Yan Chen

***Abstract*—Under frequency fitness assignment (FFA), the fitness corresponding to an objective value is its encounter frequency in fitness assignment steps and is subject to minimization. FFA renders optimization processes invariant under bijective transformations of the objective function value. On TwoMax, Jump, and Trap functions of dimension s, the classical (1 + 1)-EA with standard mutation at rate 1/s can have expected runtimes exponential in s. In our experiments, a (1 + 1)-FEA, the same algorithm but using FFA, exhibits mean runtimes that seem to scale as $s^2 \ln s$. Since Jump and Trap are bijective transformations of OneMax, it behaves identical on all three. On OneMax, LeadingOnes, and Plateau problems, it seems to be slower than the (1 + 1)-EA by a factor linear in s. The (1 + 1)-FEA performs much better than the (1 + 1)-EA on W-Model and MaxSat instances. We further verify the bijection invariance by applying the Md5 checksum computation as transformation to some of the above problems and yield the same behaviors. Finally, we show that FFA can improve the performance of a memetic algorithm for job shop scheduling.**

which are then the basis for selection. Frequency fitness assignment (FFA) [1], [2] was developed to enable algorithms to escape from local optima. In FFA, the fitness corresponding to an objective value is its encounter frequency so far in fitness assignment steps and is subject to minimization. As we discuss in detail in Section II, FFA turns a static optimization problem into a dynamic one where objective values that are often encountered will receive worse and worse fitness.

In this article, we uncover a so-far unexplored property of FFA: it is invariant under any bijective transformation of the objective function values. This is the strongest invariance known to us and encompasses all order-preserving mappings. Other examples for bijective transformations include the negation, permutation, or even encryption of the objective values. According to [3], invariance extends performance observed ...

Three Core Works

snippets of [53, 55, 56]

Frequency Fitness A

Thomas Weise, *Member, IEEE,* Mingxu Wan, Pu Wang, *Me*
Alexandre Devert, and Xin Ya

Abstract—**Metaheuristic optimization procedures such as evolutionary algorithms are usually driven by an objective function that rates the quality of a candidate solution. However, it is not clear in practice whether an objective function adequately rewards intermediate solutions on the path to the global optimum and it may exhibit deceptiveness, epistasis, neutrality, ruggedness, and a lack of causality. In this paper, we introduce the frequency fitness H, subject to minimization, which rates how often solutions with the same objective value have been discovered so far. The ideas behind this method are that good solutions are difficult to find and that if an algorithm gets stuck at a local optimum, the frequency of the objective values of the surrounding solutions will increase over time, which will eventually allow it to leave that region again. We substitute a frequency fitness assignment process (FFA) for the objective function into several different optimization algorithms. We conduct a comprehensive set of experiments: the synthesis of algorithms with genetic programming (GP), the solution of MAX-3SAT problems with genetic algorithms, classification with Memetic Genetic Programming, and numerical optimization with a (1+1) Evolution Strategy, to verify the utility of FFA. Given that they have no access to the original objective function at all, it is surprising that for some problems (e.g., the algorithm synthesis task) the FFA-based algorithm variants perform significantly better. However, this cannot be guaranteed for all tested problems. Thus, we also analyze scenarios where algorithms using FFA do not perform better or perform even worse than with the original objective functions.**

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Frequency Fitness Assignment: Optimization Without Bias for Good Solutions Can Be Efficient

Thomas Weise, Zhize Wu, Xinlu Li, Yan Chen, and Jörg Lässig

Abstract—**A fitness assignment process transforms the features (such as the objective value) of a candidate solution to a scalar fitness, which then is the basis for selection. Under frequency fitness assignment (FFA), the fitness corresponding to an objective value is its encounter frequency in selection steps and is subject to minimization. FFA creates algorithms that are not biased toward better solutions and are invariant under all injective transformations of the objective function value. We investigate the impact of FFA on the performance of two theory inspired, state-of-the-art evolutionary algorithms, the Greedy (2 + 1) GA and the self-adjusting $(1+(\lambda,\lambda))$ GA. FFA improves their performance significantly on some problems that are hard for them. In our experiments, one FFA-based algorithm exhibited mean runtimes that appear to be polynomial on the theory-based benchmark problems in our study, including traps, jumps, and plateaus. We propose two hybrid approaches that use both direct and FFA-based optimization and find that they perform well. All FFA-based algorithms also perform better on satisfiability problems than any of the pure algorithm variants.**

Index Terms—**Evolutionary algorithm (EA), FEA, frequency fitness assignment (FFA), Ising problems, jump problems, linear harmonic functions, MaxSat problem, *N*-queens problem, OneMax, Plateau problems, satisfiability, Trap function, TwoMax, *W*-model benchmark.**

fitness assignment (FFA), the fitness of a candidate solution is the absolute encounter frequency of its objective value so far during the optimization process [1]. Being subject to minimization, FFA drives the search away from already-discovered objective values and toward solutions with new qualities.

FFA breaks with one of the most fundamental concepts of heuristic optimization: FFA-based algorithms are not biased toward better solutions [2], i.e., they do not prefer better solutions over worse ones. They also are invariant under all injective transformations of the objective function value, which is the strongest invariance property of any nontrivial single-objective optimization algorithm [3].[1] Only random sampling, random walks, and exhaustive enumeration have similar properties and neither of them is considered to be an efficient optimization method.

One would expect that this comes at a significant performance penalty. Yet, FFA performed well in Genetic Programming tasks with their often rugged, deceptive, and highly epistatic landscapes [1], [4] and on a benchmark problem simulating such landscapes [5]. While the $(1+1)$ EA has exp… ntial expected runtime on problems, such as Jum… TwoMax … d Trap, the $(1+1)$ FEA, th… same al… using FFA … hibits mean runtimes that …

INTRODUCTION

ITNESS … many evo…

Discrete Benchmark Functions

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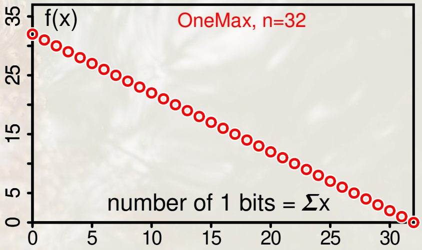

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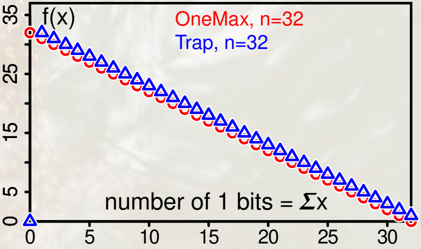

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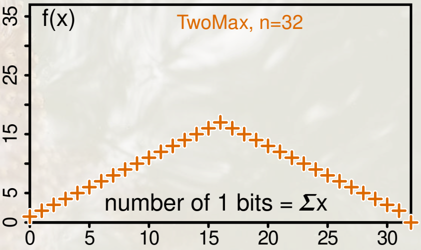

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- The $(1+1)$ FEA solves all three problems in polynomial time!

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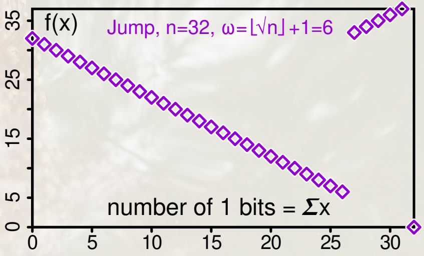

- The $(1+1)$ FEA solves all three problems in polynomial time as well as the Jump problem, where the $(1+1)$ EA also needs exponential runtime!

Maximum Satisfiability Problem

- FFA on the Maximum Satisfiability (MaxSAT) Problem, one of the most famous discrete optimization problems [53, 55, 56].

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> From Table I, we can see that **the highest number of failed runs at scale $s = 250$ of *any* algorithm using FFA is lower than the lowest number of failed runs of *any* pure algorithm at $s = 50$.** From Table II, we find that no FFA-based algorithm has a higher ERT at scale $s = 250$ than its pure variant on $s = 50$. On the scales $s \leq 75$, the FFA-based algorithms have a mean runtime which is between three and four orders of magnitude smaller that the ERT of the pure algorithms.

- Snippet of page 10 of [56] (copyright IEEE).
- Several different EAs with and without FFA

Traveling Salesperson Problem

- FFA on the Traveling Salesperson Problem (TSP): [30–32].

Traveling Salesperson Problem

- FFA on the Traveling Salesperson Problem (TSP): [30–32].
- Snippet of page 12 of [30] (copyright Springer).
- $(1+1)$ EA, $(1+1)$ FEA, Simulated Annealing (SA) w/o FFA, hybrids

In this work, we explored both the EA and SA on 56 symmetric instances from the benchmark set TSPLib (Reinelt 1991, 1995). The EA is unsuitable for this problem, but SA can find the optimum on many small and mid-sized instances. We then plug FFA into both algorithms and obtain the FEA and the FSA, respectively, which both exhibit very similar performance. **The FEA solves 27 of the instances in all of its runs, which SA can only achieve for 19 instances.** Plugging FFA into either the EA or SA thus substantially improved the number of runs in which the algorithms can find the optimum, however, both FFA-based variants suffer when the number of unique objective values is high.

• • •

Both types of **hybridization** methods significantly improve the average result quality compared to both the objective-guided and FFA-based FEA variants. **The SAFEA$_A$ discovers the optimal solutions in more runs than any other algorithm setup in our study.** It also finds the optimum most often in most instances and delivers the best approximation quality on most instances, compared to the other algorithms.

Quadratic Assignment Problem

- FFA on the Quadratic Assignment Problem (QAP): [9, 43].

Quadratic Assignment Problem

- FFA on the Quadratic Assignment Problem (QAP): [9, 43].
- Snippet of page 7 of [9] (copyright SciTePress).
- RLS w/o FFA

We find that the **FFA-based** randomized local search **FRLS does not just find better solutions than the objective-guided RLS algorithm on the vast majority of the QAPLIB instances**, it also keeps improving its current best solution for the complete computational budget of 10^8 FEs that we assigned to the runs. With this budget, **it can discover the optimal solutions of over 58% of the QAPLIB instances**. Had we assigned a larger budget – (Liang et al., 2022; Liang et al., 2024; Weise et al., 2021b; Weise et al., 2023) use 10^{10} FEs – we would likely have seen even more instances solved.

We furthermore confirm the remarkable ability of FFA to discover very diverse solutions (at least from the perspective of the objective function). It is known that on the QAP, many solutions tend to have the same objective values (Tayarani-N. and Prügel-Bennett, 2015). Yet, on some of the instances, more than half of the objective values discovered by FRLS were unique.

Job Shop Scheduling Problem

- FFA on the Job Shop Scheduling Problem (JSSP) Problem: [11, 50, 55].

Job Shop Scheduling Problem

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The **end result quality delivered by (1+1)-FEA is better in average on the abz*, ft*, la*, orb*, and yn4* instance sets, both in terms of *best* and *mean*.** On swv*, the average for *mean* is better for (1+1)-FEA, while (1+1)-EA has a slight lead in *best*. The (1+1)-EA performs better on the dmu* and ta* instances. Since these two sets are larger (holding 160 out of the 242 instances), the (1+1)-EA comes out ahead in the overall averages, but with no more than a 1.5% advantage.

- Snippet of page 10 of [50] (copyright ACM).

Algorithm Synthesis and Genetic Programming

- FFA for algorithm synthesis and Genetic Programming (GP): [52, 53].

Algorithm Synthesis and Genetic Programming

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> Fourth, we were able to confirm with great significance that **FFA** has a tremendous positive impact on the performance of GP, as it **can increase the success rate by 40%.**

- Snippet of page 7 of [52] (copyright IEEE).

Future Works

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- Apply FFA to other discrete and combinatorial problems.

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    1. Where can FFA improve the solution quality?

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Future Works

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  1. Where can FFA improve the solution quality?
  2. Where can it not do that?
  3. Why?
- Plug FFA into more optimization algorithms.
- Combine "traditional" and FFA-based optimization, i.e., create hybrid algorithms, to reap the best of both worlds.

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  2. on the Traveling Tournament Problem (TTP), we also got surprisingly good results with RLS[58] and SA.

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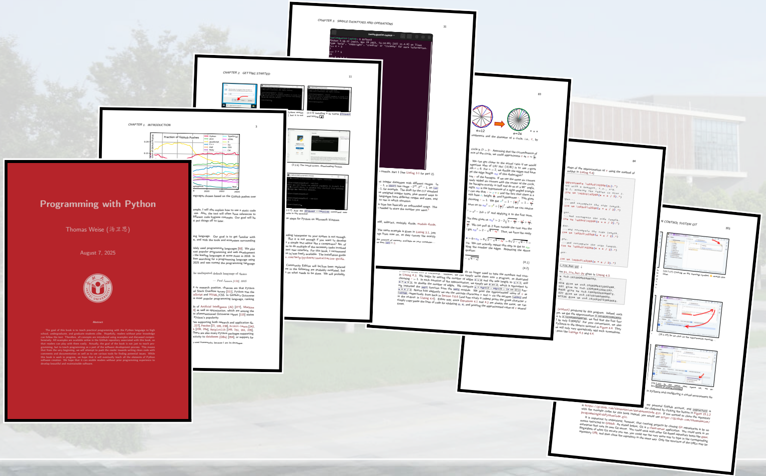
Programming with Python

We have a freely available course book on *Programming with Python* at https://thomasweise.github.io/programmingWithPython, with focus on practical software development using the Python ecosystem of tools[48].

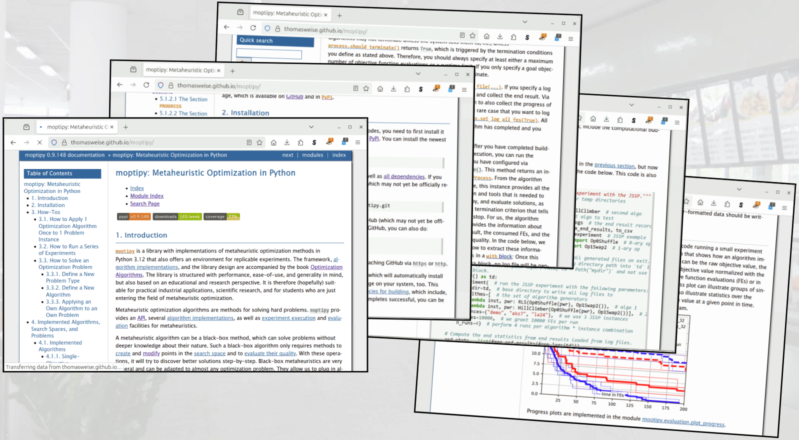


Databases

We have a freely available course book on *Databases* at https://thomasweise.github.io/databases, with actual practical examples using a real database management system (DBMS)[46].

Metaheuristic Optimization in Python: moptipy

We offer moptipy[54] a mature open source Python package for metaheuristic optimization, which implements several algorithms, can run self-documenting experiments in parallel and in a distributed fashion, and offers statistical evaluation tools.



谢谢您们！

Thank you!

Vielen Dank!

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Glossary I

$(1+1)$ EA The $(1+1)$ EA is a local search algorithm that retains the best solution x_c discovered so far during the search[6,12]. In each step, it applies a unary search operator to this best-so-far solution x_c and derives a new solution x_n. If the new solution x_n is *better or equally good* when compared with x_c, i.e., not worse, then it replaces it, i.e., is stored as the new x_c. If the search space are bit strings of length n, then the $(1+1)$ EA uses a unary search operator that flips each bit independently with probability m/n, where usually $m=1$. This operator is the main difference to randomized local search (RLS). The $(1+1)$ EA is a special case of the $(\mu+\lambda)$ evolutionary algorithm ($(\mu+\lambda)$ EA) where $\mu=\lambda=1$.

$(1+1)$ FEA The $(1+1)$ EA with FFA plugged in.

EA An *evolutionary algorithm* is a metaheuristic optimization method that maintains a population of candidate solutions, which undergo selection (where better solutions are chosen with higher probability) and reproduction (where mutation and recombination create a new candidate solution from one or two existing ones, repectively)[3,47].

$(\mu+\lambda)$ EA The $(\mu+\lambda)$ EA is an evolutionary algorithm (EA) where, in each generation, λ offspring solutions are generated from the current population of μ parent solutions. The offspring and parent populations are merged, yielding $\mu+\lambda$ solutions, from which then the best μ solutions are ratained to form the parent population of the next generation. If the search space is the bit strings of length n, then this algorithm usually applies a mutation operator flipping each bit independently with probability $1/n$.

AI Artificial Intelligence, see, e.g.,[41]

DB A *database* is an organized collection of structured information or data, typically stored electronically in a computer system. Databases are discussed in our book *Databases*[46].

DBMS A *database management system* is the software layer located between the user or application and the database (DB). The DBMS allows the user/application to create, read, write, update, delete, and otherwise manipulate the data in the DB[59].

FFA *Frequency Fitness Assignment* is a algorithm plugin for optimization methods applied to discrete or combinatorial problems with not-too-many different possible objective values. It replaces the objective values in all comparisons with their absolute encounter frequency so far during the search. FFA has successfully been applied to the QAP[9].

Glossary II

GP Genetic Programming[25,39,47,51] is the application of metaheuristic optimization, usually in form of an EAs, to a search space comprised of tree datastructures. These tree datastructures often represent programs or mathematical expressions.

JSSP The *Job Shop Scheduling Problem*[4,27] is one of the most prominent and well-studied scheduling tasks. In a JSSP instance, there are k machines and m jobs. Each job must be processed once by each machine in a job-specific sequence and has a job-specific processing time on each machine. The goal is to find an assignment of jobs to machines that results in an overall shortest makespan, i.e., the schedule which can complete all the jobs in the shortest time. The JSSP is $\mathcal{NP}$-complete[8,27].

MaxSAT The goal of satisfiability problems is to find an assignment for n Boolean variables that make a given Boolean formula $F : \{0,1\}^n \mapsto \{0,1\}$ become true. In the *Maximum Satisfiability (MaxSAT)* problem[17], F is given in conjunctive normal form, i.e., the variables appear either directly or negated in m "or" clauses, which are all combined into one "and." The objective function $f(x)$, subject to minimization, computes the number of clauses which are false under the variable setting x. If $f(x) = 0$, then all clauses of F are true, which solves the problem. The MaxSAT problem is $\mathcal{NP}$-complete[10].

moptipy is the *Metaheuristic Optimization in Python* library[54]. It has been used in several different research works, including[9,30–32,43,58,60,61]. Learn more at https://thomasweise.github.io/moptipy and https://thomasweise.github.io/moptipyapps.

Python The Python programming language[18,29,34,48], i.e., what you will learn about in our book[48]. Learn more at https://python.org.

QAP The *Quadratic Assignment Problem* is an optimization problem where the goal is to assign a set of n facilities to a set of n locations[5,9,24,33]. Such an assignment can be represented as a permutation x of the first n natural numbers, where x_i specifies the location where facility i should be placed. For each QAP, a distance matrix D is given, where D_{pq} specifies the distance from location p to location q, as well as a flow matrix F, where F_{ij} is the amount of material flowing from facility i to facility j. The objective function f then rates a permutation x as $f(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} D_{x_i x_j} F_{ij}$. The QAP is $\mathcal{NP}$-complete[42].

Glossary III

RLS Randomized local search retains the best solution x_c discovered so far during the search and, in each step, it applies a unary search operator to this best-so-far solution x_c and derives a new solution x_n. If the new solution x_n is *better or equally good* when compared with x_c, i.e., not worse, then it replaces it, i.e., is stored as the new x_c. If the search space are bit strings of length n, then RLS uses a unary search operator that flips exactly one bit. This operator is the main difference to $(1+1)$ EA.

SA Simulated Annealing is a local search that sometimes accepts a worse solution[7,20,21,38]. The probability to do so decreases over time and with the difference in objective values, i.e.,is the lower the worse the new solution is.

TSP In an instance of the *Traveling Salesperson Problem*, also known as *Traveling Salesman Problem*, a set of n cities or locations as well as the distances between them are defined[1,16,28,30,49,57]. The goal is to find the shortest round-trip tour that starts at one city, visits all the other cities one time each, and returns to the origin. The TSP is one of the most well-known $\mathcal{NP}$-hard combinatorial optimization problems[16].

TTP The *Traveling Tournament Problem* (TTP) is the combinatorial optimization problem of both efficiently and fairly organizing a tournament of n teams that play against each other in a pairwise fashion[13,58]. The efficient part boils down to arranging the games such that the total travel length is short, which is somewhat similar to the classical TSP. Initially, each team is at its home location. On each day, a team needs to travel if its scheduled game is not at its present location. On the last day, each team may need to travel back home unless their last game is a home game. The total travel length sums up the lengths of all travels over all teams. The fair part is represented in several constraints, such as doubleRoundRobin, compactness, maxStreak, and noRepeat. The TTP is $\mathcal{NP}$-hard[45].

$i..j$ with $i, j \in \mathbb{Z}$ and $i \leq j$ is the set that contains all integer numbers in the inclusive range from i to j. For example, 5..9 is equivalent to $\{5, 6, 7, 8, 9\}$

$\mathbb{N}_0$ the set of the natural numbers *including* 0, i.e., 0, 1, 2, 3, and so on. It holds that $\mathbb{N}_0 \subset \mathbb{Z}$.

$\mathcal{NP}$ is the class of computational problems that can be solved in polynomial time by a non-deterministic machine and can be verified in polynomial time by a deterministic machine (such as a normal computer)[15].

Glossary IV

$\mathcal{NP}$-complete A decision problem is $\mathcal{NP}$-complete if it is in $\mathcal{NP}$ and all problems in $\mathcal{NP}$ are reducible to it in polynomial time[15,40]. A problem is $\mathcal{NP}$-complete if it is $\mathcal{NP}$-hard and if it is in $\mathcal{NP}$.

$\mathcal{NP}$-hard Algorithms that guarantee to find the correct solutions of $\mathcal{NP}$-hard problems[8,10,27] need a runtime that is exponential in the problem scale in the worst case. A problem is $\mathcal{NP}$-hard if all problems in $\mathcal{NP}$ are reducible to it in polynomial time[15].

$\mathbf{\Omega}(g(x))$ If $f(x) = \mathbf{\Omega}(g(x))$, then there exist positive numbers $x_0 \in \mathbb{R}^+$ and $c \in \mathbb{R}^+$ such that $f(x) \geq c * g(x) \geq 0 \forall x \geq x_0$[22,23]. In other words, $\mathbf{\Omega}(g(x))$ describes a lower bound for function growth.

$\mathcal{O}(g(x))$ If $f(x) = \mathcal{O}(g(x))$, then there exist positive numbers $x_0 \in \mathbb{R}^+$ and $c \in \mathbb{R}^+$ such that $0 \leq f(x) \leq c * g(x) \forall x \geq x_0$[2,22,23,26]. In other words, $\mathcal{O}(g(x))$ describes an upper bound for function growth.

$\mathbf{\Theta}(g(x))$ If $f(x) = \mathbf{\Theta}(g(x))$, then $f(x) = \mathcal{O}(g(x))$ and $f(x) = \mathbf{\Omega}(g(x))$[22,23]. In other words, $\mathbf{\Theta}(g(x))$ describes an exact order of function growth.

$\mathbb{R}$ the set of the real numbers.

$\mathbb{R}^+$ the set of the positive real numbers, i.e., $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$.

$\mathbb{Z}$ the set of the integers numbers including positive and negative numbers and 0, i.e., ..., -3, -2, -1, 0, 1, 2, 3, ..., and so on. It holds that $\mathbb{Z} \subset \mathbb{R}$.